

ESERCIZIO DI PROGETTAZIONE
DI FILTRO NUMERICO

Lezione di Telecomunicazioni
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CORSO di TRASMISSIONE ED ELABORAZIONE
NUMERICA DEI SEGNALI - SUN
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ESEMPIO

E1

Consideriamo un filto analogico con caratteristiche passa-banda con due poli complessi coniugati a frequenza $f_0 = 3 \text{ [kHz]}$

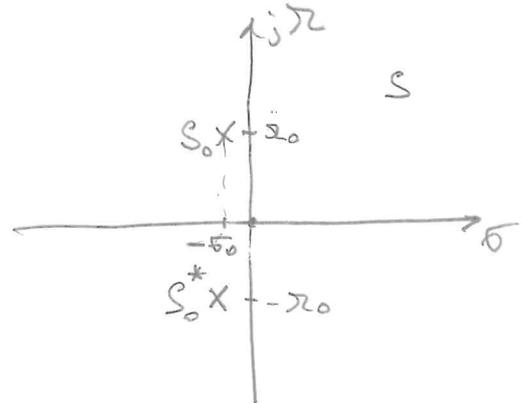
$$\omega_0 = 2\pi f_0 = 18850 \left[\frac{\text{rad}}{\text{sec}} \right],$$

$$\sigma_0 = 100 \text{ [sec}^{-1}\text{]}$$

$$H_A(s) = \frac{|s_0|^2}{(s - s_0)(s - s_0^*)}$$

$$= \frac{|s_0|^2}{s^2 - s s_0 - s s_0^* + |s_0|^2}$$

$$= \frac{|s_0|^2}{s^2 - s(s_0 + s_0^*) + |s_0|^2} = \frac{\beta_0^2}{s^2 + 2\sigma_0 s + \beta_0^2}$$



$$s_0 = -\sigma_0 + j\omega_0$$

$$\beta_0^2 = \sigma_0^2 + \omega_0^2$$

$$\beta_0 = 1.8850 \cdot 10^4$$

La risposta armonica è

$$H_A(\omega) = H_A(s) \Big|_{s=j\omega} = \frac{\beta_0^2}{-\omega^2 + j2\sigma_0\omega + \beta_0^2}$$

Modulo e fase possono essere graficati in MATLAB usando la funzione freqs

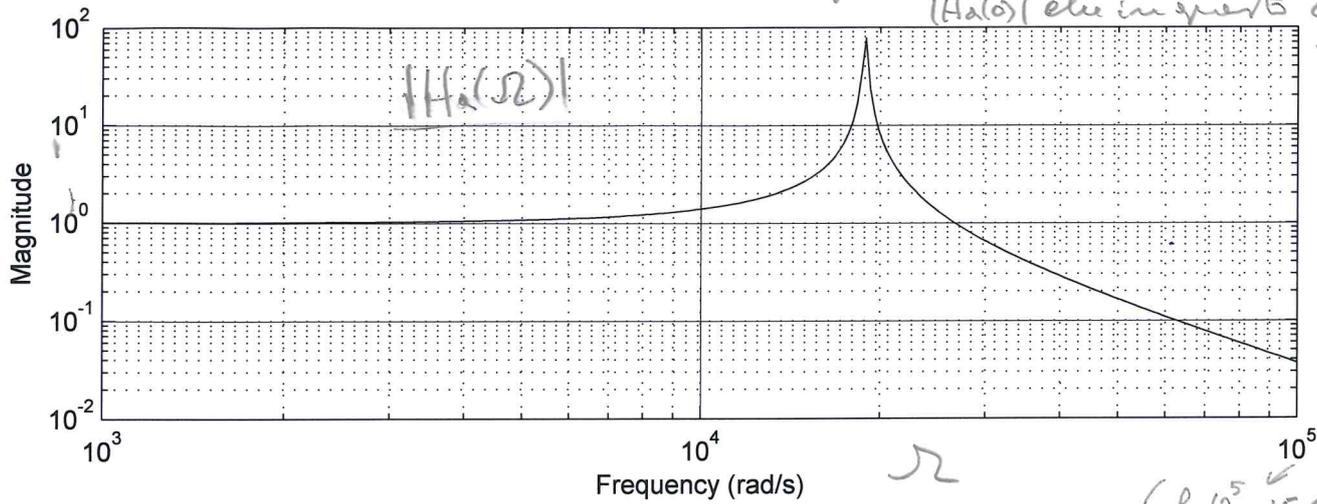
```
f0 = 3000
omega0 = 2*pi*f0
sigma0 = 100
rho = sqrt(sigma0^2 + omega0^2)
a = [1 -2*sigma0 rho^2]
b = [rho^2]
figure(1)
freqs(b,a)
```

MATLAB
SCRIPT

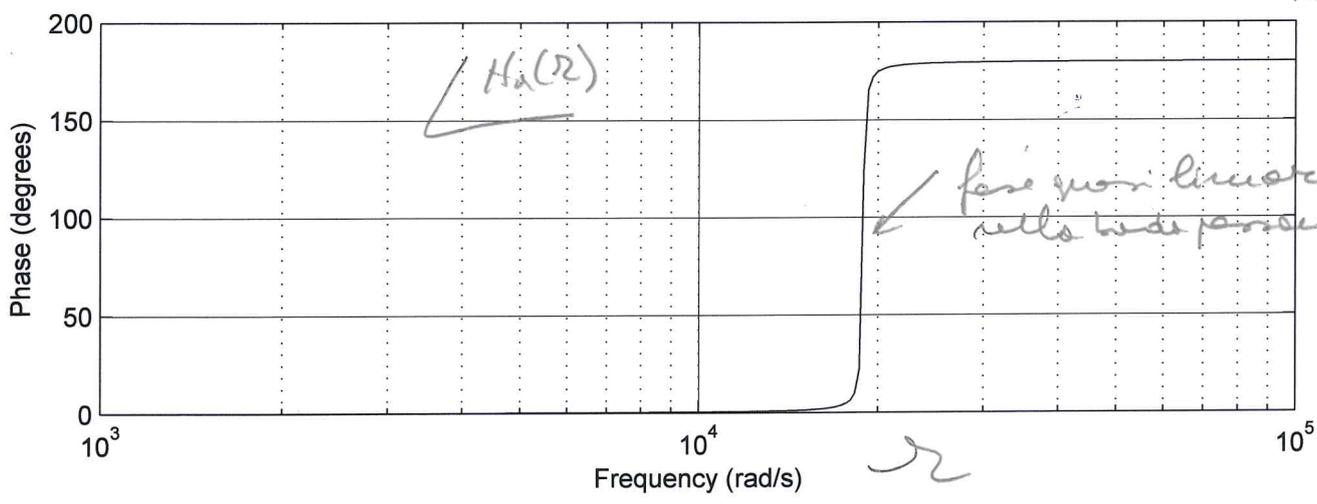
RISPOSTA DEL
FILTRO ANALOGICO
 $H_a(s)$

```
f0 = 3000
omega0 = 1.8850e+04
sigma0 = 100
rho = 1.8850e+04
a = 1.0e+08 *
0.0000 -0.0000 3.5532
b =
3.5532e+08
```

ω è in scala logaritmica
 $|H_a(\omega)|$ è anch'essa in scala
logaritmica normalizzata a
 $|H_a(0)|$ che in questo caso
è 1



$f = \frac{\omega}{2\pi} = 15.915 \text{ kHz}$

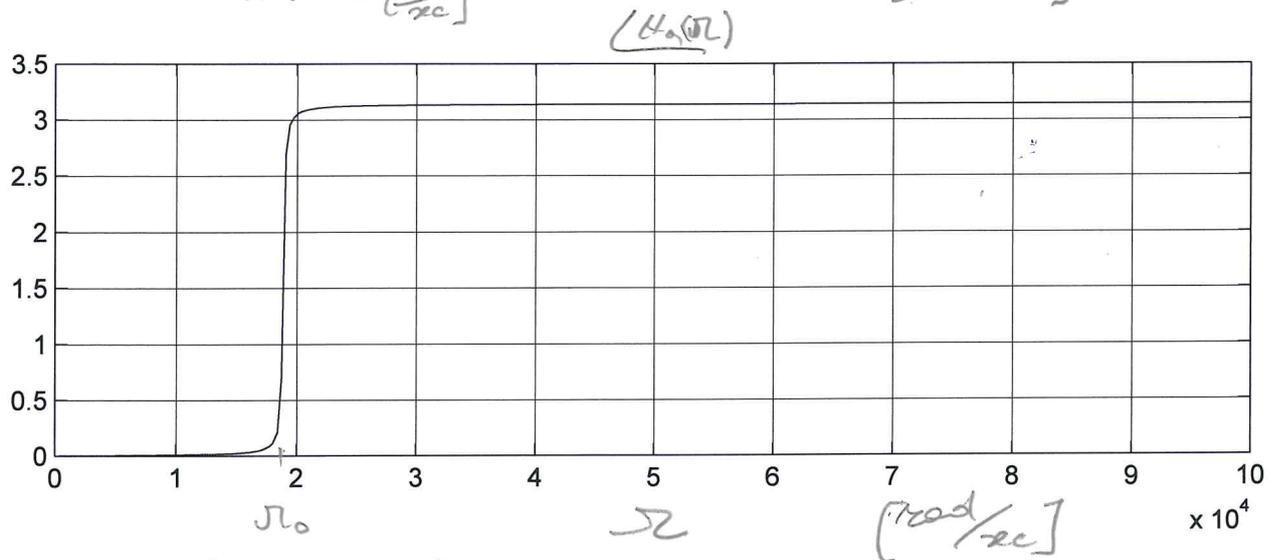
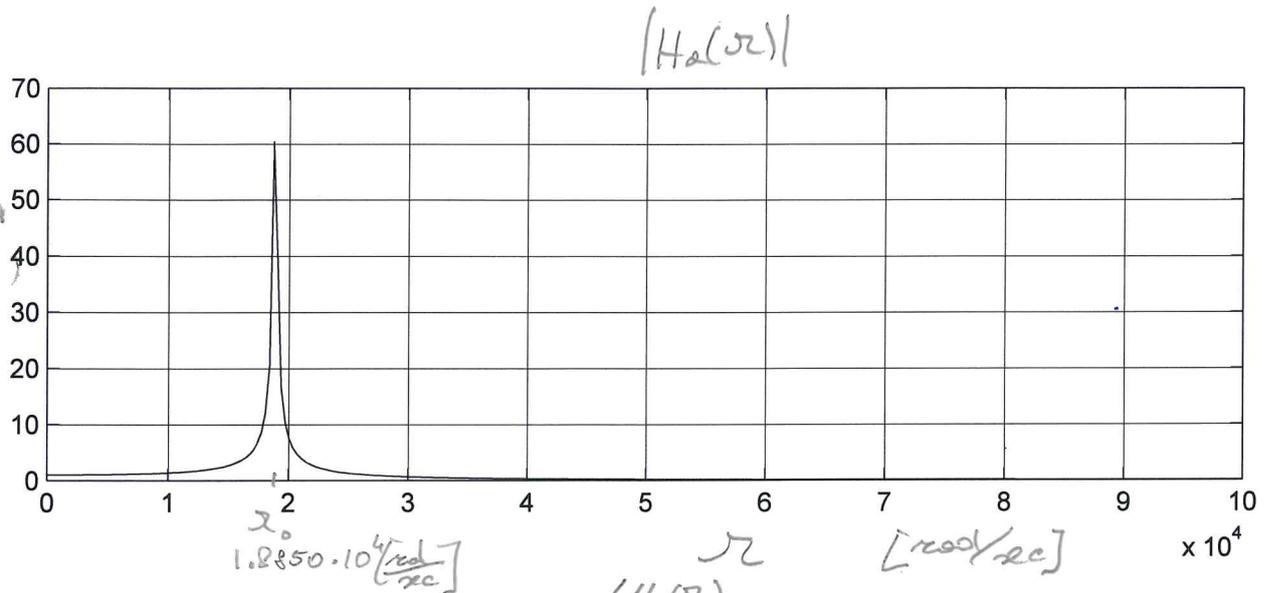


fase non lineare
nella banda passante

È utile vedere lo stesso risposta anche in scala lineare, si noti che il guadagno statico $H_a(0)$ da zero è stata calcolata $H_a(s)$ è uno.

```

om=linspace(0,100000,300); %generate 300 point on the omega axis from 0 to 10^5
H=freqs(b,a,om); % get the complex response
figure(2)
subplot(2,1,1); plot(om,abs(H)); grid on
subplot(2,1,2); plot(om, phase(H)); grid on
    
```



La scala logaritmica nelle ampiezze mostra meglio il comportamento nelle code.

