Lezioni di Telecomunicazioni
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THE MATRIX INVERSION LEMMA (LIM)

let A and B be two positive definite $M \times M$ matrices related by the equation

$$\underset{M\times M}{A} = \underset{M\times M}{B^{-1}} + \underset{M\times N}{C}\,\underset{N\times N}{D^{-1}}\,\underset{N\times M}{C^T}$$

Where D is another positive $N \times N$ matrix, and C is $M \times N$.
We can express the inverse of A as

$$A^{-1} = B - BC(D + C^TBC)^{-1}C^TB$$

The proof is straightforward showing that $A^{-1}A \overset{?}{=} I_M$

[Haykin, 1990 pp 480]. A derivation and alternative expressions can be found in
[TYLAVSKY & SOHIE, 1986]

Prova

$$A^{-1}A = (B - BC(D+C^TBC)^{-1}C^TB)(B^{-1}+CD^{-1}C^T)$$

$$= BB^{-1} + BCD^{-1}C^T - BC(D+C^TBC)^{-1}C^T\cancel{BB^{-1}} - BC(D+CBC)^{-1}C^TBCD^{-1}C^T$$

$$= I_M + BCD^{-1}C^T - BC(D+C^TBC)^{-1}[C^T + C^TBCD^{-1}C^T]$$

$$= I_M + BCD^{-1}C^T - BC(D+C^TBC)^{-1}[I + C^TBCD^{-1}]C^T$$

$$= I_M + \underbrace{BCD^{-1}C^T - BC\cancel{(D+C^TBC)^{-1}}\cancel{(D+C^TBC)}D^{-1}C^T}_{0} \overset{?}{=} I_M$$