Signal Processing and Data Fusion 4/10/2021

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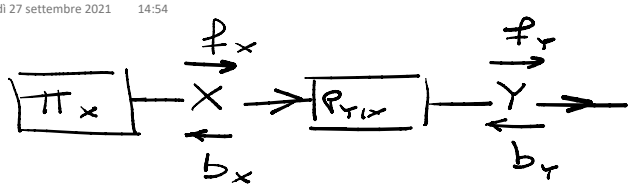
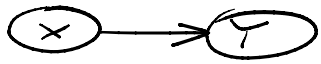


$$P_{XY}(x,y) = P_{Y|X}(y|x)\,\pi_X(x)$$



$P_{Y|X}(y|x)$, $\pi_X(x)$

$$P_Y(y) = \sum_{x\in\mathcal{X}} P_{XY}(x,y) =$$

$$= \sum_{x\in\mathcal{X}} P_{Y|X}(y|x)\,\pi_X(x)$$

$$f_X(x) = \pi_X(x)$$

$$f_Y(y) = \sum_{x\in\mathcal{X}} P_{Y|X}(y|x)\cdot \underbrace{f_X(x)}_{\pi_X(x)} = \sum_{x\in\mathcal{X}} P_{Y|X}(y|x)\cdot \pi_X(x)$$

$$P_Y(y) = \underbrace{f_Y(y)}\cdot\underbrace{b_Y(y)} = \underbrace{f_Y(y)\cdot U(y)} = f_Y(y)$$

$$\underline{f}_Y = \begin{bmatrix} 1/N_Y \\ \vdots \\ 1/N_Y \end{bmatrix}_{[N_Y\times 1]}$$

$$\underline{f}_X = \begin{bmatrix} f_X(x^1) \\ f_X(x^2) \\ \vdots \\ f_X(x^{N_X}) \end{bmatrix}_{[N_X\times 1]}$$

$$\underline{\underline{P}}_{Y|X} = \begin{bmatrix} P_{Y|X}(y^1|x^1) & P_{Y|X}(y^2|x^1) & \cdots & P_{Y|X}(y^{N_Y}|x^1) \\ & \vdots & & \\ P_{Y|X}(y^1|x^{N_X}) & P_{Y|X}(y^2|x^{N_X}) & \cdots & P_{Y|X}(y^{N_Y}|x^{N_X}) \end{bmatrix}$$

$$\left[P_{Y|X}(y^1|x^1) \;\; P_{Y|X}(y^2|x^1) \;\; \cdots \;\; P_{Y|X}(y^{N_Y}|x^1) \right]$$

$$[N_X \times N_Y]$$

$$f_Y(y^i) = \sum_{x \in \mathcal{X}} P_{Y|X}(y^i|x)\,\pi_X(x) =$$

$$= P_{Y|X}(y^i|x^1)\pi_X(x^1) + \cdots + P_{Y|X}(y^i|x^{N_X})\pi(x^{N_X})$$

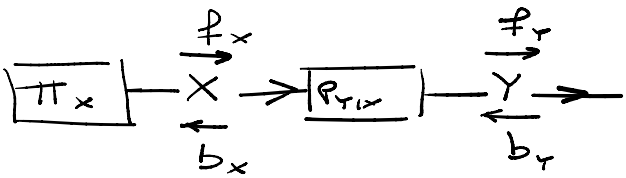
$$= P_{Y|X}(y^i|x^1) f_X(x^1) + \cdots + P_{Y|X}(y^i|x^{N_X}) f_X(x^{N_X})$$

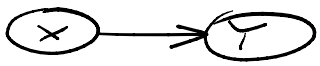
$$\Rightarrow \underline{\underline{P}}_{Y|X}[:, i] \cdot \underline{f}(x)$$

$$\underline{f}_Y = \underline{\underline{P}}^T_{Y|X}\, \underline{f}_X$$

$$\longleftrightarrow$$

$$P_{XY}(x,y) = P_{Y|X}(y|x)\,\pi_X(x)$$





Suppongo di conoscere π_X, $P_{Y|X}$ $\Rightarrow$ voglio calcolare $P_X(x)$

$$P_X(x) = \sum_{y \in \mathcal{Y}} P_{Y|X}(y|x) \cdot \pi_X(x) = \pi_X(x) \underbrace{\sum_{y \in \mathcal{Y}} P_{Y|X}(y|x)}_{1} = \pi_X(x)$$

$$P_X(x) \propto \underbrace{f_X(x)}_{\pi_X(x)} \cdot \underbrace{b_X(x)}_{U(x)} \propto \pi_X(x)$$

$$b_X(x) = \underbrace{\sum_{y \in \mathcal{Y}} P_{Y|X}(y|x) \cdot \underbrace{b_Y(y)}_{U(y)}}_{} = U(x)$$

$$\underline{b}_X = \underset{[N_X \times 1]}{\begin{bmatrix} b_X(x^1) \\ b_X(x^2) \\ \vdots \\ b_X(x^{N_X}) \end{bmatrix}} = \underset{[N_X \times N_Y]}{\underline{P}_{Y|X}} \cdot \underset{[N_Y \times 1]}{\underline{b}_Y}$$

$$= \begin{bmatrix} P_{Y|X}(y^1|x^1) & \cdots & P_{Y|X}(y^{N_Y}|x^1) \\ & \vdots & \\ P_{Y|X}(y^1|x^{N_X}) & \cdots & P_{Y|X}(y^{N_Y}|x^{N_X}) \end{bmatrix} \cdot \begin{bmatrix} 1/N_Y \\ 1/N_Y \\ \vdots \\ 1/N_Y \end{bmatrix} =$$

$$= \begin{bmatrix} 1/N_Y \, P_{Y|X}(y^1|x^1) + \cdots + 1/N_Y \, P_{Y|X}(y^{N_Y}|x^1) \\ \vdots \\ 1/N_Y \, P_{Y|X}(y^1|x^{N_X}) + \cdots + 1/N_Y \, P_{Y|X}(y^{N_Y}|x^{N_X}) \end{bmatrix} =$$

