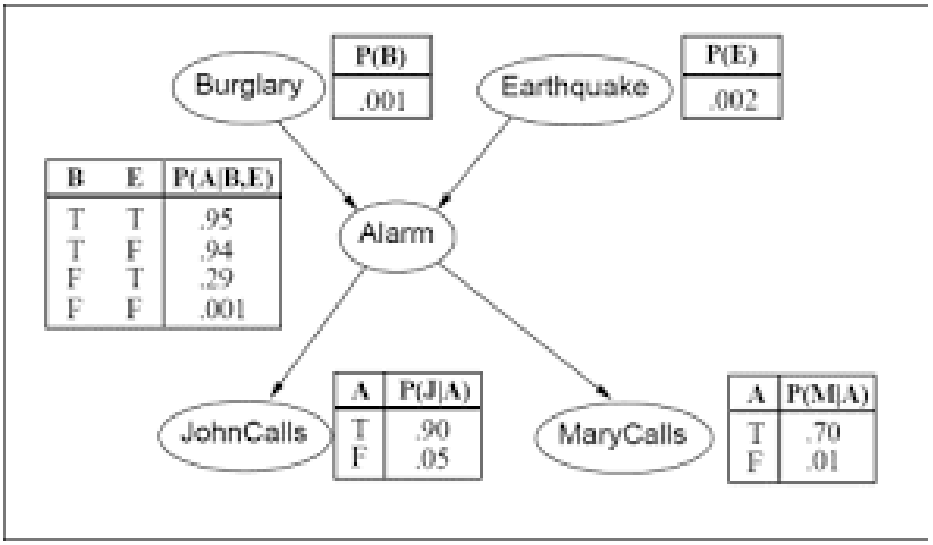
Burglary and Earthquake Example

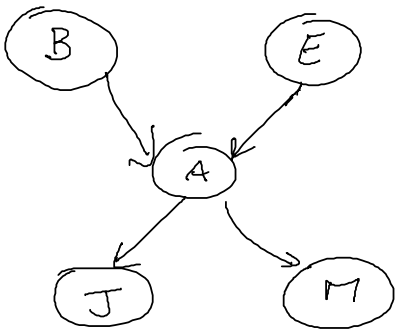
lunedì 21 dicembre 2020 10.59



FACTORIZATION:

$p(A,B,E,J,M)$

$= p(B)p(E)p(A|BE)$
$p(J|A)p(M|A)$



$A, B, E, J, M \in \{T, F\}$
Binary variables

GRAFO FATTORIALE (REDUCED NORMAL FORM)

$\pi_B = \begin{bmatrix} .01 \\ .99 \end{bmatrix}$ $\pi_E = \begin{bmatrix} .02 \\ .98 \end{bmatrix}$

$P(BE|B) = \begin{matrix} & TT & TF & FT & FF \\ T & ½ & ½ & 0 & 0 \\ F & 0 & 0 & ½ & ½ \end{matrix}$

$P(BE|E) = \begin{matrix} & TT & TF & FT & FF \\ T & ½ & 0 & ½ & 0 \\ F & 0 & ½ & 0 & ½ \end{matrix}$

$P(A|BE) = \begin{matrix} & T & F \\ TT & .95 & .05 \\ TF & .94 & .06 \\ FT & .29 & .71 \\ FF & .001 & .999 \end{matrix}$

$P(J|A) = \begin{matrix} & T & F \\ T & .90 & .10 \\ F & .05 & .95 \end{matrix}$

$P(M|A) = \begin{matrix} & T & F \\ T & .70 & .30 \\ F & .01 & .99 \end{matrix}$

QUERY 1: Mary receives a call ($J=T$) → Compute the probability of an earthquake $P(E|M=T)$

