Teoria dei Segnali 11-10-2021

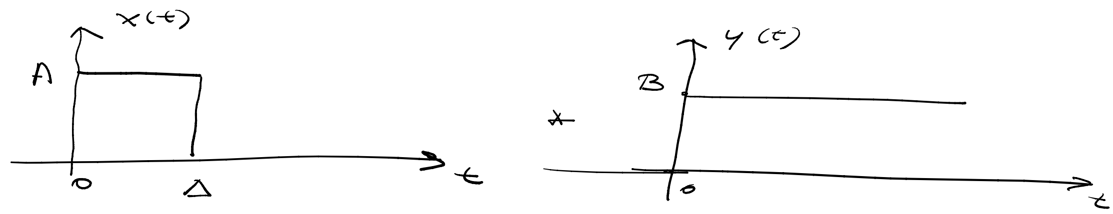
lunedì 11 ottobre 2021 08:56

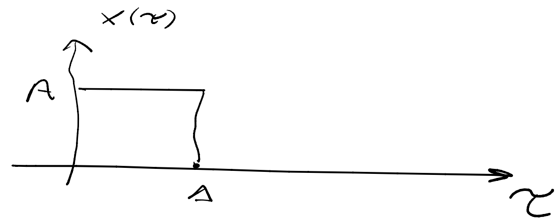
$$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$$

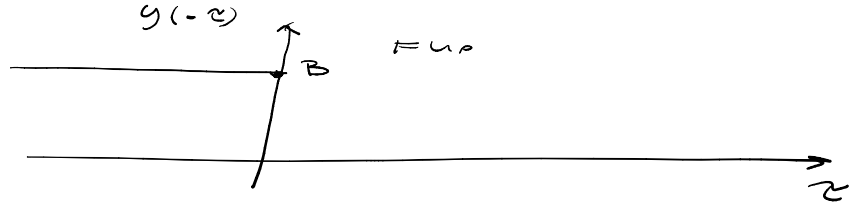
$$\underline{x(t)} * \underline{y(t)} = \underline{z(t)}$$

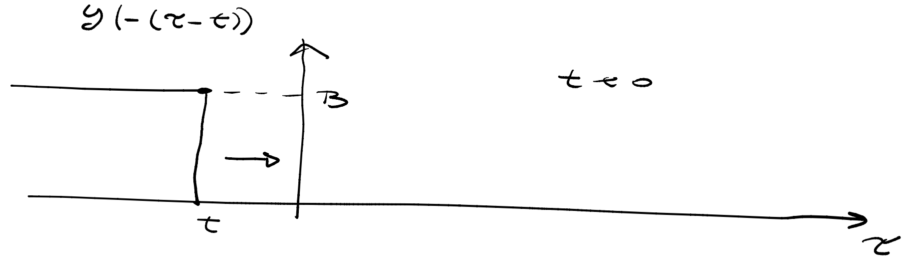
$$x(t) * y(t) = \int_{-\infty}^{+\infty} x(\tau)\, y(t-\tau)\, d\tau = \int_{-\infty}^{+\infty} x(\tau)\, \underline{y(-(\tau - t))}\, d\tau$$

esempio 1









t<0

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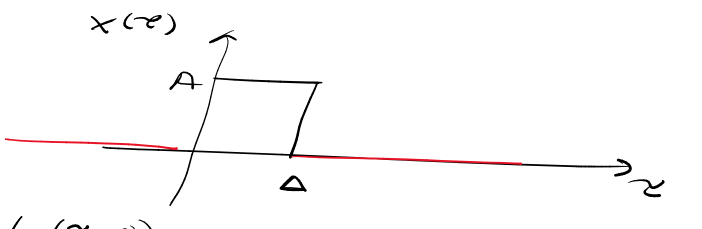
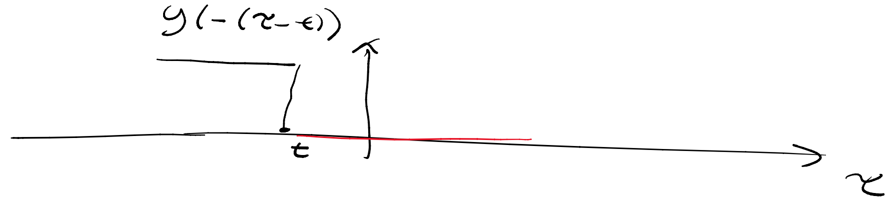
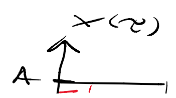
t<0





$$\int_{-\infty}^{+\infty} x(\tau)\, \underline{y(t-\tau)}\, d\tau = 0 \qquad z(t) = 0 \qquad \forall\, t<0$$