Dai grafici si vede lo scatto centrato sulle frequenze oltre gli 8kHz circa.

Scegliamo una frequenza di campionamento

E4

$$\frac{1}{T} = 20 \text{ KHz} \quad T = 0.5 \cdot 10^{-6} \text{ sec} = 0.5 \mu\text{sec}$$

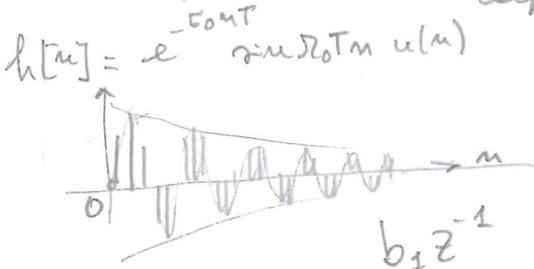
Per applicare il metodo dell'invarianza all'impulso risolviamo $H_a(s)$ alla formula 206 della tabella

$$H_a(s) = \frac{\beta_0^2}{s^2 + 2\zeta_0 s + \beta_0^2} = \frac{\beta_0^2}{s^2 + 2\zeta_0 s + \beta_0^2 - \beta_0^2 + \beta_0^2} = \frac{\beta_0^2 + \beta_0^2}{(s + \zeta_0)^2 + \beta_0^2} \frac{\beta_0}{\beta_0}$$

$$= \frac{\beta_0^2 + \beta_0^2}{\beta_0} \frac{\beta_0}{(s + \zeta_0)^2 + \beta_0^2} \leftrightarrow h_a(t) = \frac{\beta_0^2 + \beta_0^2}{\beta_0} e^{-\zeta_0 t} \sin \beta_0 t \text{ u(t)}$$

Rimuovi il coefficiente $\frac{\beta_0^2 + \beta_0^2}{\beta_0}$

Dopo campionamento e Z-transformata, dalla tabella otteniamo



$$H(z) = \frac{z e^{-\zeta_0 T} \sin \beta_0 T}{z^2 - 2z e^{-\zeta_0 T} \cos \beta_0 T + e^{-2\zeta_0 T}} = \frac{b_1 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

$a_1 = 2e^{-\zeta_0 T} \cos \beta_0 T$ $a_2 = e^{-2\zeta_0 T}$

poli: $\left\{ \begin{array}{l} p_1 = e^{-\zeta_0 T} e^{j\beta_0 T} \\ p_2 = p_1^* \end{array} \right.$
 zero in $z=0$

$$y[n] = a_1 y[n-1] + a_2 y[n-2] + b_0 x[n-1]$$

Per valutare le caratteristiche del filtro numerico risolviamo in MATLAB con uno script che valuta modulo e fase di $H(\nu) = H(z)|_{z=e^{j2\pi\nu}}$, i poli e gli zeri e la risposta impulsiva $h[n] = z^{-1}[H(z)]$.

```
%Metodo dell'invarianza all'impulso
T=1/20000; %sampling interval
B=[0 exp(-sigma*T)*sin(omega*T)]
A=[1 -2*exp(-sigma*T)*cos(omega*T) exp(-2*sigma*T)]
```

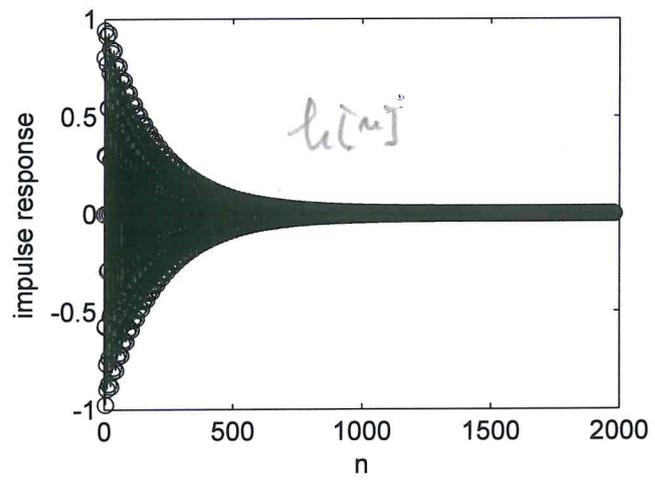
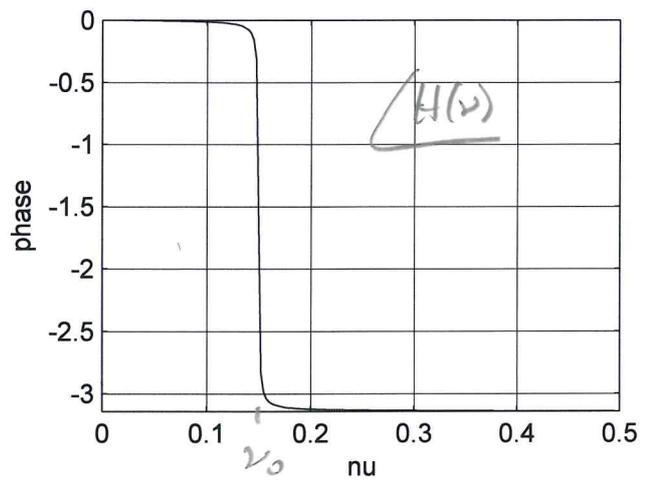
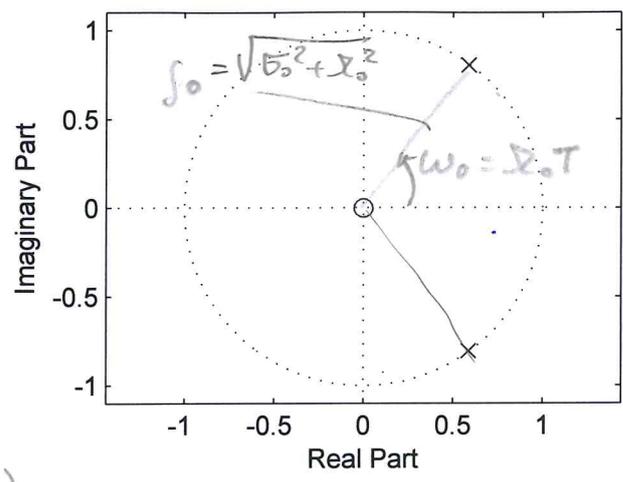
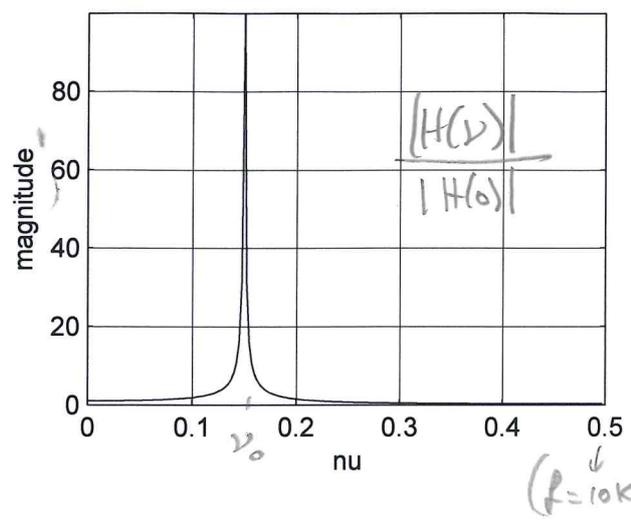
```

figure(3)
[H W]=freqz(B,A,200); %get frequency response (abscissa in rad)
[h t]=impz(B,A); %get impulse response
subplot(2,2,1)
plot(W/(2*pi),abs(H)/abs(H(1))) %plot magnitude (abscissa in norm freq.; magnitude mormalized to the value in 0)
ylabel('magnitude')
xlabel('nu')
axis([0 0.5 0 max(abs(H))])
grid on
subplot(2,2,3)
plot(W/(2*pi),phase(H)) %plot phase (abscissa in norm freq.)
ylabel('phase')
xlabel('nu')
axis([0 0.5 min(phase(H)) max(phase(H))])
grid on
subplot(2,2,2)
zplane(B,A) %compute and plot z plane
subplot(2,2,4)
stem(t,h) %plot impulse response
ylabel('impulse response')
xlabel('n')

```

(SCRIPT 1)

