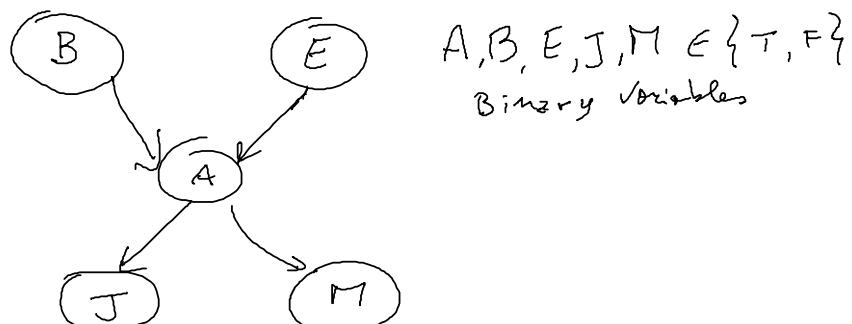


FACTORIZATION:

$$\begin{aligned} p(A, B, E, J, M) \\ = p(B)p(E)p(A|BE) \\ p(J|A)p(M|A) \end{aligned}$$



GRAFO FATTORIALE (REDUCED NORMAL FORM)

$$\pi_B = \begin{bmatrix} .01 \\ .99 \end{bmatrix}$$

$$\boxed{\pi_B} \downarrow$$

$$\begin{array}{c} \text{TT} \text{ TF} \text{ FT} \text{ FF} \\ \hline \frac{1}{2} \frac{1}{2} 0 0 \end{array} \quad \boxed{(BE)}' \quad \downarrow$$

$$\boxed{\pi_E} \downarrow$$

$$\begin{array}{c} \text{TT} \text{ TF} \text{ FT} \text{ FF} \\ \hline 0 0 \frac{1}{2} \frac{1}{2} \end{array} \quad \boxed{(BE)}^2 \quad \downarrow$$

$$\pi_A = \begin{bmatrix} .02 \\ .98 \end{bmatrix}$$

$$P(BE|E) = \begin{bmatrix} \text{TT} \text{ TF} \text{ FT} \text{ FF} \\ \hline \frac{1}{2} 0 \frac{1}{2} 0 \end{bmatrix}$$

$$\begin{array}{c} (BE)^3 \\ \boxed{P(A|BE)} \\ \downarrow \end{array}$$

$$P(A|BE) = \begin{cases} T & F \\ TT & .35 \\ TF & .34 \\ FT & .29 \\ FF & .001 \end{cases}$$

$$P(J|A) = \begin{cases} T & F \\ .90 & .10 \\ .05 & .95 \end{cases}$$

$$\begin{array}{c} \boxed{P(J|A)} \downarrow \quad \boxed{P(M|A)} \downarrow \\ J \quad M \end{array}$$

$$P(M|A) = \begin{cases} T & F \\ .70 & .30 \\ .01 & .99 \end{cases}$$

QUERY 1: Mary receives a call ($J=T$) \rightarrow Compute the probability of an earthquake $P(E|M=T)$

QUERY 1: Mary receives a call ($J=T$) \rightarrow Compute the probability of an earthquake $P(E | M=T)$

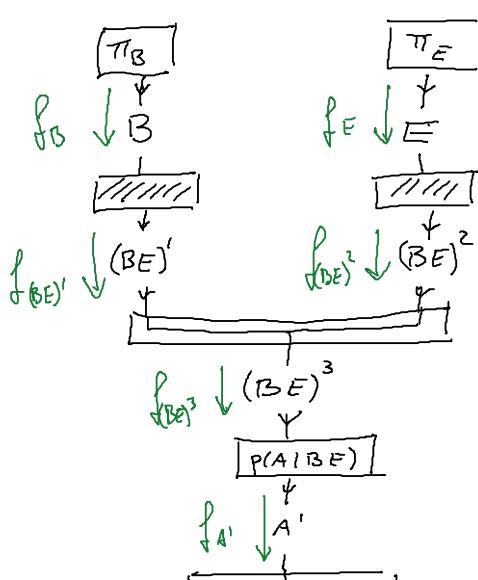
ANSWER: $p(E|J=T) = P_E \propto f_E \odot b_E = \begin{pmatrix} 0.0185 \\ 0.0726 \end{pmatrix}$ $\xrightarrow{\text{NORM}} P_E = \begin{pmatrix} 0.2033 \\ 0.7967 \end{pmatrix}$