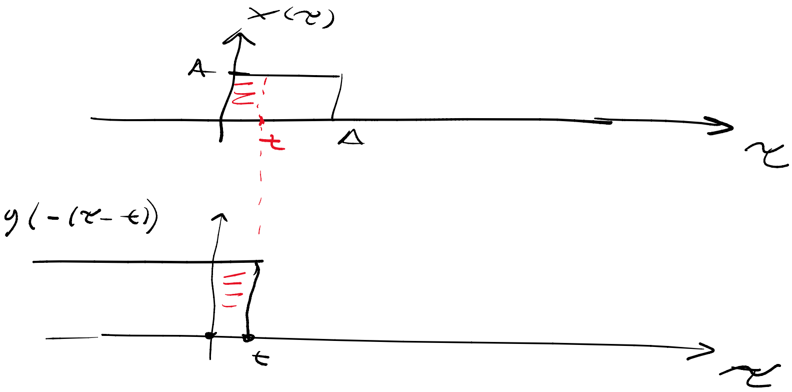
t = ...

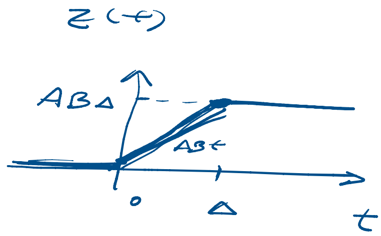


$t > 0$

$t < \Delta$

$0 < t < \Delta$

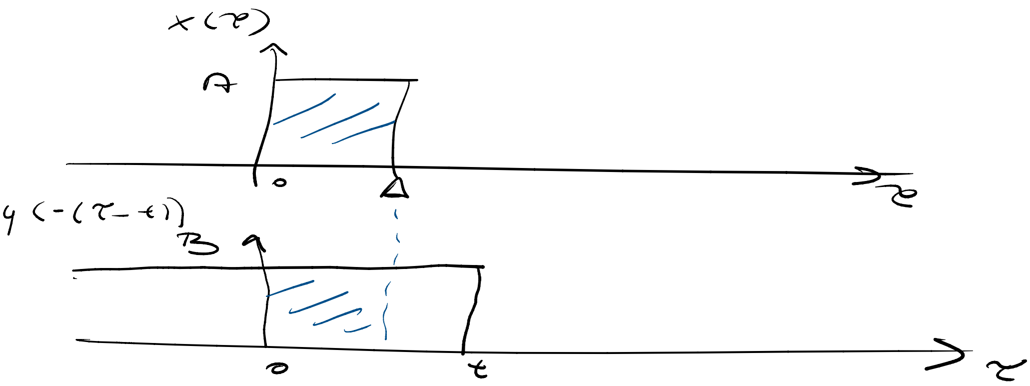




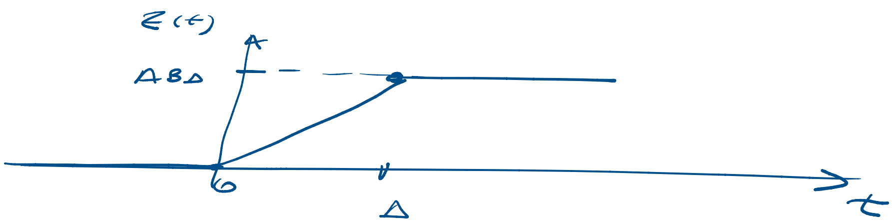
$$\int_{-\infty}^{+\infty} x(\tau)\, y(t-\tau)\, d\tau = \int_0^t x(\tau)\, y(t-\tau)\, d\tau =$$