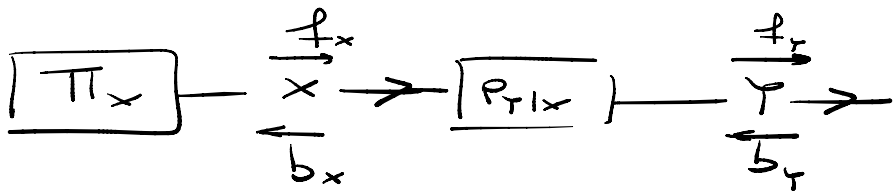
$$= \begin{bmatrix} 1/N_Y \left(\sum_{y \in \mathcal{Y}} P_{Y|X}(y|x^1) \right) \\ \vdots \\ 1/N_Y \left(\sum_{y \in \mathcal{Y}} P_{Y|X}(y|x^{N_X}) \right) \end{bmatrix} = \underset{[N_X \times 1]}{\begin{bmatrix} 1/N_Y \\ \vdots \\ 1/N_Y \end{bmatrix}} = U(x)$$

$$P_X(x) \propto f_X(x) \, b_X(x)$$

$$= \frac{f_X(x) \, b_X(x)}{}$$

$$= \frac{f_X(x)\, b_X(x)}{\sum_{\xi \in \mathcal{X}} f_X(\xi)\, b_X(\xi)}$$

$$\Longleftrightarrow$$



π_X, $P_{Y|X}$, $Y = \bar{y}$

$$P_X(x \mid Y=\bar{y}) = \frac{P_{XY}(x, Y=\bar{y})}{P_Y(Y=\bar{y})} = \frac{P_{Y|X}(Y=\bar{y} \mid x)\, \pi_X(x)}{P_Y(Y=\bar{y})}$$

$$P_X(x \mid Y=\bar{y}) \propto \underbrace{\pi_X(x)}_{\text{PRIOR}} \cdot \underbrace{P_{Y|X}(Y=\bar{y} \mid x)}_{\text{EVIDENCE}}$$

— o —

$Y = \bar{y}$ $\quad b_Y(y) = \underline{\delta}(y - \bar{y})$ $\quad$ 1 se $y = \bar{y}$, 0 altrove

$$\underline{b}_Y = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} \rightarrow \bar{y}$$

$$b_X = \sum_{y \in \mathcal{Y}} P_{Y|X}(y \mid x)\, b_Y(y) = \underbrace{\sum_{y \in \mathcal{Y}} P_{Y|X}(y \mid x)\, \delta(y - \bar{y})}$$

$$= P_{Y|X}(\bar{y} \mid x)$$

$$p_X(x) \propto f_X(x)\, b_X(x) = \pi_X(x)\, P_{Y|X}(\bar{y} \mid x)$$

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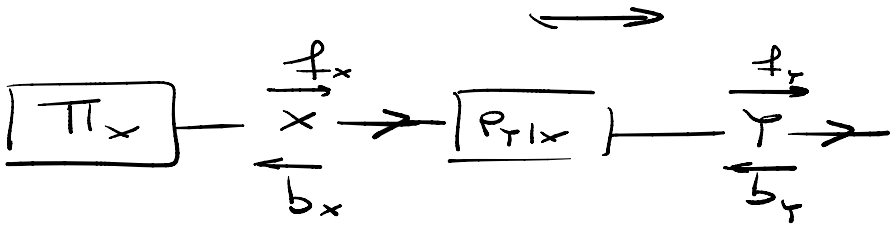
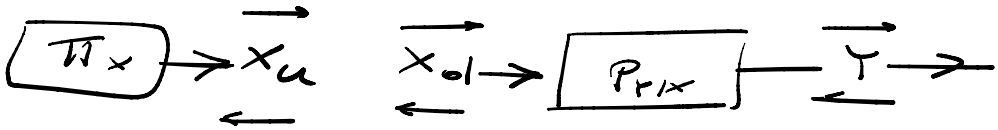
$$\Longrightarrow$$



$\underline{X=\bar{x}}$ $\qquad \underbrace{P_Y(y \mid X=\bar{x})}$ $\qquad b_Y = U(y)$



$$f_{X_d}(x) = \delta(x-\bar{x})$$

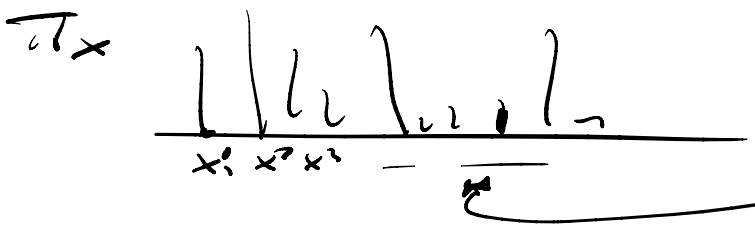
$$\underline{\underline{f_Y(y)}} = \sum_{x\in\mathcal{X}} P_{Y|X}(y|x)\cdot f_{X_d}(x) =$$

$$= \sum_{x\in\mathcal{X}} P_{Y|X}(y|x)\,\delta(x-\bar{x}) = \underline{\underline{P_Y(y|X=\bar{x})}}$$

$$P_Y(y|X=\bar{x}) \propto \underbrace{f_Y(y)}\cdot\underbrace{b_Y(y)} = P_Y(y|X=\bar{x})\cdot U(y)$$

$$P_X(x|X=\bar{x}) \propto f_{X_u}\, b_{X_u} = \Pi_X(x)\cdot\delta(x-\bar{x}) =$$

$$= \Pi_X(\bar{x})$$



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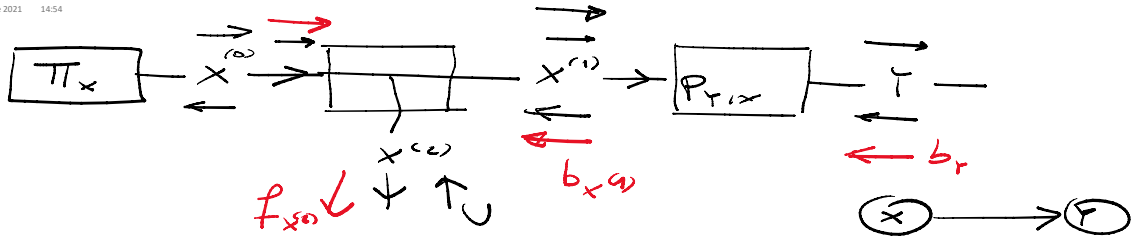

$$b_{x^{(0)}}(x) = U(x) \qquad b_Y(y) = U(y)$$

$$f_{x^{(0)}}(x) = \pi_x(x)$$

$$f_{x^{(1)}}(x) = \underbrace{f_{x^{(0)}}(x)}_{\pi_x(x)} \cdot \underbrace{b_{x^{(2)}}(x)}_{U(x)} = \pi_x(x)$$