FILTRO NUMERICO OTTENUTO CON IL METODO DELL' INVARIANZA ALL'IMPULSO



$$\nu_0 = \frac{\omega_0}{2\pi} = 0.15$$

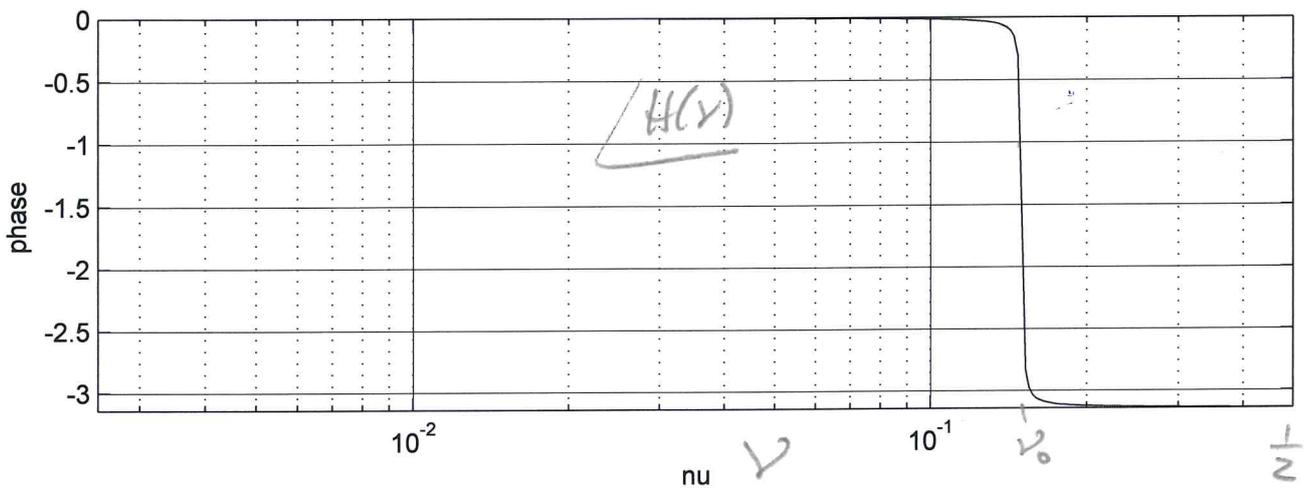
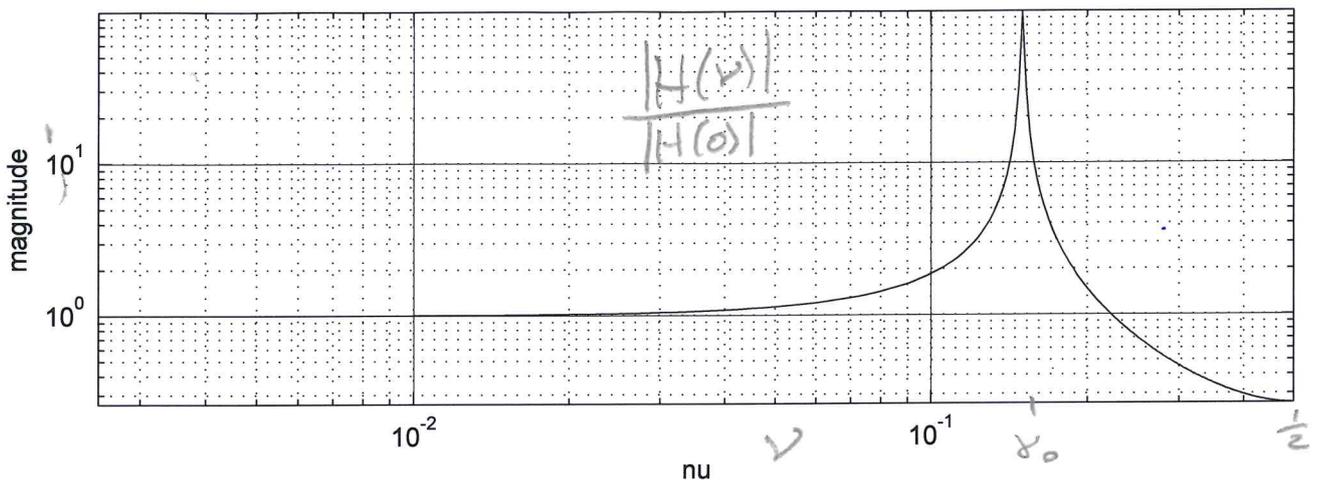
Subsequentemente riprova il grafico di $|H(\nu)|$ in scala loglog e quello di $\angle H(\nu)$ in semilogx

```

figure(4)
subplot(2,1,1)
loglog(W/(2*pi),abs(H)/abs(H(1))) %plot magnitude (abscissa in norm freq.)
ylabel('magnitude')
xlabel('nu')
axis([0 0.5 0 max(abs(H))])
grid on
subplot(2,1,2)
semilogx(W/(2*pi),phase(H)) %plot phase (abscissa in norm freq.)
ylabel('phase')
xlabel('nu')
axis([0 0.5 min(phase(H)) max(phase(H))])
grid on

```

(SCRIPT 2)



Per applicare la formula dell'invarianza al gradino deduciamo $\frac{H_0(s)}{s}$ alla formula 31 della tabella

$$\frac{H_0(s)}{s} = \frac{\sigma_0^2 + \omega_0^2}{s[(s + \sigma_0)^2 + \omega_0^2]} \xleftrightarrow{y^{-1}} \xleftrightarrow{z} \frac{z}{z-1} - \frac{z^2 - z e^{-\sigma_0 T} \cos(\omega_0 T - \theta)}{z^2 - 2\beta z + \gamma}$$

$\theta = \arctan\left(-\frac{\sigma_0}{\omega_0}\right)$

$$\xleftrightarrow{z} H(z) = 1 - \frac{z-1}{z} \frac{z^2 - \alpha z}{z^2 - 2\beta z + \gamma}$$

$$= \frac{z^2 - 2\beta z + \gamma - (z^2 - z\alpha - z + \alpha)}{z^2 - 2\beta z + \gamma} = \frac{z^2 - 2\beta z + \gamma - z^2 + \alpha z + z - \alpha}{z^2 - 2\beta z + \gamma}$$

$$= \frac{(\alpha + 1 - 2\beta)z - \alpha}{z^2 - 2\beta z + \gamma} = (\alpha + 1 - 2\beta) \frac{z - \frac{\alpha}{\alpha + 1 - 2\beta}}{z^2 - 2\beta z + \gamma}$$

$$= \frac{(\alpha + 1 - 2\beta)z^{-1} - \alpha z^{-2}}{1 - 2\beta z^{-1} + \gamma z^{-2}}$$

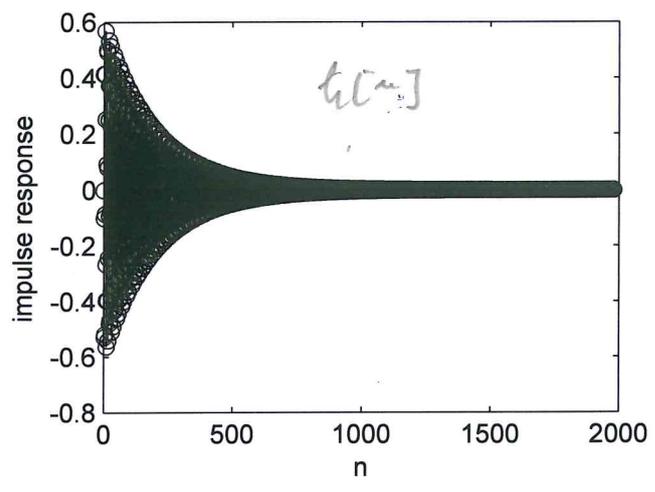
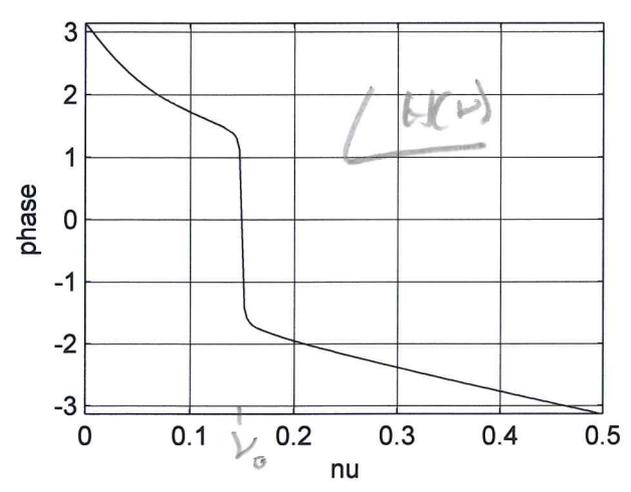
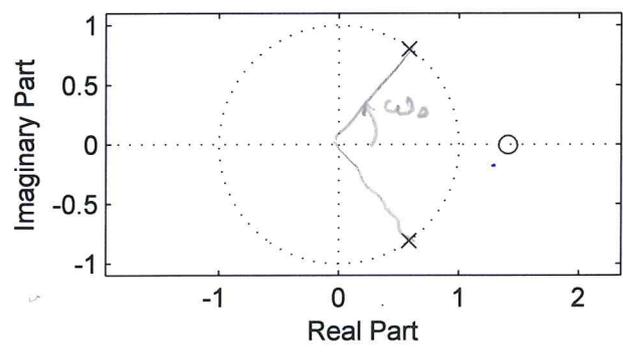
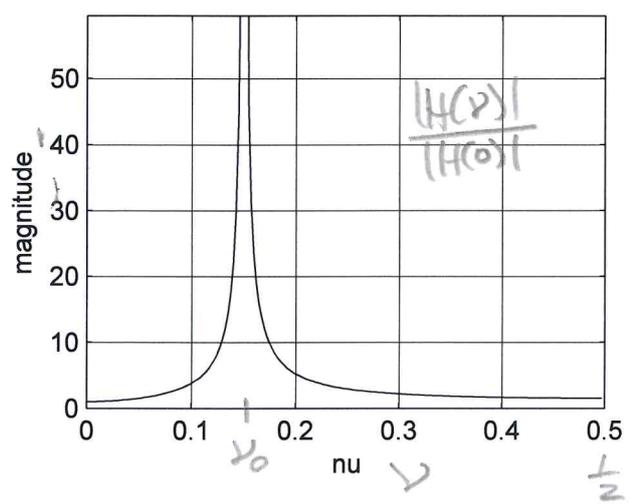
2 poli $\left\{ \begin{array}{l} e^{-\sigma_0 T} e^{j\omega_0 T} \\ e^{-\sigma_0 T} e^{-j\omega_0 T} \end{array} \right.$
 1 zero reale $\frac{\alpha}{\alpha + 1 - 2\beta}$

$$y[n] = 2\beta y[n-1] + \gamma y[n-2] + (\alpha + 1 - 2\beta)x[n] - \alpha x[n-2]$$