$$= \int_0^t AB\, d\tau = AB\tau \Big]_0^t = \underline{ABt}$$

$t > \Delta$

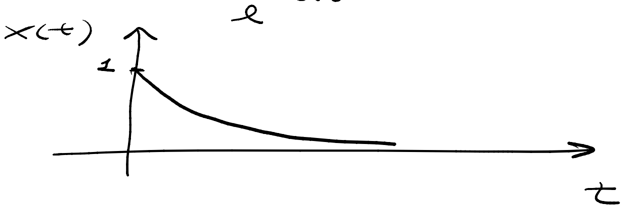


$$\int_{-\infty}^{+\infty} x(\tau)\, y(t-\tau)\, d\tau = \int_0^\Delta AB\, d\tau = AB\tau \Big]_0^\Delta = ABΔ$$

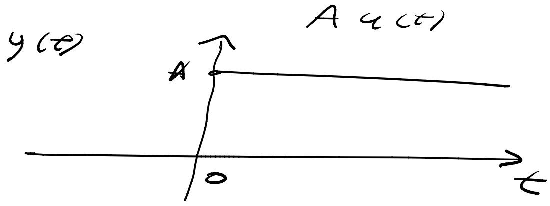


example 2

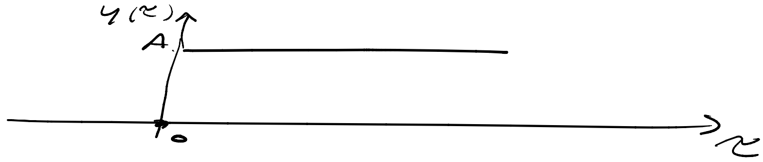
$e^{-\alpha t}$



$A\, u(t)$

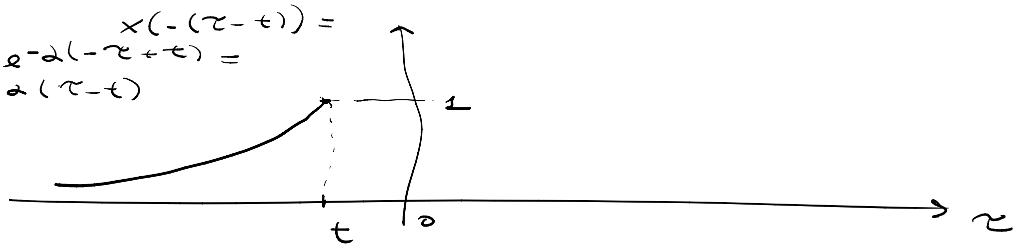


$$x(t) * y(t) = y(t) * x(t)$$

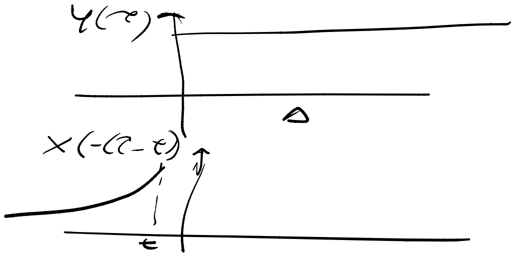


$e^{\alpha\tau} = x(-\tau)$

$x(-(\tau - t)) =$
$= e^{-\alpha(-\tau + t)} =$
$= e^{\alpha(\tau - t)}$



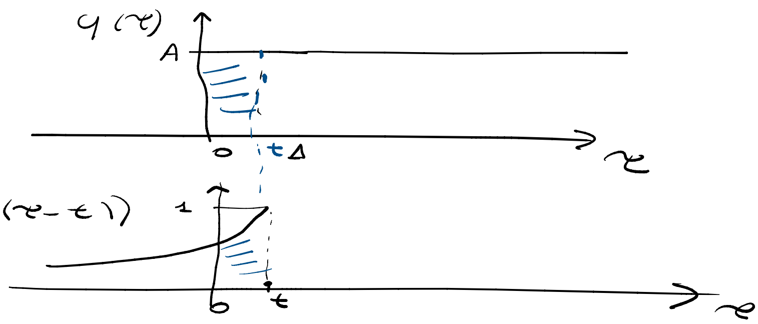
t < 0



$z(t) = 0$ se $t < 0$

t > 0

$e^{\alpha(\tau - t)} = x(-(\tau - t))$



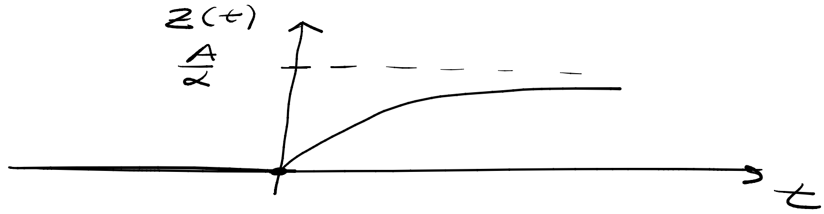