$$P_Y(y) \propto f_Y(y)\, \underbrace{b_Y(y)} = f_Y(y) \cdot U(y) = f_Y(y)$$

$$b_{x^{(1)}}(x) \propto \sum_{y \in \mathcal{Y}} P_{Y|X}(y|x)\, \underbrace{b_Y(y)}_{U(y)} = U(x)$$

$$f_{x^{(2)}}(x) \propto f_{x^{(1)}}(x) \cdot b_{x^{(1)}}(x) = U(x)\, \pi_x(x) \propto \pi_x(x)$$

$$P_{x^{(2)}}(x) \propto \underbrace{f_{x^{(2)}}(x)}\, \underbrace{b_{x^{(2)}}(x)} = \pi_x(x)$$

$$b_{x^{(0)}}(x) \propto f_{x^{(1)}}(x)\, b_{x^{(1)}}(x) = U(x) \cdot U(x) = U(x)$$

$$P_{x^{(0)}} = \underbrace{f_{x^{(0)}}(x)}_{\pi_x(x)}\, \underbrace{b_{x^{(0)}}(x)}_{U(x)} = \pi_x(x)$$

$$P_{x^{(1)}}(x) = P_{x^{(1)}}(x) = P_{x^{(0)}}(x) = \pi_x(x)$$

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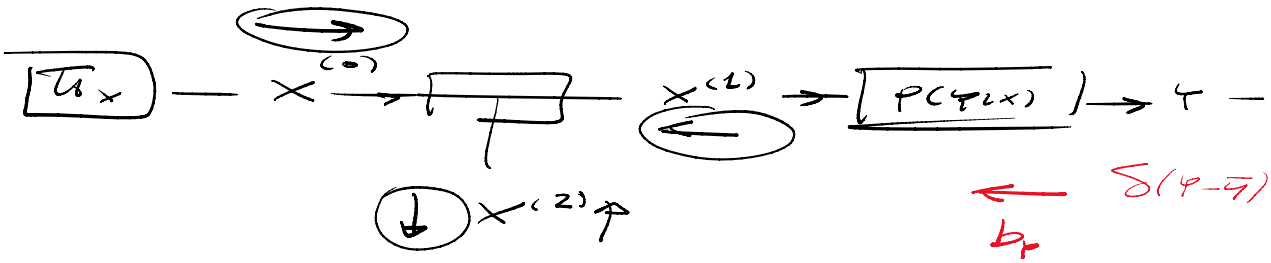
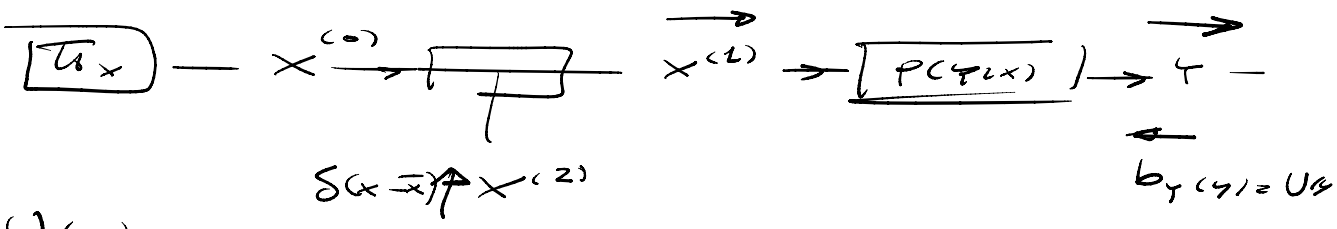


$$b_Y(y) = \delta(y - \bar{y}) \qquad b_{x^{(1)}}(x) = U(x)$$

$b_Y(y) = \delta(y-\bar{y}) \quad b_X(x) = U(x)$

$$f_{X^{(2)}} \propto f_{X^{(0)}} \cdot b_{X^{(1)}} = \underbrace{\sum_{y \in \mathcal{Y}} P_{Y|X}(y|x)\, \overbrace{b_Y(y)}^{\delta(y-\bar{y})}}_{P_{Y|X}(\bar{y}|x)} \cdot \underbrace{f_{X^{(0)}}}_{\pi_X(x)}$$

$$P_{X^{(2)}} = f_{X^{(2)}} \cdot \underbrace{b_{X^{(2)}}}_{U(x)} = P_{Y|X}(\bar{y}|x)\,\pi_X(x)$$



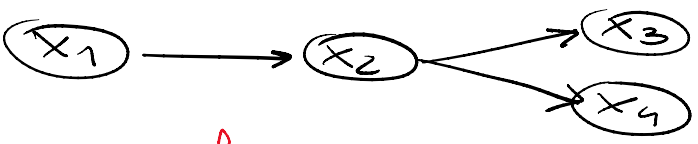
$b_Y(y) = U(y)$

$b_{X^{(2)}} = \delta(x-\bar{x})$

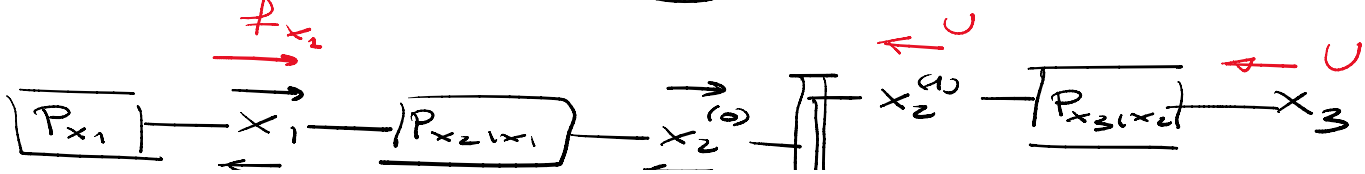
$$f_Y(y) \propto \sum_{x \in \mathcal{X}} P_{Y|X}(y|x)\, \underbrace{f_{X^{(1)}}} \propto \sum_{x \in \mathcal{X}} P_{Y|X}(y|x)\, f_{X^{(0)}}\, b_{X^{(2)}}$$