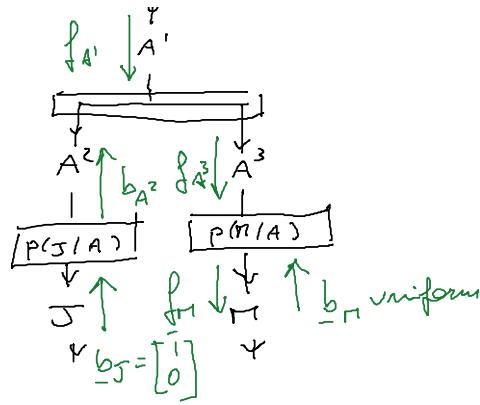
only $\approx 20\%$ prob. that an earthquake has occurred

$$\begin{aligned} f_B &= \pi_B = \begin{pmatrix} 0.01 \\ 0.99 \end{pmatrix} \\ f_{(BE)}' &= P(BE|B)f_B \\ &= \begin{pmatrix} 0.5 & 0 \\ 0.5 & 0 \\ 0 & 0.5 \\ 0 & 0.5 \end{pmatrix} \begin{pmatrix} 0.01 \\ 0.99 \end{pmatrix} \\ &= \begin{pmatrix} 0.0050 \\ 0.0050 \\ 0.4950 \\ 0.4950 \end{pmatrix} \\ f_E &= \begin{pmatrix} 0.02 \\ 0.98 \end{pmatrix} \\ f_{(BE)}^2 &= P(BE|E)f_E \\ &= \begin{pmatrix} 0.4630 \\ 0.0370 \end{pmatrix} \begin{pmatrix} 0.02 \\ 0.98 \end{pmatrix} \\ &= \begin{pmatrix} 0.0022 \\ 0.0021 \\ 0.0673 \\ 0.0034 \end{pmatrix} \\ f_{(BE)}^3 &= P(BE|BE)f_{(BE)}^2 \\ &= \begin{pmatrix} 0.0287 \\ 0.0284 \\ 0.8972 \\ 0.0457 \end{pmatrix} \begin{pmatrix} 0.0022 \\ 0.0021 \\ 0.0673 \\ 0.0034 \end{pmatrix} \\ &= \begin{pmatrix} 0.00059 \\ 0.00059 \\ 0.9360 \\ 0.0069 \end{pmatrix} \\ &\xrightarrow{\text{NORM}} b_{(BE)}^3 = \begin{pmatrix} 0.4308 \\ 0.4263 \\ 0.1360 \\ 0.0069 \end{pmatrix} \\ f_{A^1} &= P(A^1|BE)f_{(BE)}^3 \\ &= \begin{pmatrix} 0.95 & 0.05 \\ 0.94 & 0.06 \\ 0.23 & 0.71 \\ 0.001 & 0.9985 \end{pmatrix} \begin{pmatrix} 0.00059 \\ 0.00059 \\ 0.9360 \\ 0.0069 \end{pmatrix} \\ &= \begin{pmatrix} 0.8373 \\ 0.9276 \\ 0.2959 \\ 0.0151 \end{pmatrix} \\ f_{A^2} &= P(A^2|BE)f_{(BE)}^3 \\ &= \begin{pmatrix} 0.4930 \\ 0.0070 \end{pmatrix} \xrightarrow{\text{NORM.}} b_{A^2} = \begin{pmatrix} 0.5859 \\ 0.0141 \end{pmatrix} \\ f_{A^3} &= P(A^3|BE)f_{(BE)}^3 \\ &= \begin{pmatrix} 0.7 & 0.3 \\ 0.01 & 0.99 \end{pmatrix} \begin{pmatrix} 0.00059 \\ 0.00059 \\ 0.9360 \\ 0.0069 \end{pmatrix} \\ &= \begin{pmatrix} 0.0141 \\ 0.9859 \end{pmatrix} \\ &\xrightarrow{\text{NORMALIZE}} b_{A^3} = \begin{pmatrix} 0.9859 \\ 0.0141 \end{pmatrix} \\ \text{Normalize } f_{A^2} &\rightarrow b_{A^2} = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix} \text{ Uniform} \end{aligned}$$