$$z(t) = \int_{-\infty}^{+\infty} y(\tau)\, x(t-\tau)\, d\tau = \int_{-\infty}^{+\infty} A \cdot e^{\alpha(\tau - t)}\, d\tau$$

$$= A\int_0^t e^{\alpha(\tau - t)}\, d\tau = A\int_0^t e^{\alpha\tau} \cdot e^{-\alpha t}\, d\tau =$$

$$= A e^{-\alpha t} \int_0^t e^{\alpha\tau}\, d\tau = A e^{-\alpha t} \left[\frac{e^{\alpha\tau}}{\alpha}\right]_0^t =$$

$$= \frac{A}{\alpha} e^{-\alpha t}\left[e^{\alpha t} - 1\right] = \frac{A}{\alpha}\left[1 - e^{-\alpha t}\right]$$

$$= \frac{A}{\alpha} e^{-\alpha t}\left[e^{\alpha t} - 1\right] = \frac{A}{\alpha}\left[1 - e^{-\alpha t}\right]$$



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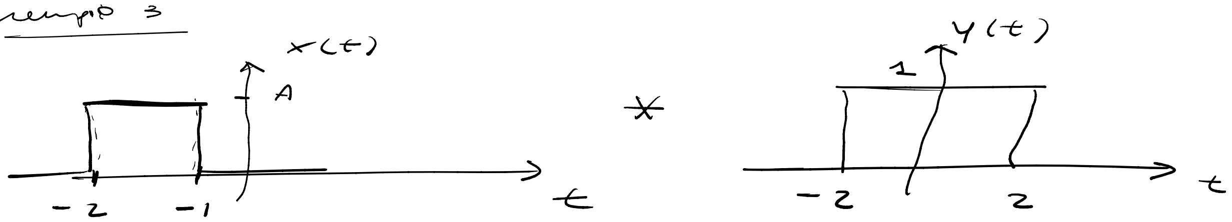
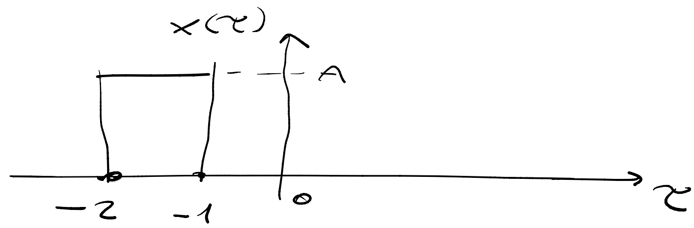
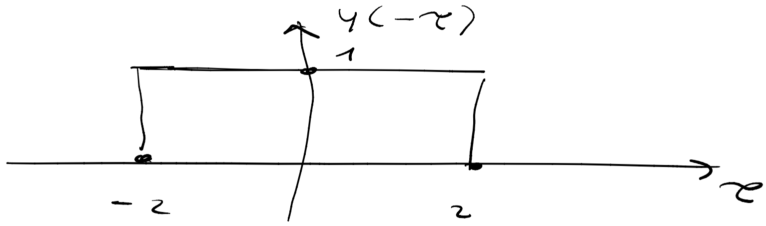
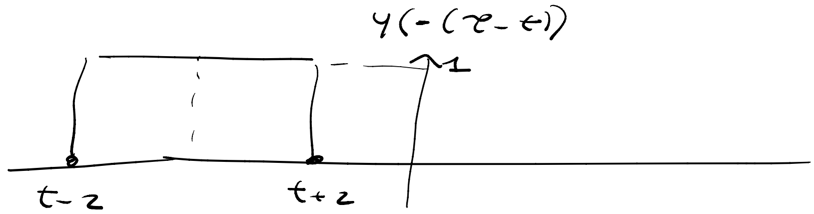
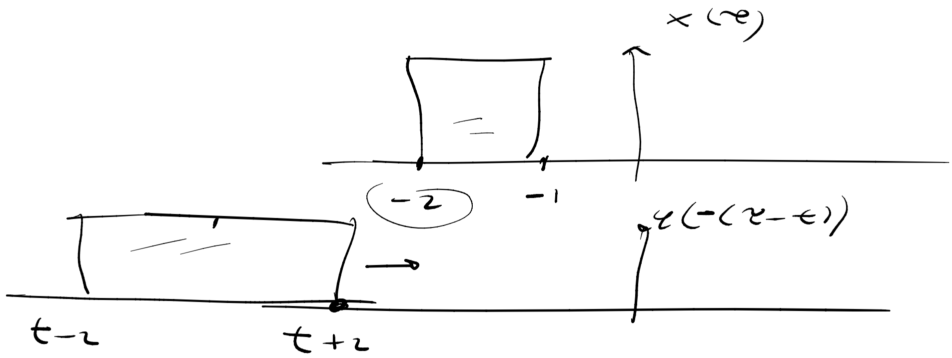
ejemplo 3