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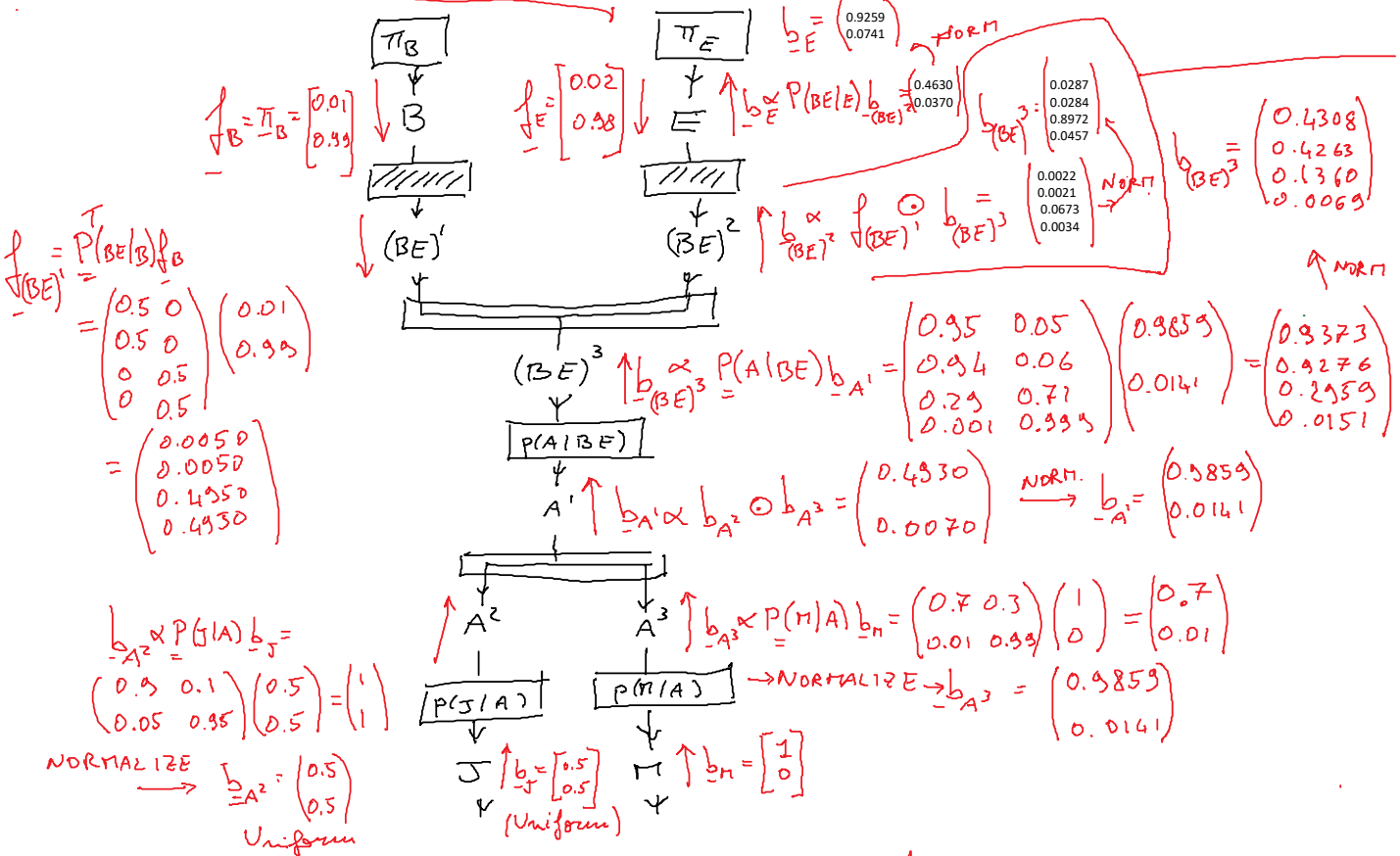
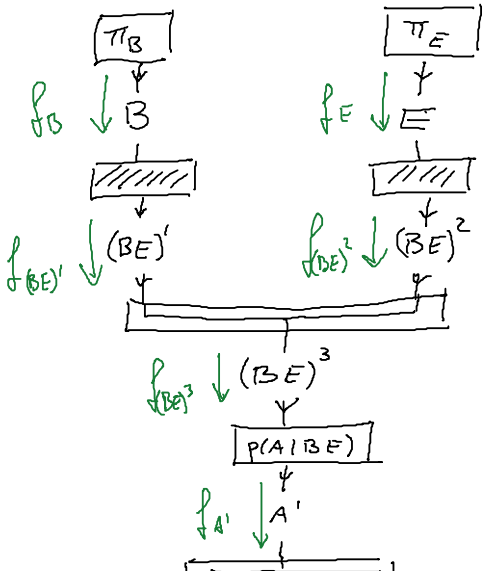
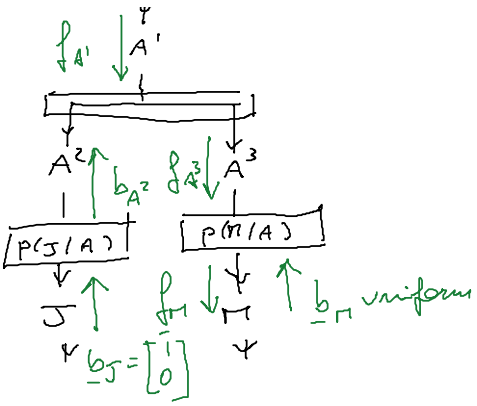
ANSWER : $p(E|J=T) = \underline{p}_E \propto \underline{f}_E \odot \underline{b}_E = \begin{pmatrix}0.0185\\0.0726\end{pmatrix}$ NORM → $\underline{p}_E = \begin{pmatrix}0.2033\\0.7967\end{pmatrix}$ only ≈ 20 % prob. that an earthquake has occurred