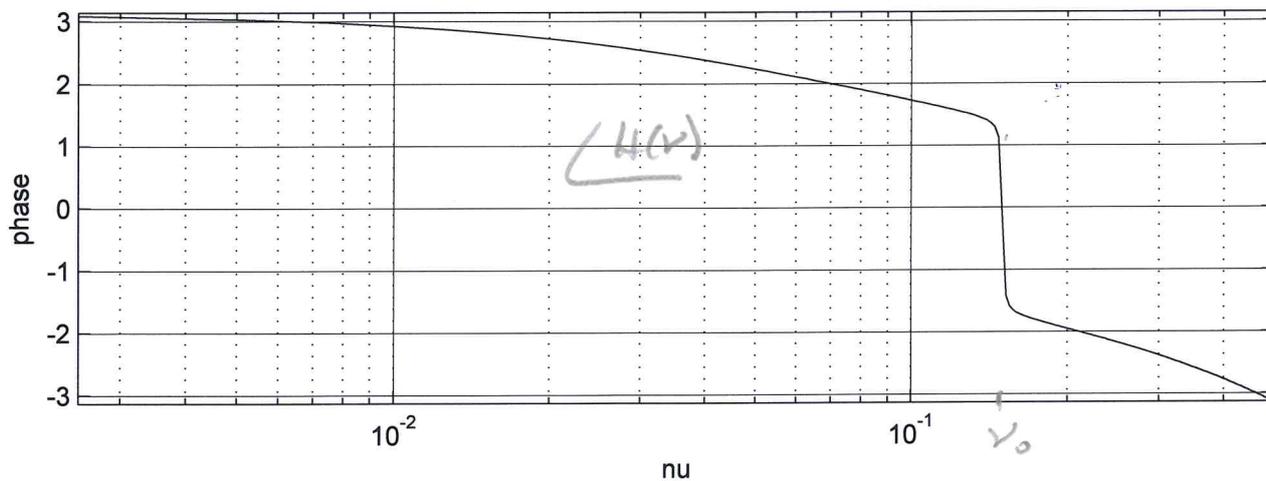
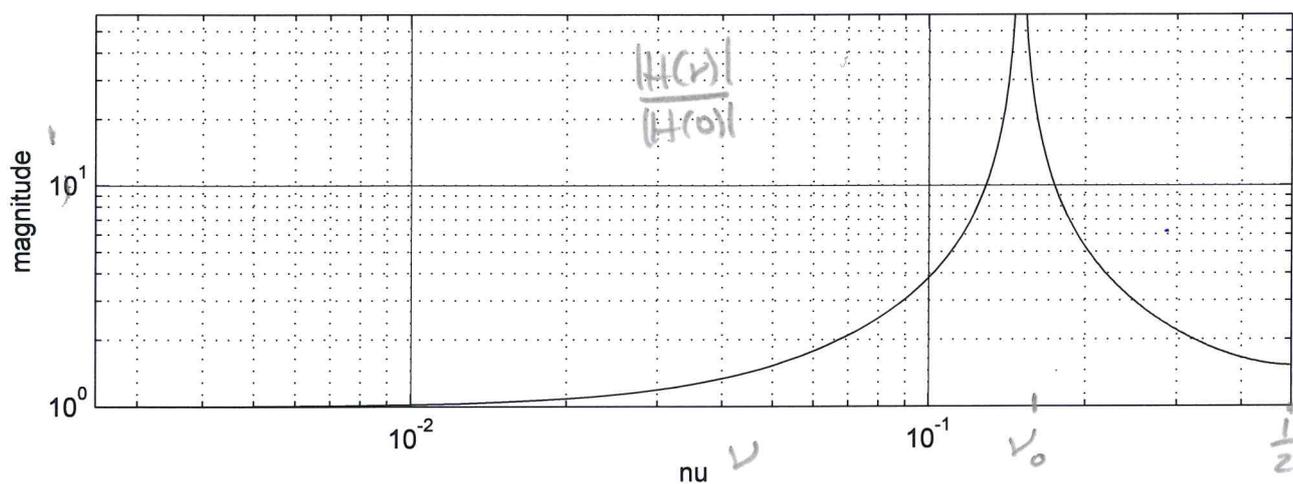
```
%metodo dell'invarianza al gradino
theta=atan(- sigma0 / omega0)
alpha=exp(-sigma0*T)*sec(theta)*cos(omega0*T - theta)
beta=exp(-sigma0*T)*cos(omega0*T)
gamma=exp(-2*sigma0*T)
A=[1 -2*beta gamma]
B=[0 alpha+1-2*beta -alpha]
```

gli stessi script $\sqrt{1e2}$ delle pagine precedenti sono utilizzati per ottenere i plot che seguono.

FILTRO NUMERICO RESULTANTE DALLA TECNICA DELL'INVARIANZA AL GRADINO



(INVARIANZA AL GRADINO)



Per applicare la formula dell'invarianza allorompa E10
 risolviamo $\frac{H_A(s)}{s^2}$ alla formula 407 della tabella

$$\frac{H_A(s)}{s^2} = \frac{\sigma_0^2 + \lambda_0^2}{s^2 [(s + \sigma_0)^2 + \lambda_0^2]} = \frac{1}{\sigma_0^2 + \lambda_0^2} \frac{(\sigma_0^2 + \lambda_0^2)^2}{s^2 [(s + \sigma_0)^2 + \lambda_0^2]} \xleftarrow{y} x$$

$$\xleftarrow{z} \frac{[(\sigma_0^2 + \lambda_0^2)T + 2\sigma_0]z - 2\sigma_0 z^2}{(z-1)^2} + \frac{2\sigma_0 [z^2 - z e^{-\sigma_0 T} \cos \theta \cos(\lambda_0 T - \theta)]}{z^2 - 2z e^{-\sigma_0 T} \cos \lambda_0 T + e^{-2\sigma_0 T}}$$

$$\theta = \tan^{-1} \frac{\lambda_0^2 - \sigma_0^2}{2\sigma_0 \lambda_0}$$

$$H(z) = \frac{1}{\sigma_0^2 + \lambda_0^2} \frac{[(\sigma_0^2 + \lambda_0^2)T + 2\sigma_0]z - 2\sigma_0 z^2}{(z-1)^2}$$

$$+ \frac{(z-1)^2}{Tz} \frac{2\sigma_0 [z^2 - z e^{-\sigma_0 T} \cos \theta \cos(\lambda_0 T - \theta)]}{z^2 - 2z e^{-\sigma_0 T} \cos \lambda_0 T + e^{-2\sigma_0 T}}$$

$$= \frac{1}{\sigma_0^2 + \lambda_0^2} \left\{ (\sigma_0^2 + \lambda_0^2) + \frac{2\sigma_0}{T} - \frac{2\sigma_0}{T} z + \frac{2\sigma_0}{T} \frac{(z-1)^2}{z} \frac{z^2 - z e^{-\sigma_0 T} \cos \theta \cos(\lambda_0 T - \theta)}{z^2 - 2z e^{-\sigma_0 T} \cos \lambda_0 T + e^{-2\sigma_0 T}} \right\}$$

$$= 1 + \frac{2\sigma_0}{T(\sigma_0^2 + \lambda_0^2)} - \frac{2\sigma_0}{T(\sigma_0^2 + \lambda_0^2)} z + \frac{2\sigma_0}{T(\sigma_0^2 + \lambda_0^2)} \frac{(z-1)^2 (z-\alpha)}{z^2 - 2z\beta + \gamma}$$

$$= \frac{[(1+\delta) - \delta z](z^2 - 2z\beta + \gamma) + \delta(z^3 - \alpha z^2 + z - \alpha - 2z^2 + 2\alpha z)}{(z^2 - 2z\beta + \gamma)}$$

$$= \frac{(1+\delta)z^3 - 2\beta(1+\delta)z^2 + (1+\delta)\gamma - \delta z^3 + 2\delta\beta z - \delta\gamma z + \delta z^3 - \alpha\delta z^2 + \delta z - \alpha\delta - 2\delta z^2 + 2\alpha\delta z}{z^2 - 2z\beta + \gamma}$$

$$= \frac{(1+\delta - \alpha\delta - 2\delta)z^2 + (-2\beta(1+\delta) + 2\delta\beta - \delta\gamma + \delta + 2\alpha\delta)z + (1+\delta)\gamma - \alpha\delta}{z^2 - 2z\beta + \gamma}$$

$$= \frac{\frac{b_0}{(1-\delta-\alpha\delta)} z^2 + (-2\beta - \delta\gamma + \delta + 2\alpha\delta)z + (1+\delta)\gamma - \alpha\delta}{z^2 - 2z\beta + \gamma}$$

$$= \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

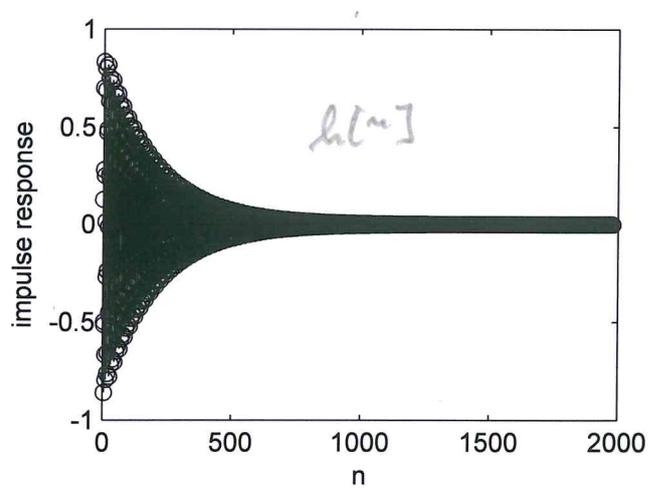
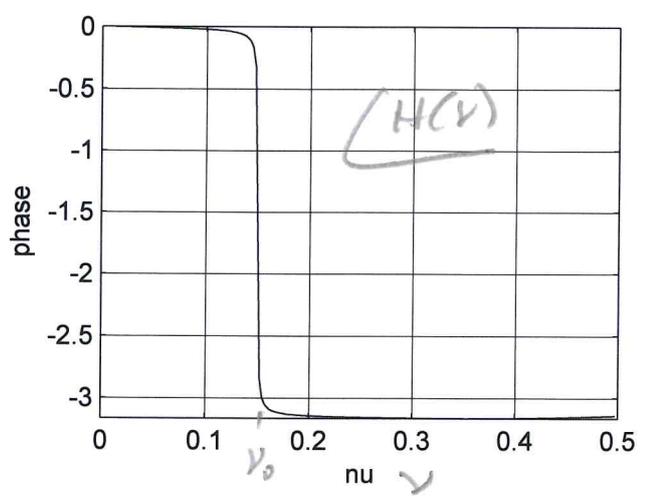
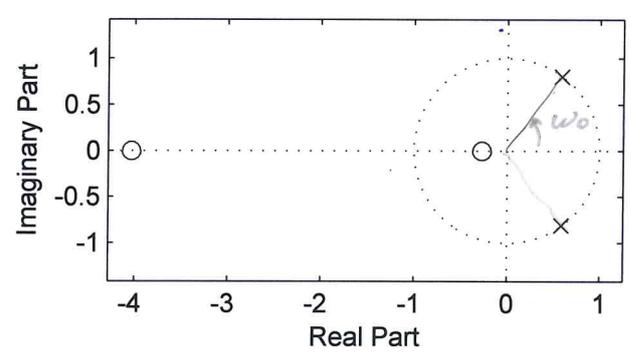
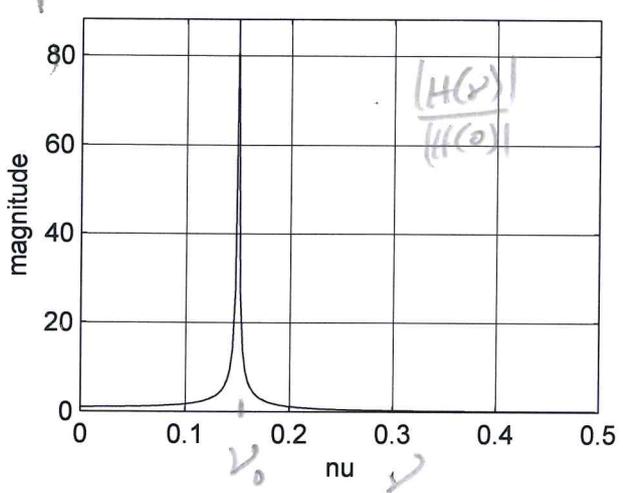
$$y[n] = -a_1 y[n-1] - a_2 y[n-2] + b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$$

$$z_{poli} \begin{cases} e^{-\gamma_0 T} e^{j\omega_0 T} \\ e^{-\gamma_0 T} e^{-j\omega_0 T} \end{cases}$$