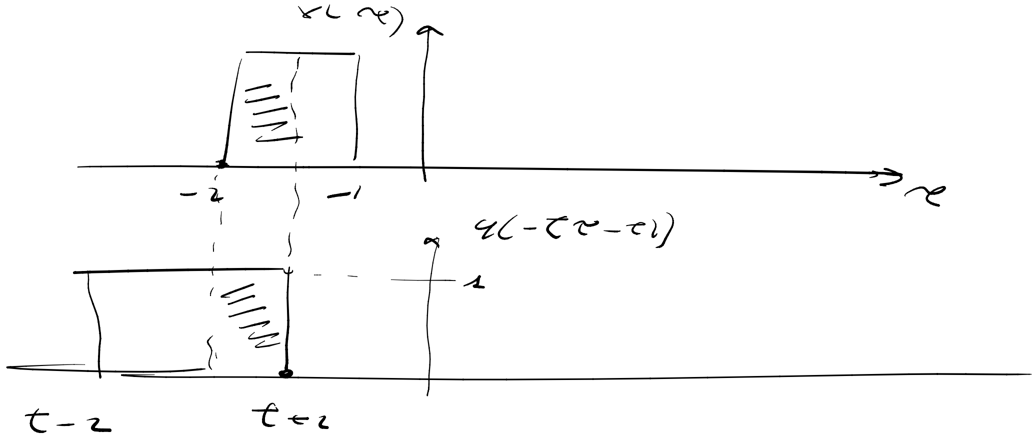








$$t + 2 < -2 \Rightarrow \underline{\underline{t < -4}} \qquad z(t) = 0 \quad \forall\, t < -4$$

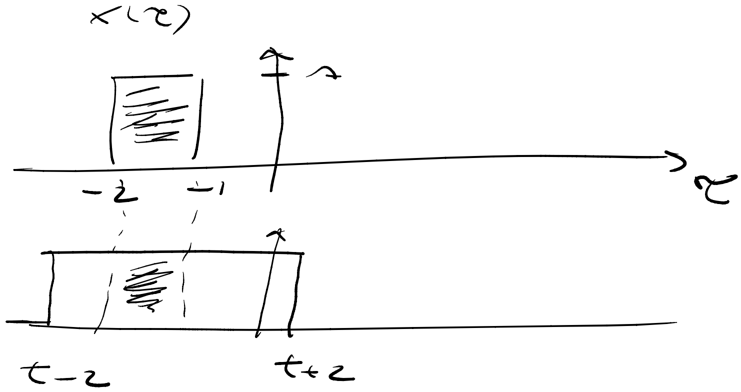


$t+2 > -2 \qquad t+2 < -1$

$-2 < t+2 < -1 \Leftrightarrow -4 < t < -3$

$$z(t) = \int_{-\infty}^{+\infty} x(\tau)\, y(t-\tau)\, d\tau = \int_{-2}^{t+2} A \cdot 1\, d\tau = A\tau \Big]_{-2}^{t+2} = A(t+2) + 2A =$$

