Lezione 2

venerdì 24 settembre 2021 08.53

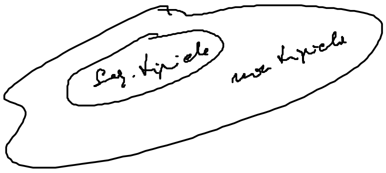
$\boxed{S} - S[k] \quad \mathcal{A} = \{a_1, a_2 \dots a_M\}$

$\Pi = \{p_1, p_2 \dots p_M\}$

$$H(\Pi) = \sum_{i=1}^{M} p_i \log_2 \frac{1}{p_i} \quad \text{bit.}$$

$S[1]\ S[2]\ S[3]\ S[4] \dots\dots\dots S[m]$

$$H(\Pi) \le \log_2 M$$



AEP

$\#$ seq. tipiche $\simeq 2^{mH(\Pi)}$

$P_e\{\tau_\varepsilon^m\} \simeq 1$

seq. tipiche $\simeq$ equiprobabili.

$\{0, 1\}$

$\boxed{S} - S[k]$

$\underbrace{x\,x\,x\,x\,x}$ → efficiente con un codice a lunghezza variabile

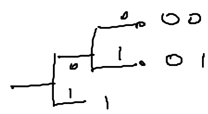
$p_1 \quad a_1 \longrightarrow 00..$

$p_i \quad \vdots \longrightarrow \underbrace{01, \dots}_{l_i}$

$p_M \quad a_M \longrightarrow 1 1 1 0 0$

$$\log_2 \frac{1}{p_i} < l_i < \log_2 \frac{1}{p_i} + 1$$

$1|0\ 0|1|1|1|0\ 0|0\ 1|1|1|1|1|1|1|$



$$\sum_{i=1}^{M} 2^{-l_i} < 1$$

DIS DI KRAFT-McMILLAN

soddisfa la dis. di Kraft-M.M.

$\mathcal{A}^N = \underbrace{\mathcal{A} \times \mathcal{A} \times \mathcal{A} \dots \times \mathcal{A}}_{N}$

$\Pi^N = \Pi \otimes \Pi \otimes \dots \otimes \Pi$

$\{p_1^N, p_2^N, \dots\dots p_{M^N}^N\}$

$$p_i^N \log_2 \frac{1}{p_i^N} < p_i l_i^N < p_i^N \log_2 \frac{1}{p_i^N} + p_i$$

$$\underbrace{\sum_{i=1}^{M^N} \log_2 \frac{1}{p_i^N}}_{H(\Pi^N)} < \underbrace{\sum_{i=1}^{M^N} p_i l_i^N}_{\bar{L}_N} < \underbrace{\sum_{i=1}^{M^N} p_i^N \log_2 \frac{1}{p_i^N}}_{H(\Pi^N)} + \sum_{i=1}^{M^N} p_i$$

$\nearrow 1$

$$N H(\Pi) \le \bar{L}_N \le N H(\Pi^N) + 1$$

$$H(\Pi) \le \frac{\bar{L}_N}{N} \le H(\Pi) + \frac{1}{N}$$

$$N \to \infty \qquad \frac{\bar{L}_N}{N} \to H(\pi)$$

$\mathcal{A} = \{a_1, a_2, a_3\}$ 00 01 10

$\pi = \{0.1, 0.2, 0.7\}$

$B_2 = \log_2 3 = 1.58$ bit.

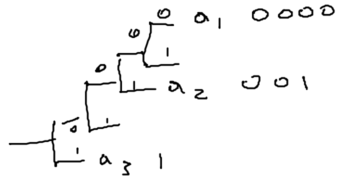
$H(\pi) = 1.1568$

$N = 1$

$$l_1 = \left\lceil \log_2 \frac{1}{p_1} \right\rceil = 4$$

$$l_2 = \left\lceil \log_2 \frac{1}{p_2} \right\rceil = 3$$

$$l_3 = \left\lceil \log_2 \frac{1}{p_3} \right\rceil = 1$$



$\bar{L}_1 \doteq 0.1\, l_1 + 0.2\, l_2 + 0.3\, l_3 \simeq 1.7$ bit

$N = 2$

$\mathcal{A}^2 = \mathcal{A} \times \mathcal{A} = \{a_1a_1, a_1a_2, a_1a_3, a_2a_1, a_2a_2, a_2a_3, a_3a_1, a_3a_2, a_3a_3\}$

$n^N = 3^2 = 9$

$\pi = \pi \otimes \pi =$ p2 =

0.0100 0.0200 0.0700 0.0200 0.0400 0.1400 0.0700 0.1400 0.4900

```
l2 =

   7   6   4   6   5   3   4   3   2

>> Lm2=p2*l2'

Lm2 =

   2.8900
```

$\bar{L}_2 = 2.89$

$\frac{\bar{L}_2}{2} = 1.4450$

```
p3=kron(p,p2)

p3 =

 Columns 1 through 17

  0.0010  0.0020  0.0070  0.0020  0.0040  0.0140  0.0070  0.0140
0.0490  0.0020  0.0040  0.0140  0.0040  0.0080  0.0280  0.0140
0.0280

 Columns 18 through 27

  0.0980  0.0070  0.0140  0.0490  0.0140  0.0280  0.0980  0.0490
0.0980  0.3430

>> l3=ceil(log2( 1 ./ p3))

l3 =

  10   9   8   9   8   7   8   7   5   9   8   7   8   7   6   7   6   4
8   7   5   7   6   4   5   4   2

>> Lm3=(1/3)* p3*l3'

Lm3 =

  1.3577
```

$\frac{\bar{L}_3}{3} = 1.3577$

CONVERGENZA AL CODICE EFFICIENTE PIÙ RAPIDA.