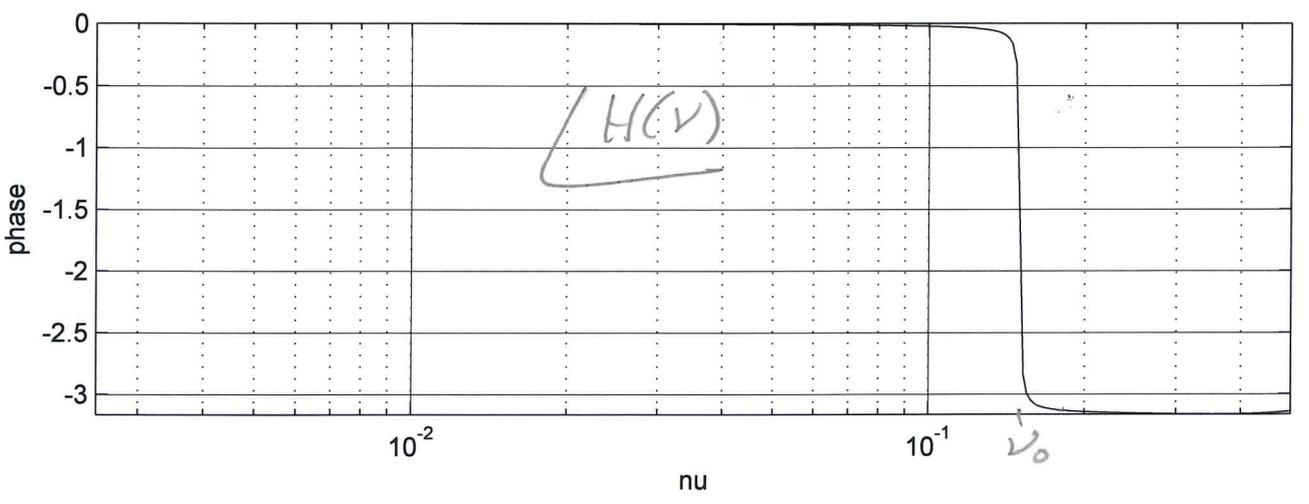
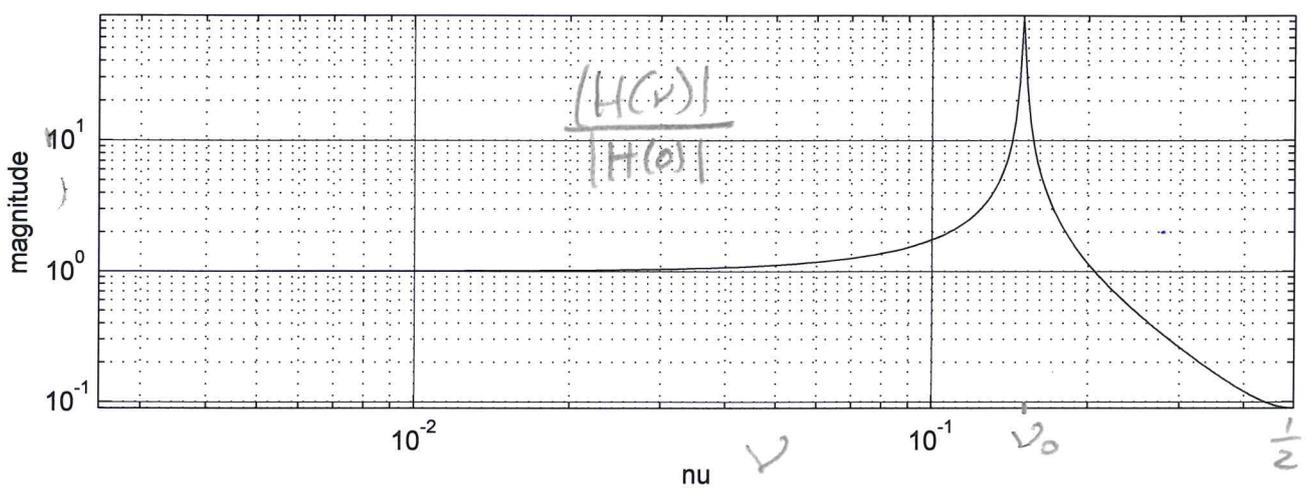
z_{zeri}

%medodo dell'invarianza alla rampa

```
theta=atan((omega0^2 - sigma0^2) / (2*omega0*sigma0))
alpha=exp(-sigma0*T)*sec(theta)*cos(omega0*T - theta)
beta=exp(-sigma0*T)*cos(omega0*T)
gamma=exp(-2*sigma0*T)
delta=(2*sigma0)/(T*(sigma0^2+omega0^2))
A=[1 -2*beta gamma]
B=[1-delta-alpha*delta -2*beta-delta*gamma+delta+2*alpha*delta (1+delta)*gamma-alpha*delta]
```



(inversione alle risonanze)



Metodo per approssimazione delle derivate (APPROSSIMAZIONE CAUSALE) E13

$$H(z) = H_a(s) \Big|_{s = \frac{1-z^{-1}}{T}} = \frac{\beta_0^2}{\left(\frac{z-1}{Tz}\right)^2 + 2\zeta_0\left(\frac{z-1}{Tz}\right) + \beta_0^2}$$

$$= \frac{\beta_0^2}{\frac{z^2 + 1 - 2z}{T^2 z^2} + \frac{2\zeta_0}{T} - \frac{2\zeta_0}{T} \frac{1}{z} + \beta_0^2} = \frac{\beta_0^2}{\frac{z^2 + 1 - 2z + 2\zeta_0 T z^2 - 2\zeta_0 T z + \beta_0^2 T^2 z^2}{T^2 z^2}}$$

$$= \frac{\beta_0^2 T^2 z^2}{(1 + 2\zeta_0 T + \beta_0^2 T^2) z^2 - 2(1 + \zeta_0 T) z + 1}$$

$$= \frac{\beta_0^2 T^2}{(1 + 2\zeta_0 T + \beta_0^2 T^2)} z^2 = \frac{b_0}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

$$y[m] = -a_1 y[m-1] - a_2 y[m-2] + b_0 x[m]$$

} 2 poli
} 2 zeri in zero

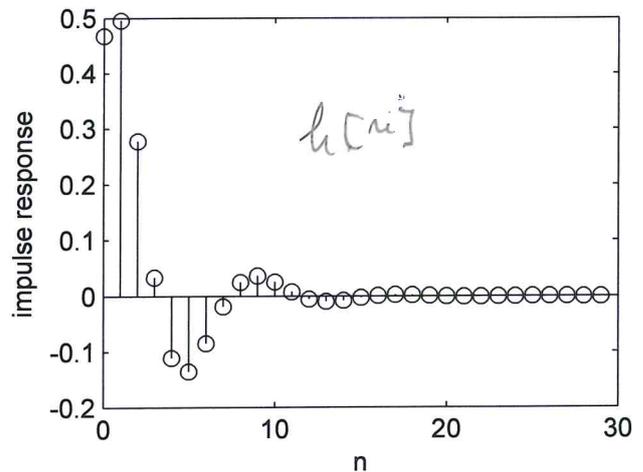
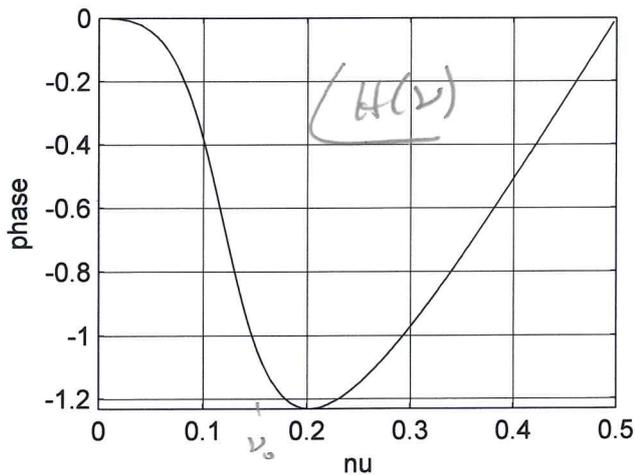
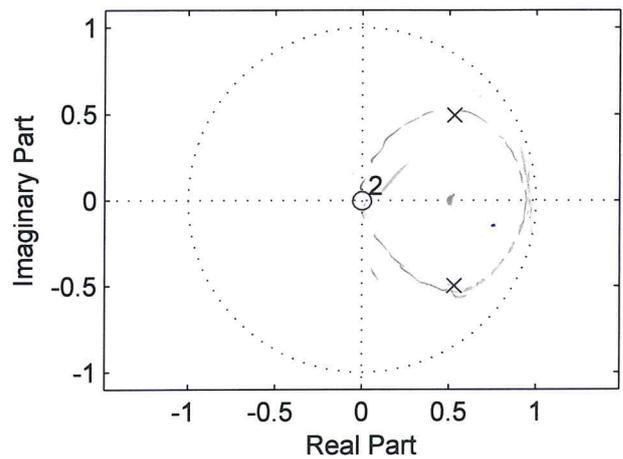
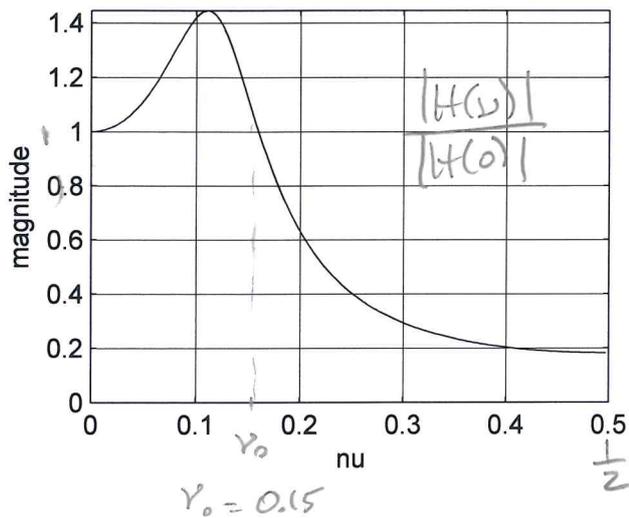
%Metodo della approssimazione delle derivate 1 (APPR. CAUSALE)

A = [1 - 2*(1+sigma0*T)/(1+2*sigma0*T+rho^2*T^2) 1/(1+2*sigma0*T+rho^2*T^2)]
 B = [(rho^2*T^2)/(1+2*sigma0*T+rho^2*T^2)]