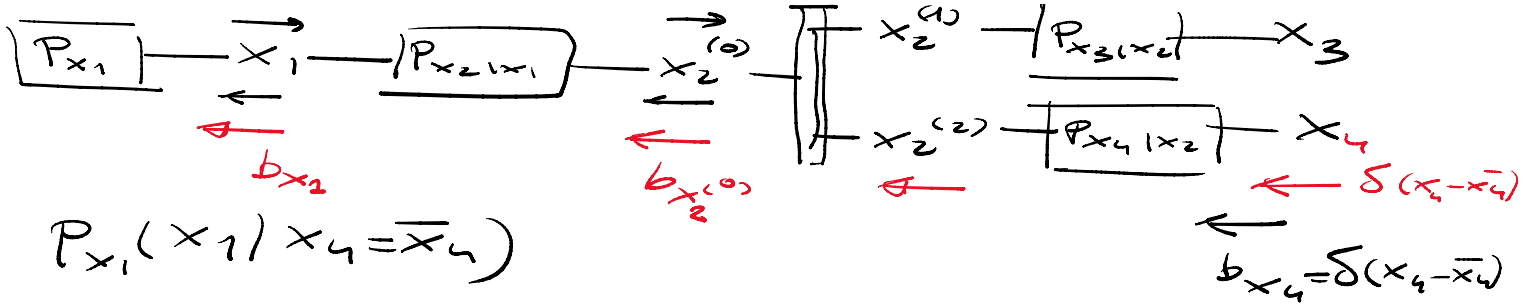
$$= \sum_{x \in \mathcal{X}} P_{Y|X}(y|x)\, \underbrace{\pi_X(x)\,\delta(x-\bar{x})} =$$

$$= P_{Y|X}(y|\bar{x})\, \pi_X(\bar{x})$$



$$P_{X_1X_2X_3X_4} = P_{X_1} P_{X_2|X_1} P_{X_3|X_2} P_{X_4|X_2}$$





$$P_{x_1}(x_1 / x_4=\bar{x}_4)$$

$$P(x_1 | x_4=\bar{x}_4) = f_{x_1}(x)\, b_{x_1}(x) = p(x_1) \cdot$$

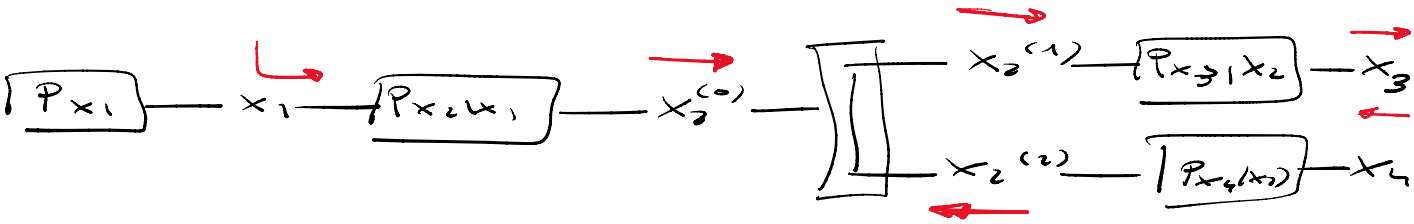
$$\cdot \sum_{x_2 \in \mathcal{X}_2} P(x_2|x_1) b_{x_2^{(0)}} = p(x_1) \cdot \sum_{x_2} P(x_2|x_1) \underbrace{b_{x_2^{(1)}}}_{U} \underbrace{b_{x_2^{(2)}}}$$

$$= p(x_1) \sum_{x_2} P(x_2|x_1) \underbrace{\sum_{x_4} P(x_4|x_2)\, \delta(x_4-\bar{x}_4)}$$

$$= p(x_1) \sum_{x_2} P(x_2|x_1) \cdot P(\bar{x}_4 / x_2)$$

$$P(x_1 | x_4=\bar{x}_4) \propto \sum_{x_2 x_3} \underline{P(x_1 x_2 x_3 \bar{x}_4)} = \ldots \text{ stesso}$$

$$P(x_3 | \underline{x_1=\bar{x}_1})$$



$$P_{x_3} = f_{x_3} b_{x_3} \qquad f_{x_3} \propto \sum_{x_2} P(x_3|x_2)\, f_{x_2^{(1)}}(x_2)$$

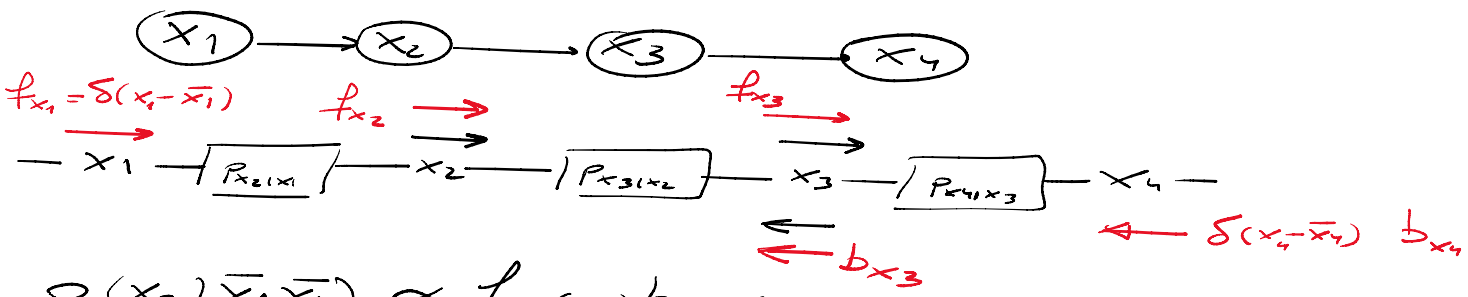
$$= \sum_{x_2} P(x_3|x_2)\, f_{x_2^{(0)}} b_{x_2^{(2)}} = \sum_{x_2} P(x_3|x_2) \Big(\sum_{x_1} P(x_2|x_1)\, \delta(x_1-\bar{x}_1) \Big)$$

$$\underbrace{\Big(\sum_{x_4} P(x_4|x_2)\, \underbrace{b_{x_4}}_{U} \Big)}_{U(x_2)} = \sum_{x_2} P(x_3|x_2)\, P(x_2|\bar{x}_1) \cdot U(x_2)$$

$$= \sum_{x_2} p(x_3|x_2)\, P(x_2|\bar{x}_1)$$

$$p(x_3|x_1=\bar{x}_1) \propto \sum_{x_2 x_4} \underbrace{p(\bar{x}_1 x_2 x_3 x_4)}$$