ALG. DI HUFFMAN

$\mathcal{A} = \{a_1, a_2, a_3\}$

$\pi = \{0.1, 0.2, 0.7\}$

00 a_1 0.1

01 a_2 0.2

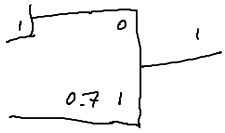
$\pi = \{0.1, 0.2, 0.7\}$

$B_z = \log_2 3 = 1.58$ bit.

$H(\pi) = 1.1568$

0 1 a_2 0.2

1 a_3 0.7



$\underline{l} = \{2, 2, 1\}$

$\overline{L} = 0.1 \cdot 2 + 0.2 \cdot 2 + 0.7 \cdot 1 = 1.3$ bit.

$A^2 = A \times A = \{ \overset{a_1^2}{a_1 a_1}, \overset{a_2^2}{a_1 a_2}, \overset{a_3^2}{a_1 a_3}, \overset{a_4^2}{a_2 a_1}, \overset{a_5^2}{a_2 a_2}, \overset{a_6^2}{a_2 a_3}, \overset{a_7^2}{a_3 a_1}, \overset{a_8^2}{a_3 a_2}, \overset{a_9^2}{a_3 a_3} \}$

$\pi = \pi \otimes \pi =$ p2 =

0.0100 0.0200 0.0700 0.0200 0.0400 0.1400 0.0700 0.1400 0.4900

$n^N = 3^2 = 9$

000110 a_1^2 0.01

000111 a_2^2 0.02

00010 a_4^2 0.02

0000 a_5^2 0.04

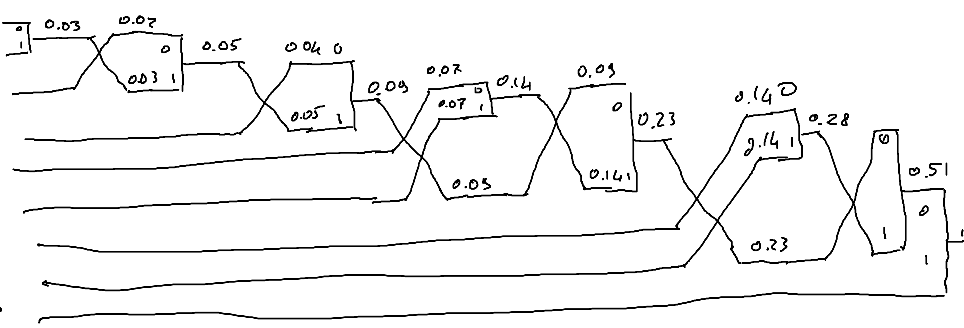
0010 a_7^2 0.07

0011 a_3^2 0.07

010 a_8^2 0.14

011 a_6^2 0.14

1 a_9^2 0.49



$l_2 = \{6, 6, 4, 5, 4, 3, 4, 3, 1\}$

```
l2=[6 6 4 5 4 3 4 3 1]

l2 =

   6   6   4   5   4   3   4   3   1

>> Lm2=(1/2)* p2*l2'

Lm2 =

   1.1650
```

$\frac{\overline{L}_2}{2} = 1,1650$

ABC Codice ternario

$A = \{a_1, a_2 \dots a_n\}$

$\pi = \{p_1 p_2 \dots p_n\}$

$H(\pi)$

$\frac{\overline{L}_N}{\log_2 3 \, N} \longrightarrow H(\pi)$

Es. $A = \{a_1, a_2, a_3, a_4, a_5, a_6\}$

$\pi: \{0.1, 0.2, 0.3, 0.35, 0.01, 0.04\}$

BAA a_5 0.01

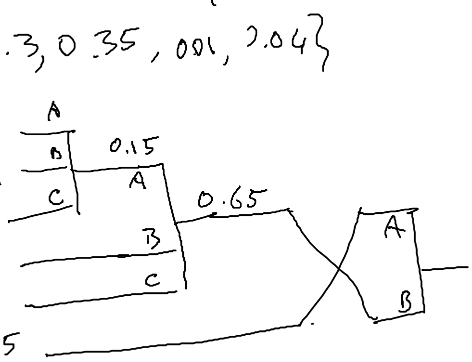
BAB a_6 0.04

BAC a_1 0.1

BB a_2 0.2

BC a_3 0.3

A a_4 0.35



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SORGENTI A SIMBOLI DIPENDENTI

$S[K]$

$\ldots S[K-3]\ S[K-2]\ \underbrace{S[K-1]\ S[K]\ S[K+1]\ S[K+2]}\ \ldots\ldots$

$$H(S^N) \neq \cancel{N\,H(S)}$$

ENTROPY RATE
TASSO ENTROPICO

$$\lim_{N\to\infty} \frac{H(S^N)}{N} = H(S)$$

$$x\ x\ x\ x\ x\ \underbrace{x\ x\ x\ x}_{N}\ x\ x\ x$$

$$p_i^N \log_2 \frac{1}{p_i^N} < p_i^N l_i^N < p_i^N \log_2 \frac{1}{p_i^N} + p_i^N$$

$$\Sigma \downarrow$$

$$H(\Pi^N) \le \overline{L}_N \le H(\Pi^N) + 1$$

$$\frac{H(\Pi^N)}{N} \le \frac{\overline{L}_N}{N} \le \frac{H(\Pi^N)}{N} + \frac{1}{N}$$

$$N \to \infty \quad \frac{\overline{L}_N}{N} \to H(S)$$

HUFFMAN
pro : Efficiente
contro : codebook