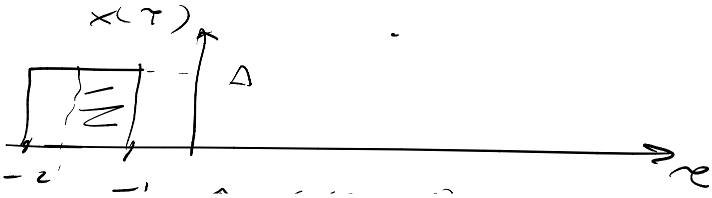
$$= At + 4A = A(t+4)$$

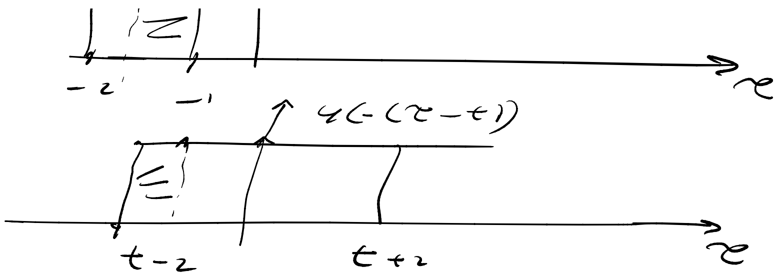


$\begin{cases} t+2 > -1 \\ t-2 < -2 \end{cases} \qquad \begin{cases} t > -3 \\ t < 0 \end{cases} \qquad \underline{-3 < t < 0}$

$$z(t) = \int_{-\infty}^{+\infty} x(\tau)\, y(t-\tau)\, d\tau = \int_{-2}^{-1} A\, d\tau = A\tau \Big]_{-2}^{-1} = A(-1+2) = A =$$

$-3 < t < 0 \Rightarrow z(t) = A$



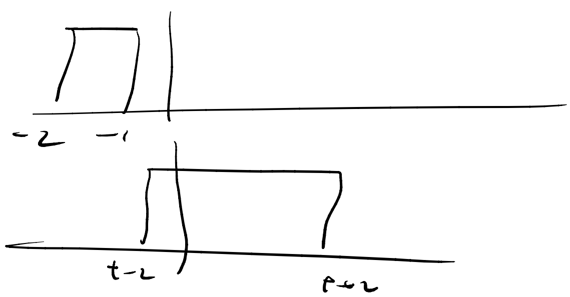


$$\begin{cases} t-2 > -2 \\ t-2 < -1 \end{cases} \Rightarrow \begin{cases} t > 0 \\ t < 1 \end{cases} \qquad \underline{0 < t < 1}$$