$$= \sum_{x_2 x_4} p(x_2|\bar{x}_1)\, p(\bar{x}_1)\, p(x_3|x_2)\, p(x_4|x_2)$$



$$P_{x_3}(x_3|\bar{x}_1 \bar{x}_4) \propto f_{x_3}(x_3)\, b_{x_3}(x_3)$$

$$f_{x_3}(x_3) = \sum_{x_2 \in \mathcal{X}_2} P_{x_3|x_2}(x_3|x_2)\, f_{x_2}(x_2) =$$

$$= \sum_{x_2 \in \mathcal{X}_2} P_{x_3|x_2}(x_3|x_2) \underbrace{\sum_{x_1 \in \mathcal{X}_1} P_{x_2|x_1}(x_2|x_1)\, \overset{\delta(x_1-\bar{x}_1)}{f_{x_1}} =}$$

$$= \sum_{x_2 \in \mathcal{X}_2} P_{x_3|x_2}(x_3|x_2)\, P_{x_2|x_1}(x_2|\bar{x}_1)$$

$$b_{x_3}(x_3) = \underbrace{\sum_{x_4 \in \mathcal{X}_4} P_{x_4|x_3}(x_4|x_3)\, \underbrace{b_{x_4}(x_4)}_{\delta(x_4-\bar{x}_4)} =}$$

$$= P_{x_4|x_3}(\bar{x}_4|x_3)$$

$$P_{x_3}(x_3) = f_{x_3}(x_3)\, b_{x_3}(x_3) = \sum_{x_2 \in \mathcal{X}_2} P_{x_3|x_2}(x_3|x_2)\, P_{x_2|x_1}(x_2|\bar{x}_1)$$

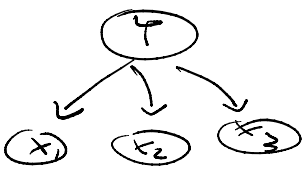
$$P_{x_4|x_3}(\bar{x}_4|x_3)$$

$$\underset{x_3}{P}(x_3|\bar{x}_1\,\bar{x}_4) = \sum_{x_2\in\mathcal{X}_2} \underline{P_{X_4|X_3}(\bar{x}_4|x_3)}\; P_{X_3|X_2}(x_3|x_2)\; \underline{P_{X_2|X_1}(x_2|\bar{x}_1)}$$

$$= \underbrace{\sum_{x_2\in\mathcal{X}_2} \underbrace{\sum_{x_1\in\mathcal{X}_1} P_{X_2|X_1}(x_2|x_1)\,\underbrace{\delta(x_1-\bar{x}_1)}_{f_{x_1}}}_{f_{x_2}}\; P_{X_3|X_2}(x_3|x_2)}_{f_{x_3}}$$

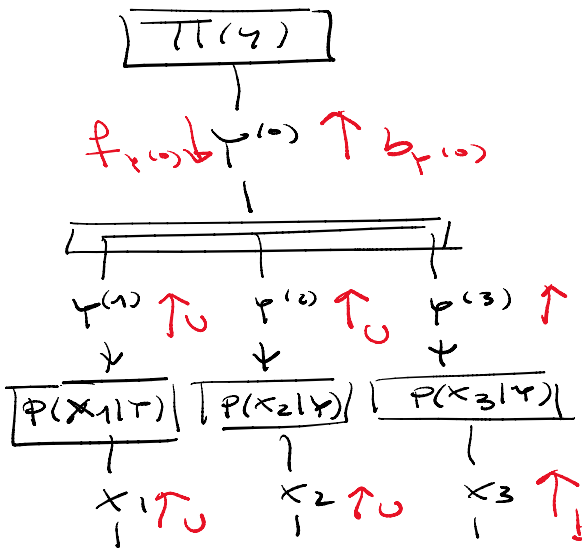
$$\underbrace{\sum_{x_4\in\mathcal{X}_4} P_{X_4|X_3}(x_4|x_3)\,\underbrace{\delta(x_4-\bar{x}_4)}_{b_{x_4}}}_{b_{x_3}}$$

$$=$$



$$p(y)\,p(x_1|y)\,p(x_2|y)\,p(x_3|y)$$

$$p(y|\,x_3=\bar{x}_3)$$



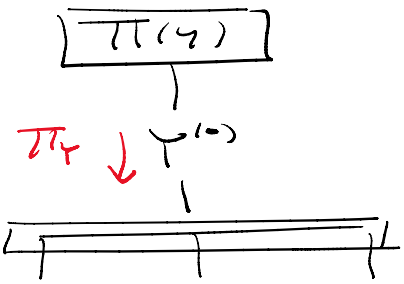
$f_{Y(0)} = \pi(y)$

$b_{Y(0)} = p(\bar{x}_3|y)$

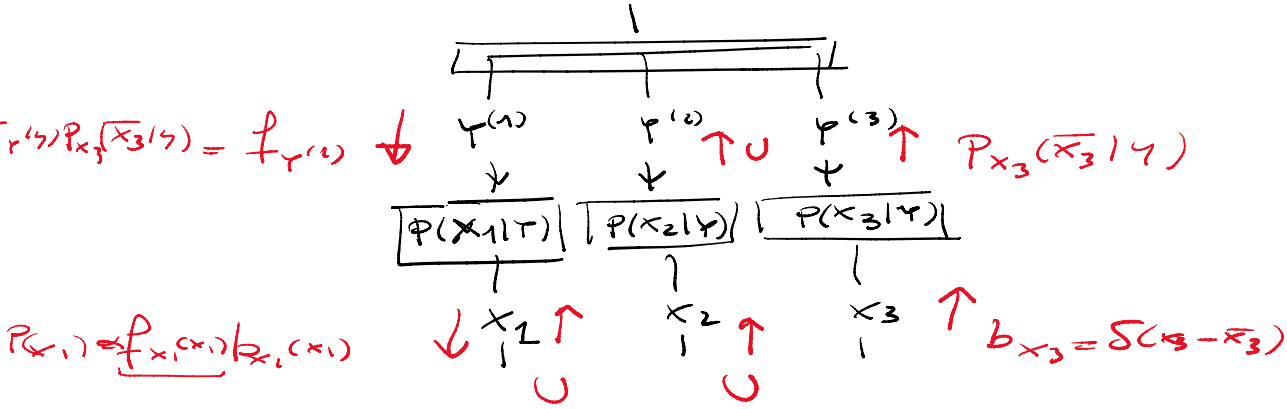
$P_{Y(0)} = \pi(y)\,p(\bar{x}_3|y)$

$f_{Y(0)}$, $b_{Y(0)}$, $\sum_{x_3} p(x_3|Y)\,\delta(x_3-\bar{x}_3) = p(\bar{x}_3|y)$, $b_{x_3} = \delta(x_3-\bar{x}_3)$