A =
 1.0000 -1.0588 0.5268

B =
 0.4679

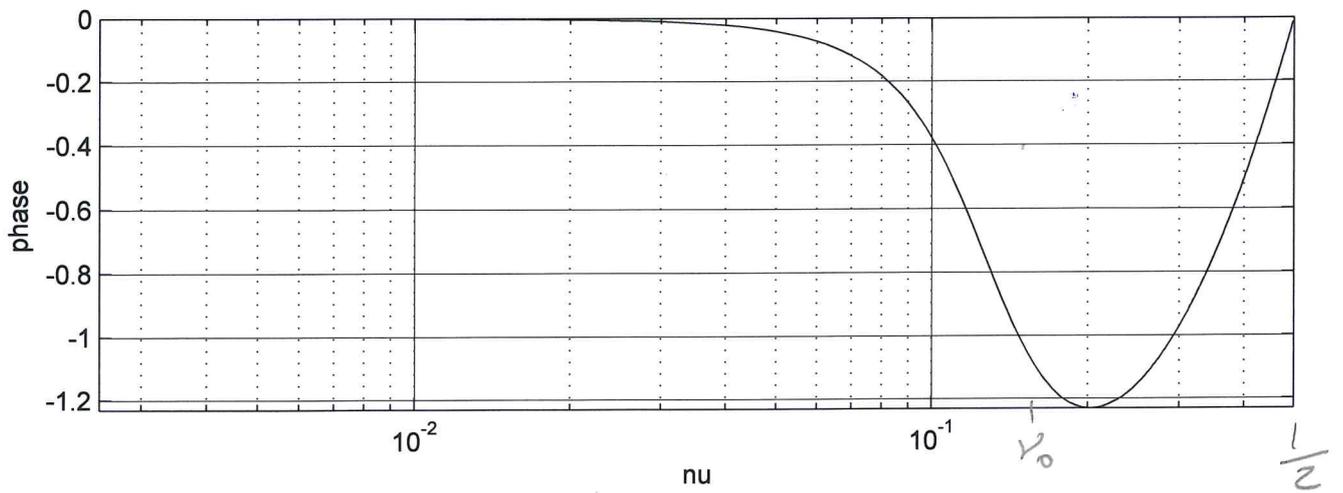
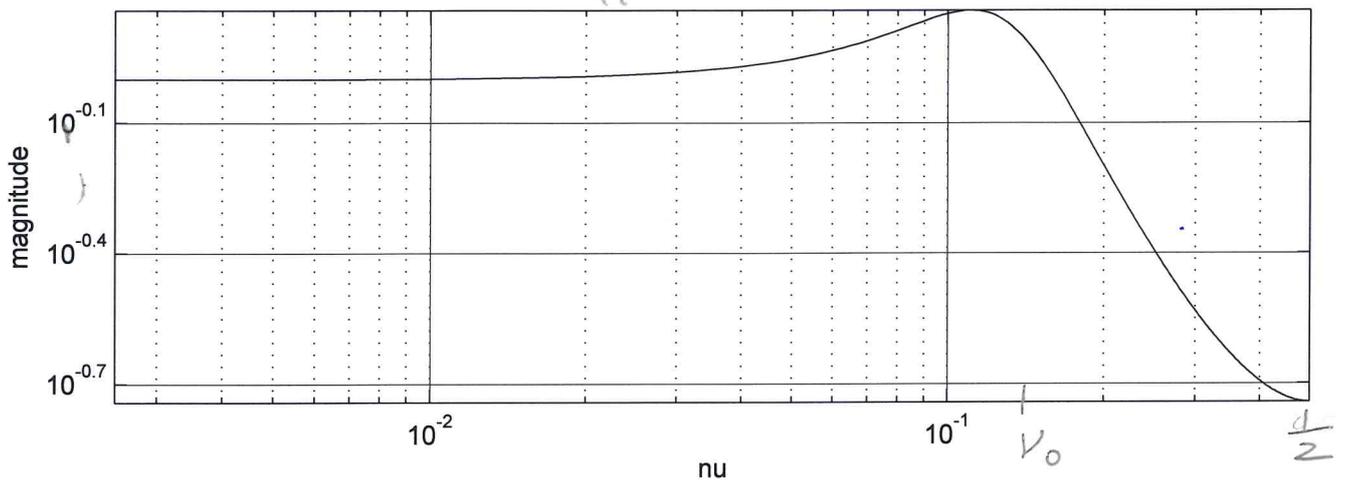
FILTRO NUMERICO DALLA APPROSSIMAZIONE DELLE DERIVATE (APPR. CAUSALE)



Filtro poco selettivo. I poli non possono essere troppo vicini al cerchio unitario a quella frequenza. Si tratta di una trasferimento più adatta a filtri passa-basso o esente a frequenze di campionamento molto elevate e quindi a caratteristiche passanti a basse frequenze.

App. derivate 1

$$\frac{|H(\nu)|}{|H(0)|}$$



Metodo per approssimazione delle derivate 2 (App. automatica)

E16

$$H(z) = \bar{H}_A(s) \Big|_{s = \frac{z-1}{T}} = \frac{\beta_0^2}{\left(\frac{z-1}{T}\right)^2 + 2\zeta\omega_0 \left(\frac{z-1}{T}\right) + \omega_0^2}$$

$$= \frac{\beta_0^2}{\frac{z^2 + 1 - 2z}{T^2} + \frac{2\zeta\omega_0 z - 2\zeta\omega_0}{T} + \omega_0^2} = \frac{T^2 \beta_0^2}{z^2 + 1 - 2z + 2\zeta\omega_0 T z - 2\zeta\omega_0 T + \omega_0^2 T^2}$$

$$= \frac{\omega_0^2 z^2 - 2(1 - \zeta\omega_0 T)z - 2\zeta\omega_0 T + \omega_0^2 T^2}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

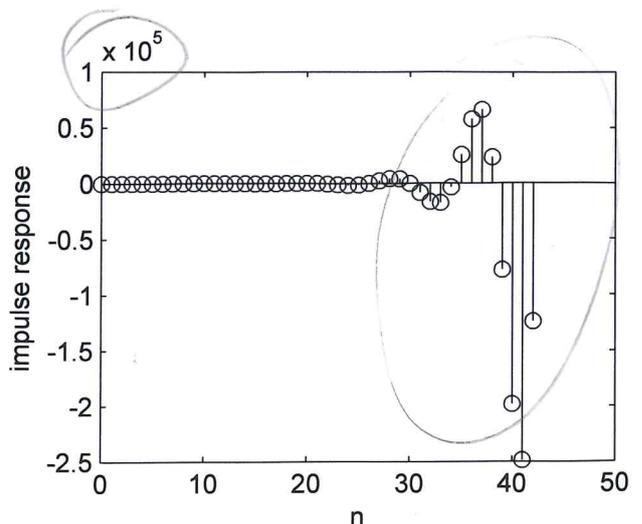
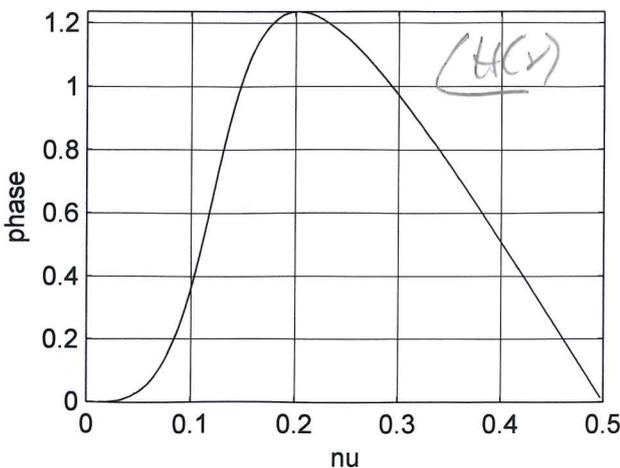
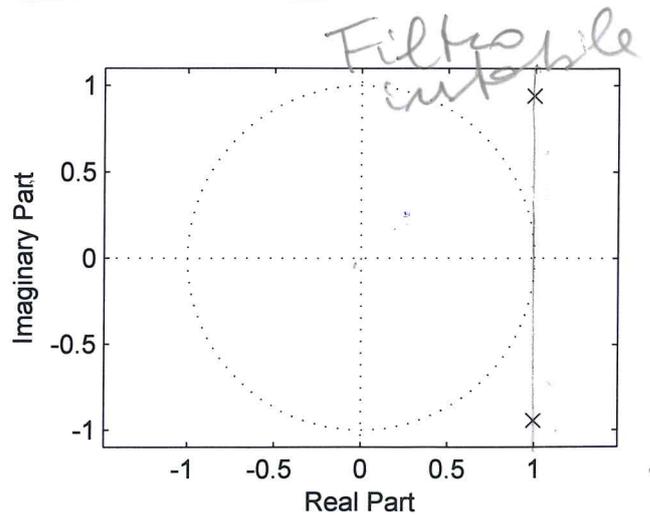
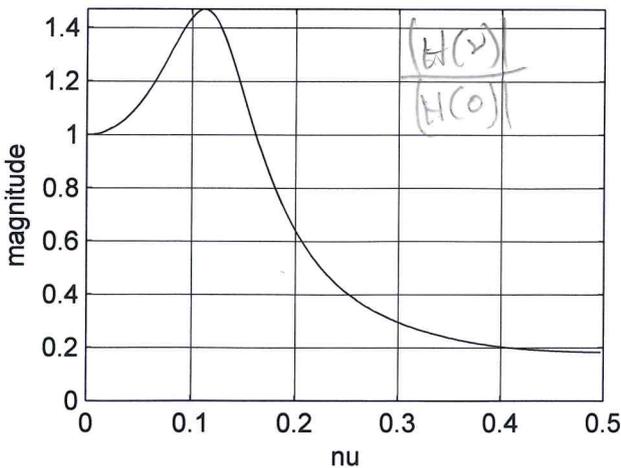
$$y[n] = -a_1 y[n-1] - a_2 y[n-2] + b_0 x[n-2]$$

%Metodo della approssimazione delle derivate 2

A=[1 -2*(1-sigma0*T) 1-2*sigma0*T+rho^2*T^2]
B=[0 0 rho^2*T^2]

A =
1.0000 -1.9900 1.8783
B =
0 0 0.8883

Anche se lo ripeto in
risposta potrebbe essere
superiore, il filtro è
instabile (poli fuori
dal cerchio unitario)



Se l'obiettivo del filtro numerico è quello di realizzare un passa-banda attorno alla frequenza di 3000 Hz, bisogna realizzare il filtro analogico prima della trasformazione.