$$z(t) = \int_{-\infty}^{+\infty} x(\tau)\, y(t-\tau)\, d\tau =$$

$$= \int_{t-2}^{-1} A\, d\tau = A\tau\Big]_{t-2}^{-1} = A(-1-t+2) =$$

$$= A(1-t)$$



$t-2 > -1 \Rightarrow \underline{t > 1}$ $\qquad \underline{z(t) = 0}$

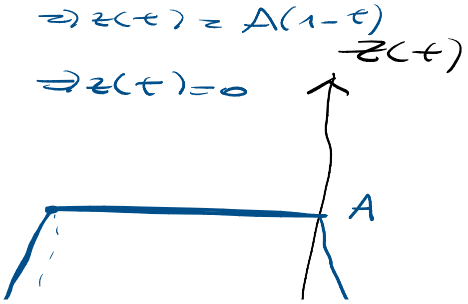
Se $t < -4 \Rightarrow z(t) = 0$

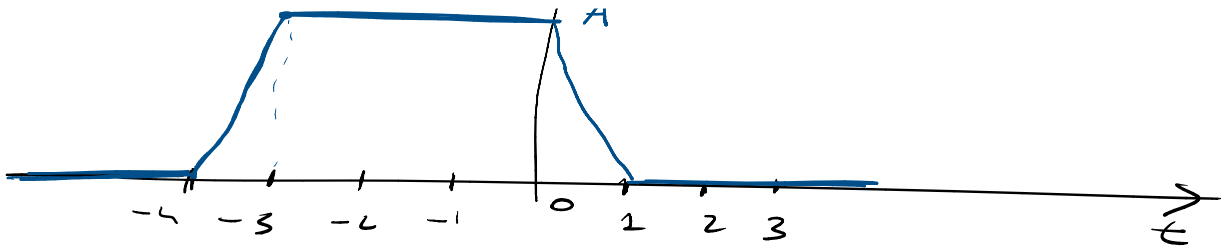
$-4 < t < -3 \Rightarrow z(t) = A(t+4)$

$-3 < t < 0 \Rightarrow z(t) = A$

$0 < t < 1 \Rightarrow z(t) = A(1-t)$

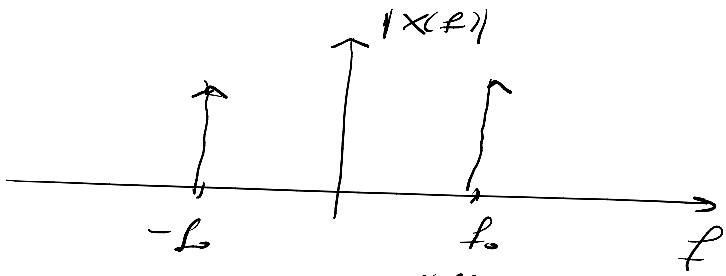
$t > 1 \Rightarrow z(t) = 0$

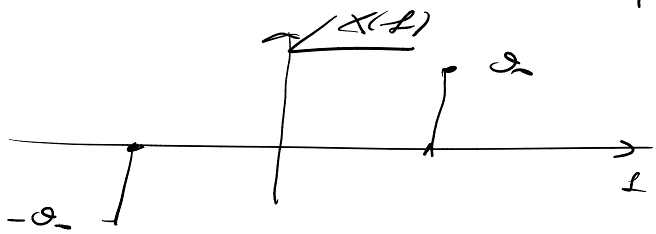




$$x(t) = A_1 \cos(2\pi f_2 t + \theta_1) + A_2 \cos(2\pi f_2 t + \theta_2)$$

$$\mathcal{F}\left[A\cos(2\pi f_0 t + \theta_0)\right] = \frac{A}{2} e^{j\theta_0} \delta(f - f_0) + \frac{A}{2} e^{-j\theta_0} \delta(f + f_0)$$

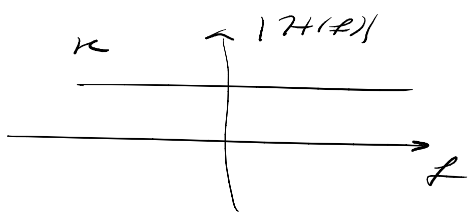




$$X(f) = \frac{A_1}{2}\left[e^{j\theta_1}\delta(f - f_1) + e^{-j\theta_1}\delta(f + f_1)\right] +$$

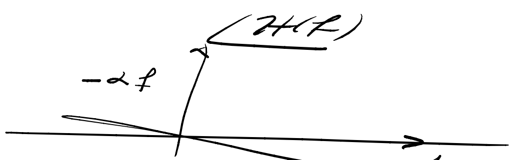
$$+ \frac{A_2}{2}\left[e^{j\theta_2}\delta(f - f_2) + e^{-j\theta_2}\delta(f + f_2)\right]$$

$$|H(f)| = k \qquad \angle H(f) = -\alpha f$$



$$Y(f) = H(f)X(f) =$$

$$= |H(f)|\,|X(f)|\, e^{j(\angle H(f) + \angle X(f))}$$



$$= \frac{A_1 k}{2}\left[e^{j(\theta_1 - \alpha f_1)}\delta(f - f_1) + e^{j(-\theta_1 + \alpha f_1)}\delta(f + f_1)\right] +$$

$$+\frac{A_2 k}{2}\left[e^{j(\theta_2-\alpha f_2)}\delta(f-f_2)+e^{j(-\theta_2+\alpha f_2)}\delta(f+f_2)\right]$$

$$=\frac{A_1 k}{2}\left[e^{j(\theta_1-\alpha f_1)}\delta(f-f_1)+e^{-j(\theta_1-\alpha f_1)}\delta(f+f_1)\right]$$

$$+\frac{A_2 k}{2}\left[e^{j(\theta_2-\alpha f_2)}\delta(f-f_2)+e^{-j(\theta_2-\alpha f_2)}\delta(f+f_2)\right]$$