$$p(x_1|x_3=\bar{x}_3)$$



π_Y



$$\sum_y P(x_1|y)\,\pi_y(y)\,p(\bar{x}_3|y)$$

$$p(x_1|x_3=\bar{x}_3) = \frac{p(x_1,\bar{x}_3)}{p(\bar{x}_3)} \propto p(x_1,\bar{x}_3)$$

$$\underbrace{p(x_1|x_3=\bar{x}_3)} \propto \sum_{x_2 y} \underbrace{p(x_1 x_2 \bar{x}_3 y)} = \sum_{x_2 y} p(x_1 x_2 \bar{x}_3 y)$$

$$= \sum_{x_2 y} p(x_1|y)\,p(x_2|y)\,p(\bar{x}_3|y)\,p(y) =$$

$$= \sum_y p(x_1|y)\,p(\bar{x}_3|y)\cdot p(y) \underbrace{\sum_{x_2} p(x_2|y)}_{1} =$$

$$= \sum_y p(x_1|y)\,p(\bar{x}_3|y)\,p(y)$$

$$\underbrace{p(x_1,x_2)}$$

$$p(x_1) = \sum_{x_2} p(x_1,x_2)$$

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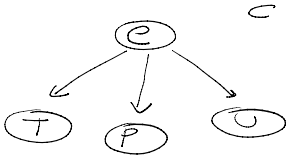
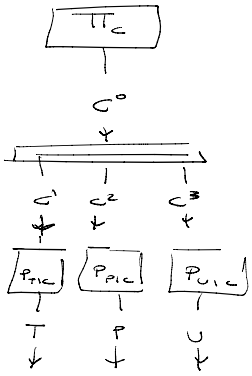


C (clima) $= \{b, B\}$
bello, Brutto

$T = \{b, a\}$

$P = \{b, a\}$

$U = \{b, a\}$



$$p(c) = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

$P_{T|c}$

c \ T	b	a
b	0.3	0.7
B	0.6	0.4

$P_{P|c}$

c \ P	b	a
b	0.2	0.8
B	0.5	0.5

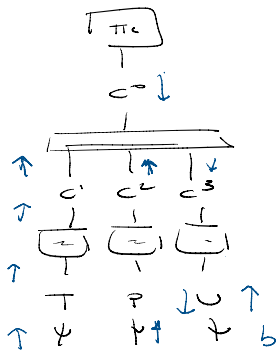
$P_{U|c}$

c \ U	b	a
b	0.4	0.6
B	0.7	0.3

$$P_{C,T,U,P}(c, t, u, p) = p(c)\, p(t|c)\, p(p|c)\, p(u|c)$$

esempio

$T = a \Rightarrow P(U | T = a)$



$$\underline{b}_{c^1} \propto \underline{\underline{P}}_{T|c} \cdot \underline{b}_T = \begin{bmatrix} 0.3 & 0.7 \\ 0.6 & 0.4 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.7 \\ 0.4 \end{bmatrix}$$

$$\underline{b}_{c^2} \propto \underline{\underline{P}}_{P|c} \cdot \underline{b}_P = \begin{bmatrix} 0.2 & 0.8 \\ 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

$$\underline{f}_{c^3} \propto \underline{b}_{c^1} \odot \underline{b}_{c^2} \odot \underline{f}_{c^0} = 0.5 \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

$b_T = \delta(t - a)$

$$\underline{b}_T = \begin{array}{c} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{array} \begin{array}{c} b \\ a \end{array}$$

$$b_U = U(u) = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \odot \begin{bmatrix} 0.7 \\ 0.4 \end{bmatrix} \odot \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 0.175 \\ 0.1 \end{bmatrix}$$

$$\underline{f}_U = \underline{\underline{P}}_{U|c}^T \cdot \underline{f}_{c^3} = \begin{bmatrix} 0.4 & 0.7 \\ 0.6 & 0.3 \end{bmatrix} \odot \begin{bmatrix} 0.175 \\ 0.1 \end{bmatrix} =$$

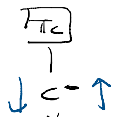
$$= \begin{bmatrix} 0.14 \\ 0.135 \end{bmatrix}$$

$$\underline{P}_U = \underline{f}_U \odot \underline{b}_U \propto \begin{bmatrix} 0.14 \\ 0.135 \end{bmatrix} \xrightarrow{\text{NORMAL}} \begin{bmatrix} 0.5091 \\ 0.4909 \end{bmatrix}$$

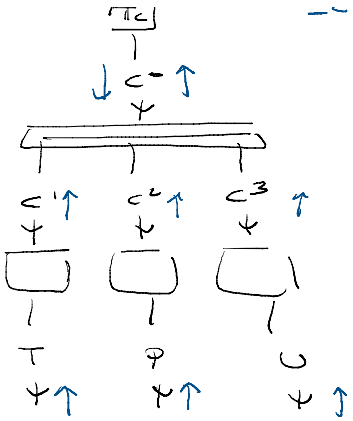
$$\begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_N \end{bmatrix} = \begin{bmatrix} \dfrac{e_1}{\sum_{j=1}^{N} e_i} \\ \dfrac{e_2}{\sum_{j=1}^{N} e_j} \end{bmatrix} \qquad e_i \longrightarrow \frac{e_i}{\sum_{j=1}^{N} e_j}$$

NORMALIZZ.