PREWARPING

L'obiettivo è ottenere un passa-banda numerico centrato a

$$\nu_0 = \frac{3000}{20000} = 0.15 \quad \text{ovvero a } \omega_0 = 2\pi \nu_0 = 0.9425 \text{ [rad]}$$

(La pulsazione analogica senza warping sarebbe)

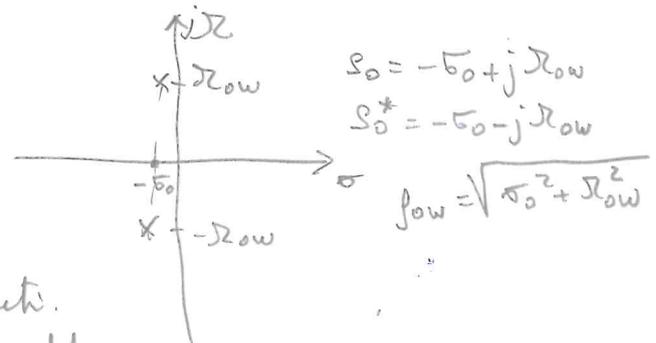
$$\Omega_0 = \frac{\omega_0}{T} = 1.8850 \cdot 10^4 \text{ [rad/sec]}$$

Con il warping invece abbiamo

$$\Omega_{ow} = \frac{2}{T} \operatorname{tg} \frac{\omega_c}{2} = 2.0381 \cdot 10^4 \text{ [rad/sec]}$$

Quindi il filtro analogico da cui partire è

$$H_{ow}(s) = \frac{\beta_0^2}{s^2 + 2\zeta_0 s + \beta_{ow}^2}$$



La frequenza di centro banda è leggermente spostata in avanti.

Applicando la trasform. bilineare otteniamo

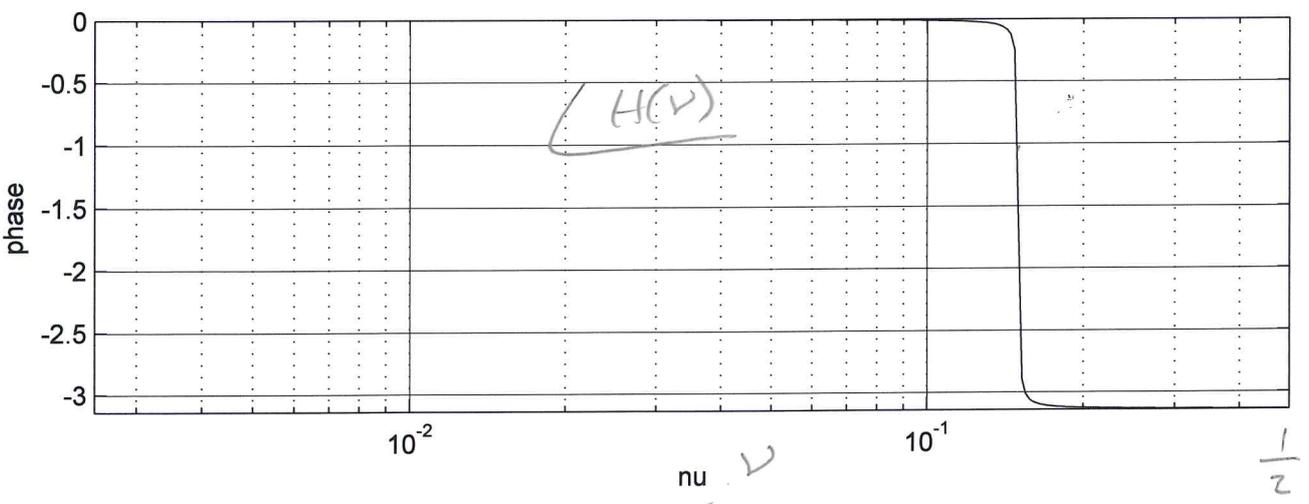
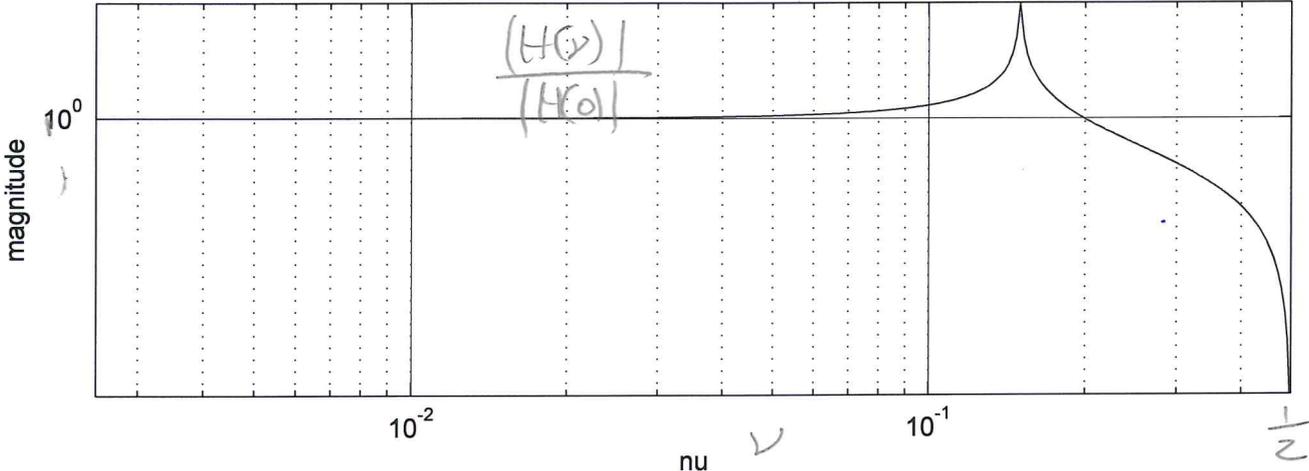
$$H(z) = H_{ow}(s) \Big|_{s = \frac{z-1}{T} \frac{z+1}{z-1}} = \frac{\beta_{ow}^2}{\frac{4}{T^2} \frac{(z-1)^2}{(z+1)^2} + 2\zeta_0 \frac{z-1}{T} \frac{z+1}{z-1} + \beta_{ow}^2}$$

$$= \frac{\beta_{ow}^2 T^2}{4(z-1)^2 + 4\zeta_0 T(z+1)(z-1) + \beta_{ow}^2 T^2 (z+1)^2}$$

$$= \frac{\beta_{ow}^2 T^2 z^2 + 2\zeta_0 \beta_{ow}^2 T^2 z + \beta_{ow}^2 T^2}{4z^2 + 4 - 8z + 4\zeta_0 T z^2 - 4\zeta_0 T + \beta_{ow}^2 T^2 z^2 + \beta_{ow}^2 T^2 + 2\beta_{ow}^2 T^2 z}$$

$$= \frac{\beta_{ow}^2 T^2 z^2 + 2\zeta_0 \beta_{ow}^2 T^2 z + \beta_{ow}^2 T^2}{(4 + 4\zeta_0 T + \beta_{ow}^2 T^2) z^2 + (-8 + 2\zeta_0 \beta_{ow}^2 T^2) z + 4 - 4\zeta_0 T + \beta_{ow}^2 T^2}$$

FILTRO NUMERICO OTTENUTO MEDIANTE
TRASFORMAZIONE BILINEARE



$$H(z) = \frac{b_0 z^2 + \frac{2b_1 z}{4+4\sigma_0 T + \rho_{0w}^2 T^2} + \frac{b_2}{4+4\sigma_0 T + \rho_{0w}^2 T^2}}{z^2 + \frac{(-8+2\rho_{0w}^2 T^2)}{4+4\sigma_0 T + \rho_{0w}^2 T^2} z + \frac{4-4\sigma_0 T + \rho_{0w}^2 T^2}{4+4\sigma_0 T + \rho_{0w}^2 T^2}}$$

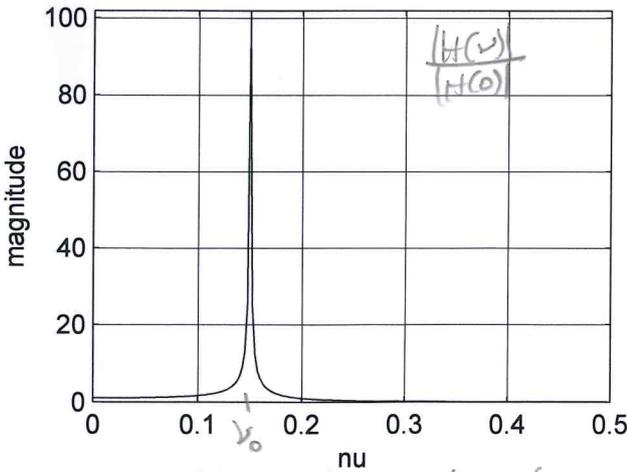
$$= \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{z^2 + a_1 z^{-1} + a_2 z^{-2}}$$

$$y[n] = -a_1 y[n-1] - a_2 y[n-2] + b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$$

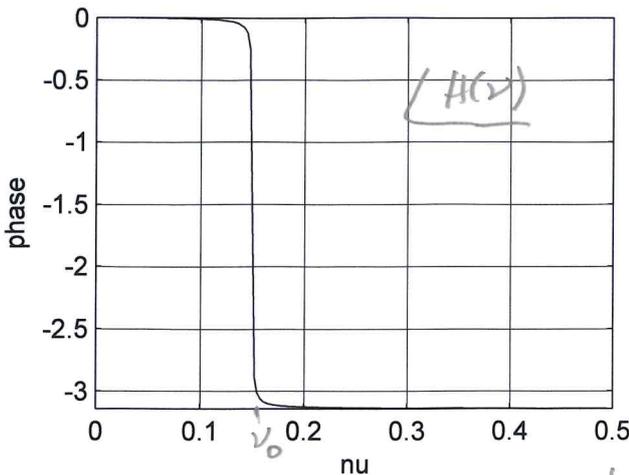
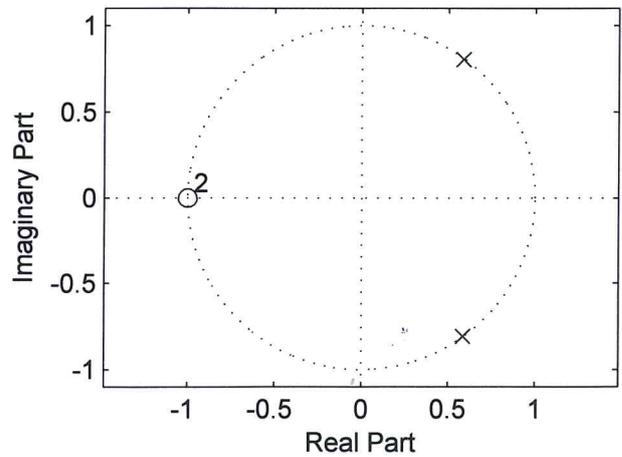
%metodo della trasformazione bilineare
 $\omega_0 = (2/T) * \tan(\pi * \nu_0)$
 $\rho_{0w} = \sqrt{\sigma_0^2 + \omega_0^2}$
 $k = 4 + 4 * \sigma_0 * T + \rho_{0w}^2 * T^2$
 $A = [1 \quad (-8 + 2 * \rho_{0w}^2 * T^2) / k \quad (4 - 4 * \sigma_0 * T + \rho_{0w}^2 * T^2) / k]$
 $B = [(\rho_{0w}^2 * T^2) / k \quad (2 * \rho_{0w}^2 * T^2) / k \quad (\rho_{0w}^2 * T^2) / k]$

2 poli stabili
 2 poli coincidenti
 $a = z = -1$

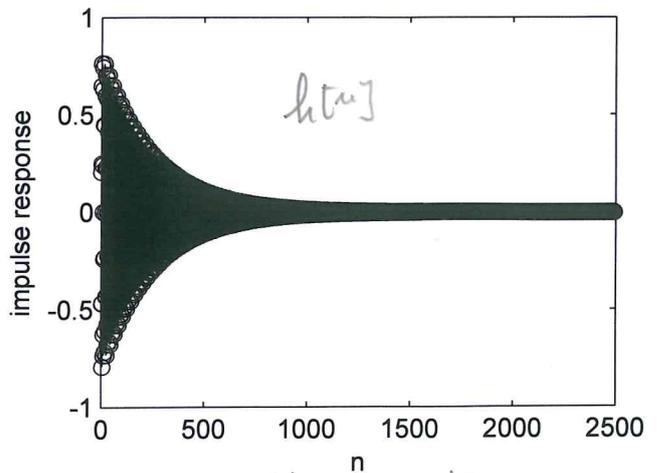
$\omega_0 = 2.0381e+04$
 $\rho_{0w} = 2.0381e+04$
 $k = 5.0585$
 $A =$
 1.0000 -1.1709 0.9921
 $B =$
 0.2053 0.4106 0.2053



eccellente risposta di ampiezza.