NOTE: Normalization can be avoided everywhere except at the end for computation of the answer P_E .

QUERY 2: John receives a call \Rightarrow Prob. that also Mary receives a call ($J=T$) $P(M|J=T)$





ANSWER: $p(\pi|J=T) = p_M \propto f_M \odot b_M \propto f_M \rightarrow p_M = \begin{pmatrix} 0.1671 \\ 0.8329 \end{pmatrix}$

```
MATLAB Script:
%Burglary/Earthquake Example
PjB=[0.01 0.99];
PIE=[0.02 0.98];
PBEdB=[0.5 0.5 0 0 0 0.5];
PBEdE=[0.5 0.5 0 0 0.5 0.5];
PAdeB=[0.95 0.05; 0.94 0.06; 0.29 0.71; 0.001 0.999];
PAdeE=[0.9 0.1; 0.05 0.95];
PMda=[0.7 0.3; 0.01 0.99];
%input
bj=[1 0];
bM=[0.5 0.5]; %Uniform
%message propagation
fb=PjB;
fe=PIE;
fBE1=PBEdB*fB; %non normalized
fBE2=PBEdE*fE; %non normalized
fBE3=fBE1.* fBE2; %non normalized
fA1=PAdeB*fBE3; %non normalized
bA2=Pjda*b1; %
fA3=fA1.* bA2;
fM=PMda*fA3;
pM=fM.* bM
%normalize
pM=pM/sum(pM)
```

(cut-and-paste in
Matlab)

```
MATLAB Script:
%Burglary/Earthquake Example
PjB=[0.5 0.5];
PIE=[0.5 0.5];
PBEdB=[0.5 0.5 0 0 0 0.5];
PBEdE=[0.5 0.5 0; 0 0.5 0.5];
PAdeB=[0.95 0.05; 0.94 0.06; 0.29 0.71; 0.001 0.999];
PAdeE=[0.9 0.1; 0.05 0.95];
PMda=[0.7 0.3; 0.01 0.99];
%input
bj=[1 0];
bM=[0.5 0.5]; %Uniform
%message propagation
fb=PjB;
fe=PIE;
fBE1=PBEdB*fB; %non normalized
fBE2=PBEdE*fE; %non normalized
fBE3=fBE1.* fBE2; %non normalized
fA1=PAdeB*fBE3; %non normalized
bA2=Pjda*b1; %
fA3=fA1.* bA2;
fM=PMda*fA3;
pM=fM.* bM
%normalize
pM=pM/sum(pM)
```

The result may seem surprising giving only 17% probability to a call to Mary. The reason is that there is a strong (negative) prior on A because B and E are not very likely.

If we assume π_B and π_E uniform, then

$$f_{(BE)^3} = \text{Uniform} \begin{pmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{pmatrix} \rightarrow f_A = P(A|BE)^T f_{(BE)^3}$$

$$= \begin{pmatrix} 0.95 & 0.94 & 0.25 & 0.01 \\ 0.05 & 0.06 & 0.71 & 0.999 \end{pmatrix} \begin{pmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \end{pmatrix} = \begin{pmatrix} 0.5453 \\ 0.4548 \end{pmatrix}$$

\rightarrow $P(\pi|J=T) = \begin{pmatrix} 0.6694 \\ 0.3306 \end{pmatrix}$ It is more likely that Mary receives a call

P₁/A