$T = a, P = a \quad P(C | T = a, P = a)$



$$\underline{b}_{c^1} \propto \underline{\underline{P}}_{T|c} \underline{b}_T = \begin{bmatrix} 0.3 & 0.7 \\ 0.6 & 0.4 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.7 \\ 0.4 \end{bmatrix}$$



$$\begin{bmatrix} 0.6 & 0.4 \end{bmatrix} \begin{bmatrix} 1 \end{bmatrix} = \begin{bmatrix} 0.4 \end{bmatrix}$$

$$\underline{b}_{c^2} \propto \underline{P}_{P|C}\, \underline{b}_P = \begin{bmatrix} 0.2 & 0.8 \\ 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.8 \\ 0.5 \end{bmatrix}$$

$$\underline{f}_{c^3} \propto \underline{b}_{c^1} \odot \underline{b}_{c^2} \odot \underline{f}_{c^0} = \begin{bmatrix} 0.7 \\ 0.4 \end{bmatrix} \begin{bmatrix} 0.8 \\ 0.5 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

$$= 0.5 \begin{bmatrix} 0.56 \\ 0.20 \end{bmatrix}$$

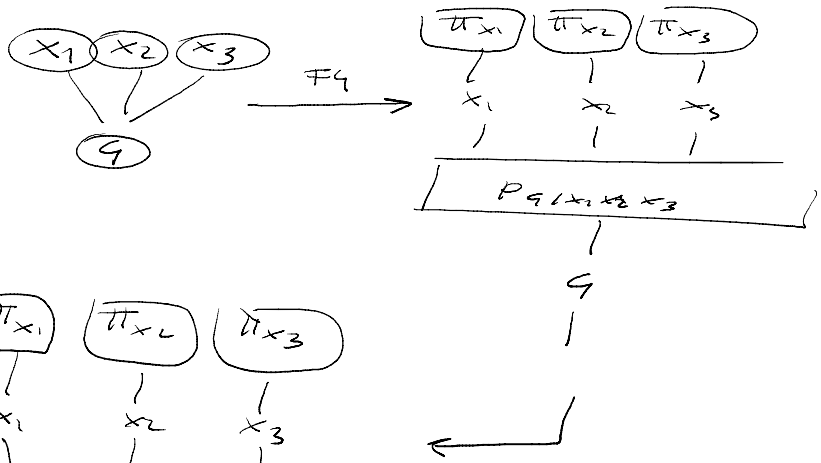
$b_T = \delta(t-a)$ $\quad b_P = \delta(p-a)$ $\quad b_U = U(u)$ $\quad \underline{b}_{c^3} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$

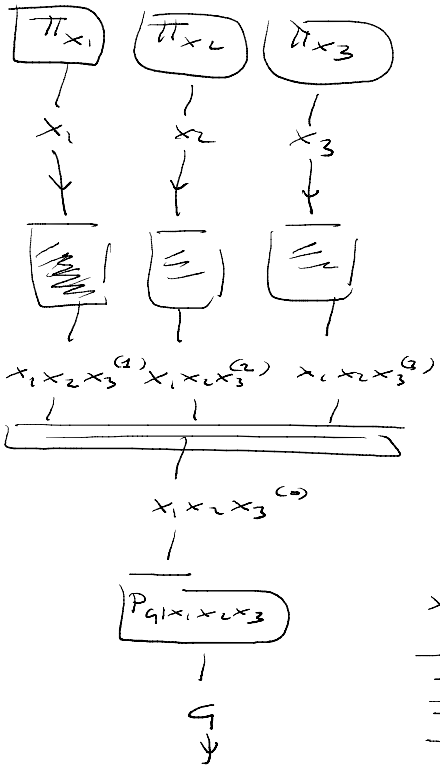
$$\underline{b}_{c^0} \propto \underline{b}_{c^1} \odot \underline{b}_{c^2} \odot \underline{b}_{c^3} =$$

$$= \begin{pmatrix} 0.7 \\ 0.4 \end{pmatrix} \begin{pmatrix} 0.8 \\ 0.5 \end{pmatrix} \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix} = 0.5 \begin{bmatrix} 0.56 \\ 0.20 \end{bmatrix}$$

$$\underline{P}(C \mid T=a, P=a) = \underline{b}_{c^0} \odot \underline{f}_{c^0} = 0.5 \begin{bmatrix} 0.56 \\ 0.20 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \propto \begin{bmatrix} 0.56 \\ 0.20 \end{bmatrix}$$

$$\xrightarrow{\text{Norm}} \begin{bmatrix} 0.7368 \\ 0.2632 \end{bmatrix}$$





$$P_{x_1x_2x_3|x_1}$$

$$P_{x_2|x_1x_2x_3}$$

$x_1x_2x_3$ \ x_1	T	F	x_2		
T T T	1	0		1	0
T T F	1	0		1	0
T F T	1	0		0	1

$G \downarrow$

T T F	1	0	1	0
T F T	1	0	0	1
T F F	1	0	0	1
F T T	0	1	1	0
F T F	0	1	1	0
F F T	0	1	0	1
F F F	0	1	0	1

$P_{x_2|x_1x_2x_3}$ $\quad$ $P_{x_3|x_1x_2x_3}$

1
1
0
0
1
1
0
0

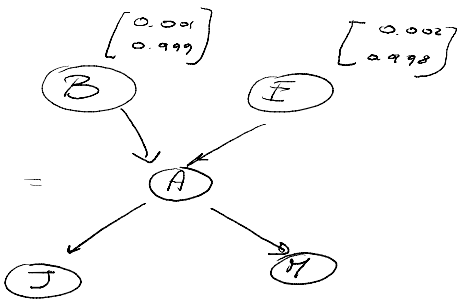
$\underline{\underline{P}}_{x_1x_2x_3|x_1}$

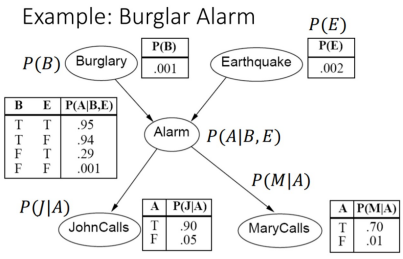
x_1 \ $x_1x_2x_3$	T T T	T T F	T F T	T F F	F T T	F T F	F F T	F F F
T	1/4	1/4	1/4	1/4	0	0	0	0
F	0	0	0	0	1/4	1/4	1/4	1/4

$$\underline{\underline{P}}_{x_1x_2x_3|x_1} = \frac{1}{4} \cdot \underline{\underline{P}}^T_{x_1|x_1x_2x_3}$$

$$\underline{\underline{P}}_{x_1x_2x_3|x_2} = \frac{1}{4} \underline{\underline{P}}^T_{x_2|x_1x_2x_3}$$

$$\underline{\underline{P}}_{x_1x_2x_3|x_3} = \frac{1}{4} \underline{\underline{P}}_{x_3|x_1x_2x_3}$$