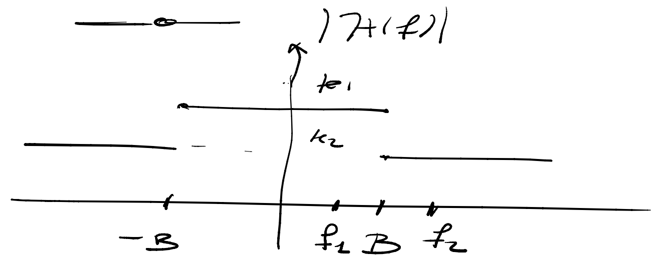
$$y(t)=A_1 k\cos(\underline{2\pi f_1 t}+\theta_1-\alpha f_1)+A_2 k\cos(2\pi f_2 t+\theta_2-\alpha f_2)$$

$$=A_1 \underline{k}\cos\left(2\pi f_1\underbrace{\left(t-\frac{\alpha}{2\pi}\right)}+\theta_2\right)+A_2 \underline{k}\cos\left(2\pi f_2\underbrace{\left(t-\frac{\alpha}{2\pi}\right)}+\theta_2\right)$$

$$=\underline{k}\cdot x\left(t-\underline{\frac{\alpha}{2\pi}}\right) \qquad t_0=\frac{\alpha}{2\pi}$$

$$|H(f)|=\begin{cases}k_1 & |f|<B\\ k_2\end{cases}$$

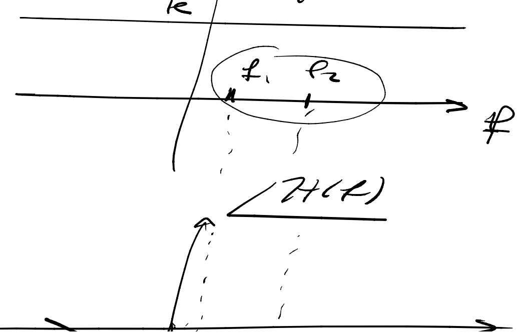
$$\angle H(f)=-\alpha f$$



$$Y(t)=A_1 k_1\cos\left(2\pi f_1\underbrace{\left(t-\frac{\alpha}{2\pi}\right)}+\theta_1\right)$$

$$+A_2 k_2\cos\left(2\pi f_2\underbrace{\left(t-\frac{\alpha}{2\pi}\right)}+\theta_2\right)$$

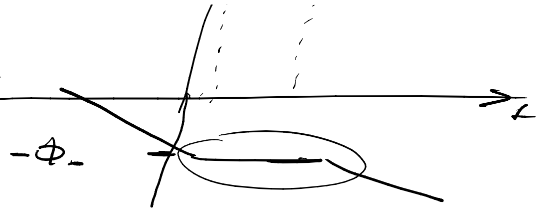
$$|H(f)|=k \qquad \angle H(f)=-\underline{\Phi_0}$$



$$y(t)=A_1 k\cos(2\pi f_1 t+\theta_1-\Phi_0)+$$

$$+A_2 k\cos(2\pi f_2 t+\theta_2-\Phi_0)$$

$$= A_1 \underline{k} \cos\left(2\pi f_1 \left(t - \frac{\Phi_0}{2\pi f_1}\right) + \vartheta_1\right) +$$



$$+ A_2 \underline{k} \cos\left(2\pi f_2 \left(t - \frac{\Phi_0}{2\pi f_2}\right) + \vartheta_2\right)$$

$$-\frac{\Phi_0}{2\pi f_1} \qquad -\frac{\Phi_0}{2\pi f_2}$$