NOTE: Normalization can be avoided everywhere except at the end for computation of the answer $\underline{p}_E$.

QUERY 2 : John receives a call ($J=T$) ⇒ Prob. that also Mary receives a call $P(M|J=T)$





ANSWER: $p(M|J=T) = p_M \propto f_M \odot b_M \propto f_M \xrightarrow{\text{NORMALIZE}} p_M = \begin{pmatrix} 0.1671 \\ 0.8329 \end{pmatrix}$

MATLAB Script:

```
%Burglary/Earthquake Example
PiB=[0.01 0.99]';
PiE=[0.02 0.98]';
PBEdB=[0.5 0.5 0 0; 0 0 0.5 0.5];
PBEdE=[0.5 0 0.5 0; 0 0.5 0 0.5];
PAdBE=[0.95 0.05; 0.94 0.06; 0.29 0.71; 0.001 0.999];
PJdA=[0.9 0.1; 0.05 0.95];
PMdA=[0.7 0.3; 0.01 0.99];
%input
bJ=[1 0]';
bM=[0.5 0.5]'; %Uniform
%mesaage propagation
fB=PiB;
fE=PiE;
fBE1=PBEdB'*fB;  %non normalized
fBE2=PBEdE'*fE;  %non normalized
fBE3=fBE1 .* fBE2; %non normalized
fA1=PAdBE'*fBE3; %non normalized
bA2=PJdA*bJ; % "
fA3=fA1 .* bA2;
fM=PMdA'*fA3;
pM=fM .* bM
%normalize
pM=pM/sum(pM)
```

(cut-and-paste in Matlab)

The result may seem surprising giving only 17% probability to a call to Mary. The reason is that there is a strong (negative) prior on A because B and E are not very likely.

If we assume π_B and π_E uniform, then

$$f_{(BE)^3} = \text{Uniform} \begin{pmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{pmatrix} \rightarrow f_{A'} = P(A|BE)^T f_{(BE)^3}$$

$$= \begin{pmatrix} 0.95 & 0.94 & 0.29 & 0.01 \\ 0.05 & 0.06 & 0.71 & 0.999 \end{pmatrix} \begin{pmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \end{pmatrix} = \begin{pmatrix} 0.5453 \\ 0.4548 \end{pmatrix}$$

MATLAB Script:

```
%Burglary/Earthquake Example
PiB=[0.5 0.5]';
PiE=[0.5 0.5]';
PBEdB=[0.5 0.5 0 0; 0 0 0.5 0.5];
PBEdE=[0.5 0 0.5 0; 0 0.5 0 0.5];
PAdBE=[0.95 0.05; 0.94 0.06; 0.29 0.71; 0.001 0.999];
PJdA=[0.9 0.1; 0.05 0.95];
PMdA=[0.7 0.3; 0.01 0.99];
%input
bJ=[1 0]';
bM=[0.5 0.5]'; %Uniform
%mesaage propagation
fB=PiB;
fE=PiE;
fBE1=PBEdB'*fB;  %non normalized
fBE2=PBEdE'*fE;  %non normalized
fBE3=fBE1 .* fBE2; %non normalized
fA1=PAdBE'*fBE3; %non normalized
bA2=PJdA*bJ; % "
fA3=fA1 .* bA2;
fM=PMdA'*fA3;
pM=fM .* bM
%normalize
pM=pM/sum(pM)
```

$p_M = \begin{pmatrix} 0.6694 \\ 0.3306 \end{pmatrix}$ It is more likely that Mary receives a call