$$P_{ABEJM} = P(B)\,P(E)\,P(A|BE)\,P(J|A)\,P(M|A)$$

π_B

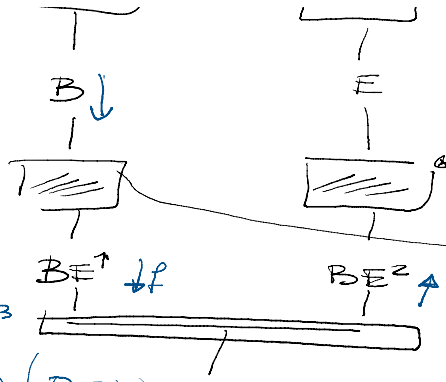
B

π_E

E

$P_{BE|E} =$

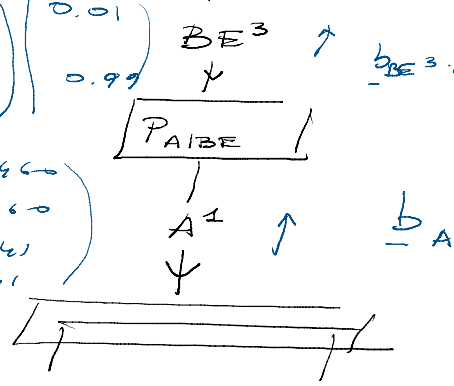
E \ BE	TT	TF	FT	FF
T	1/2	0	1/2	0



$$P_{BE|E} = \begin{array}{c|cccc} E & & & & \\ T & 1/2 & 0 & 1/2 & 0 \\ F & 0 & 1/2 & 0 & 1/2 \end{array}$$

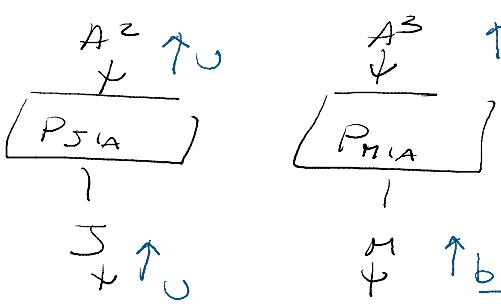
$$P_{BE|B} = \begin{array}{c|cccc} B \backslash BE & TT & TF & FT & FF \\ T & 1/2 & 1/2 & 0 & 0 \\ F & 0 & 0 & 1/2 & 1/2 \end{array}$$

$$f_{BE'} = \underline{P}^T_{BE|B} \cdot f_B = \begin{pmatrix} 0.5 & 0 \\ 0.5 & 0 \\ 0 & 0.5 \\ 0 & 0.5 \end{pmatrix} \begin{pmatrix} 0.01 \\ 0.99 \end{pmatrix} \simeq \begin{pmatrix} 0.0460 \\ 0.0460 \\ 0.4541 \\ 0.4541 \end{pmatrix}$$



$$\underline{b}_{BE^3} \propto \underline{P}_{A|BE} \cdot b_{A'} = \begin{pmatrix} 0.95 & 0.05 \\ 0.94 & 0.06 \\ 0.29 & 0.71 \\ 0.001 & 0.999 \end{pmatrix} \begin{pmatrix} 0.9858 \\ 0.0142 \end{pmatrix} \simeq \begin{pmatrix} 0.4307 \\ 0.4263 \\ 0.1360 \\ 0.0070 \end{pmatrix}$$

$$\underline{b}_{A'} = \underline{b}_{A^2} \odot \underline{b}_{A^3} = \begin{pmatrix} 0.4930 \\ 0.0071 \end{pmatrix} \longrightarrow \begin{pmatrix} 0.9858 \\ 0.0142 \end{pmatrix}$$



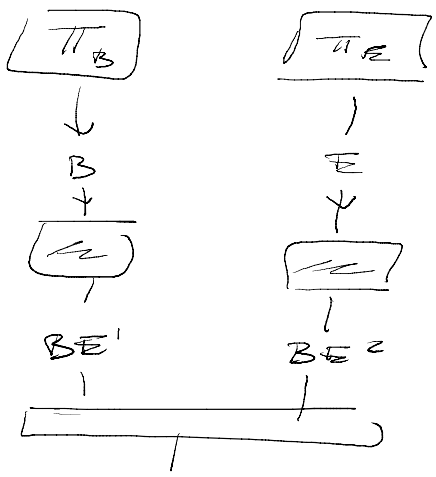
$$\underline{b}_{A^3} \propto \underline{P}_{M|A} \cdot \underline{b}_M = \begin{pmatrix} .70 & .30 \\ .01 & 0.99 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} .70 \\ .01 \end{pmatrix} \xrightarrow{\sim} \begin{pmatrix} 0.9857 \\ 0.0149 \end{pmatrix}$$

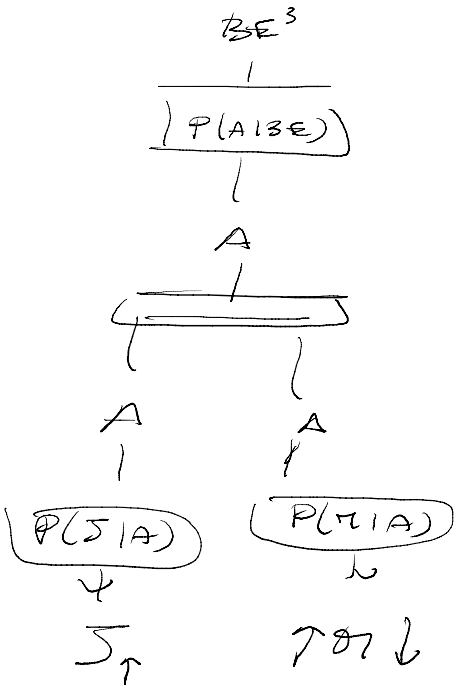
$$\underline{b}_M = \begin{array}{c} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{array} \begin{array}{c} T \\ F \end{array}$$

$\underline{M = T}$ $\quad p(E \mid M = T) = \underline{p}_E \propto \underline{f}_E \odot \underline{b}_E$

$$\underline{b}_{BE^2} \propto \underline{f}_{BE'} \odot \underline{b}_{BE^3} = \_\_$$

$$\underline{b}_E \propto \underline{P}_{BE|E} \cdot \underline{b}_{BE} \xrightarrow{\sim}$$





$$P(\underline{M} \mid J=T) =$$

$$\underline{p}_M \propto \underline{f}_M \odot \underline{b}_M$$

$$\underline{b}_J = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \underline{b}_M = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$