La freq. di centro-banda ν_0 è esattamente dove desiderato nonostante il warping non essere dell'ordine delle frequenze.



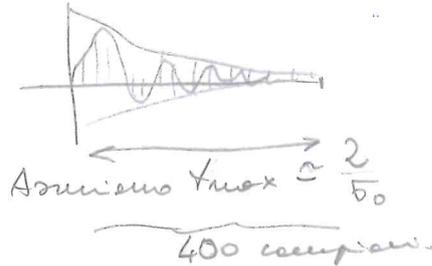
Per completare la corollata di tecniche di progetto, proponiamo anche un FIR ottenuto come campionamento della risposta impulsiva. E20

Precediamo la coppia $H_a(s) \leftrightarrow h_a(t)$

$$H_a(s) = \frac{\beta_0^2}{s^2 + 2\sigma_0 s + \beta_0^2} \leftrightarrow h_a(t) = \frac{\beta_0^2 + \sigma_0^2}{\sigma_0} e^{-\sigma_0 t} \sin \sigma_0 t$$

$$t_{max} = \frac{2}{\sigma_0} = \frac{2}{100} = 0.02 \text{ [sec]}$$

$$N = \left\lceil \frac{t_{max}}{T} \right\rceil = 200$$



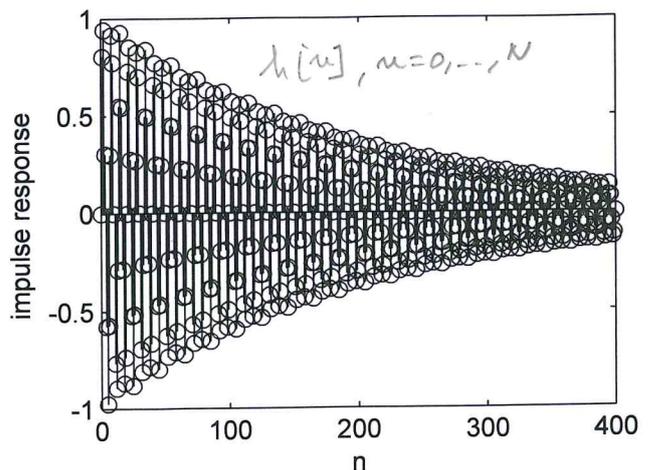
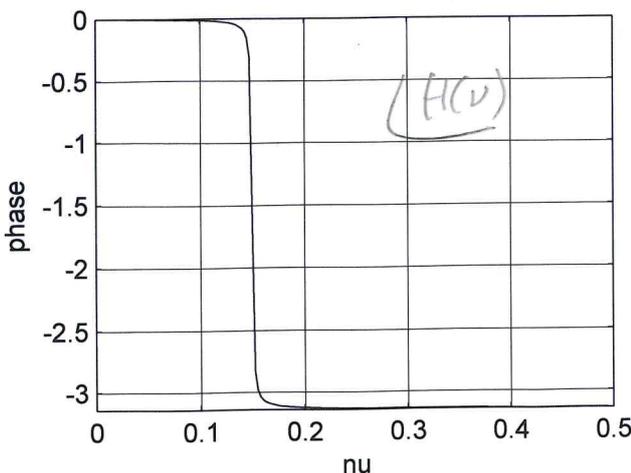
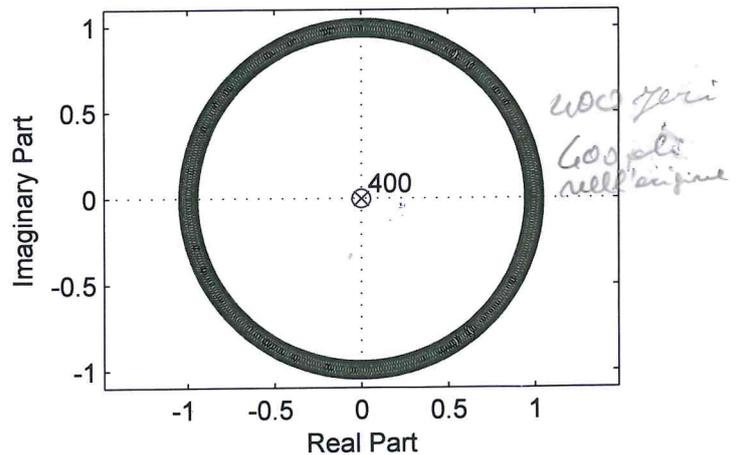
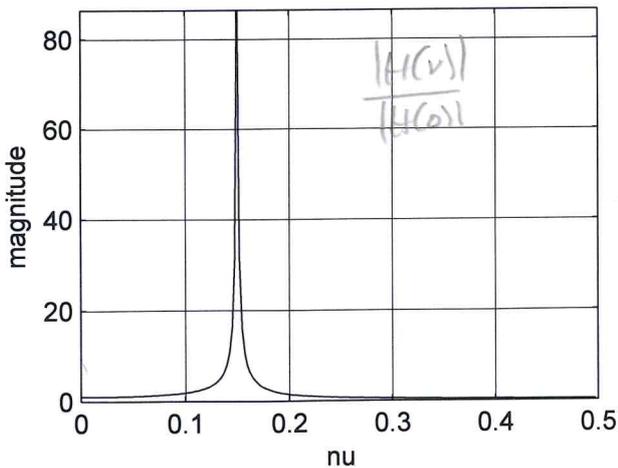
$$h[n] = b_n = e^{-\sigma_0 n T} \sin \sigma_0 n T \quad n=0, \dots, N$$

rimoviamo il coefficiente per avere valori centrollati

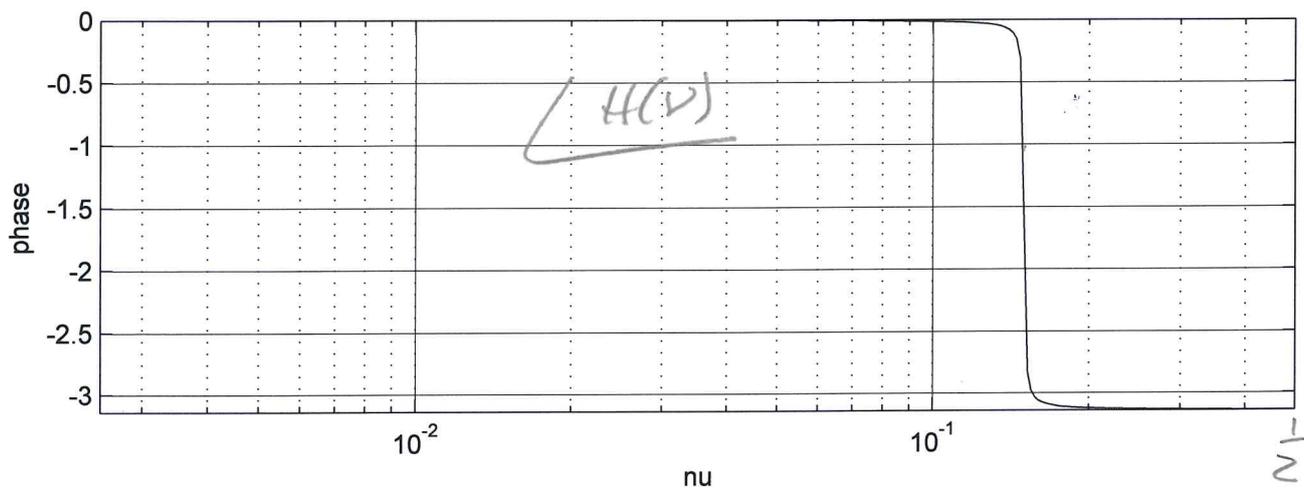
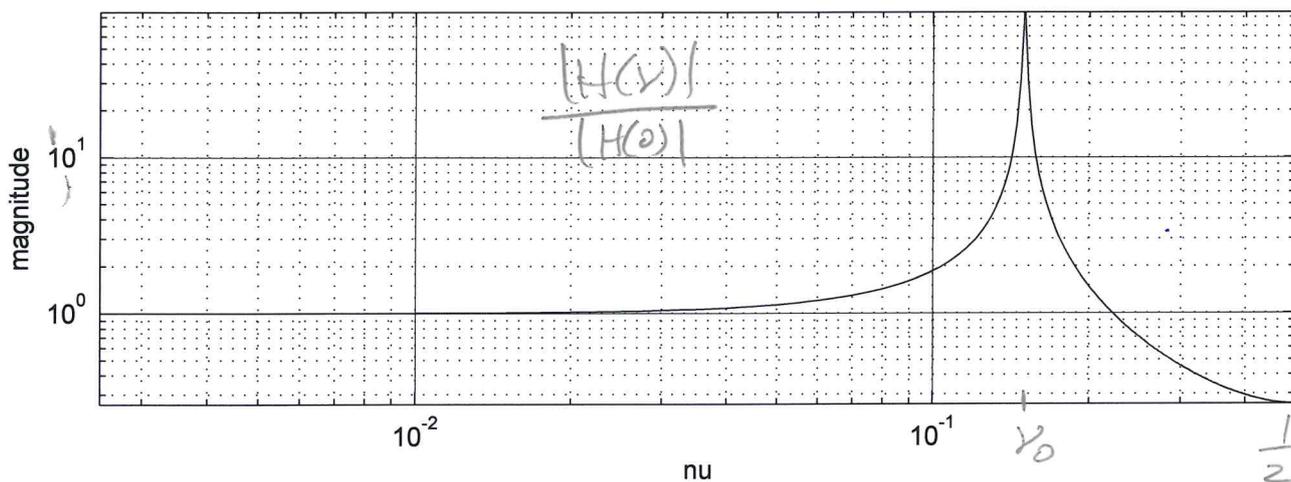
```

) %metodo del FIR dal campionamento e troncamento della risposta impulsiva
tmax=2/sigma0
N=floor(tmax/T)
B=[]
for n=0:N
B=[B exp(-sigma0*T*n)*sin(omega0*T*n)]
end
A=[1]
    
```

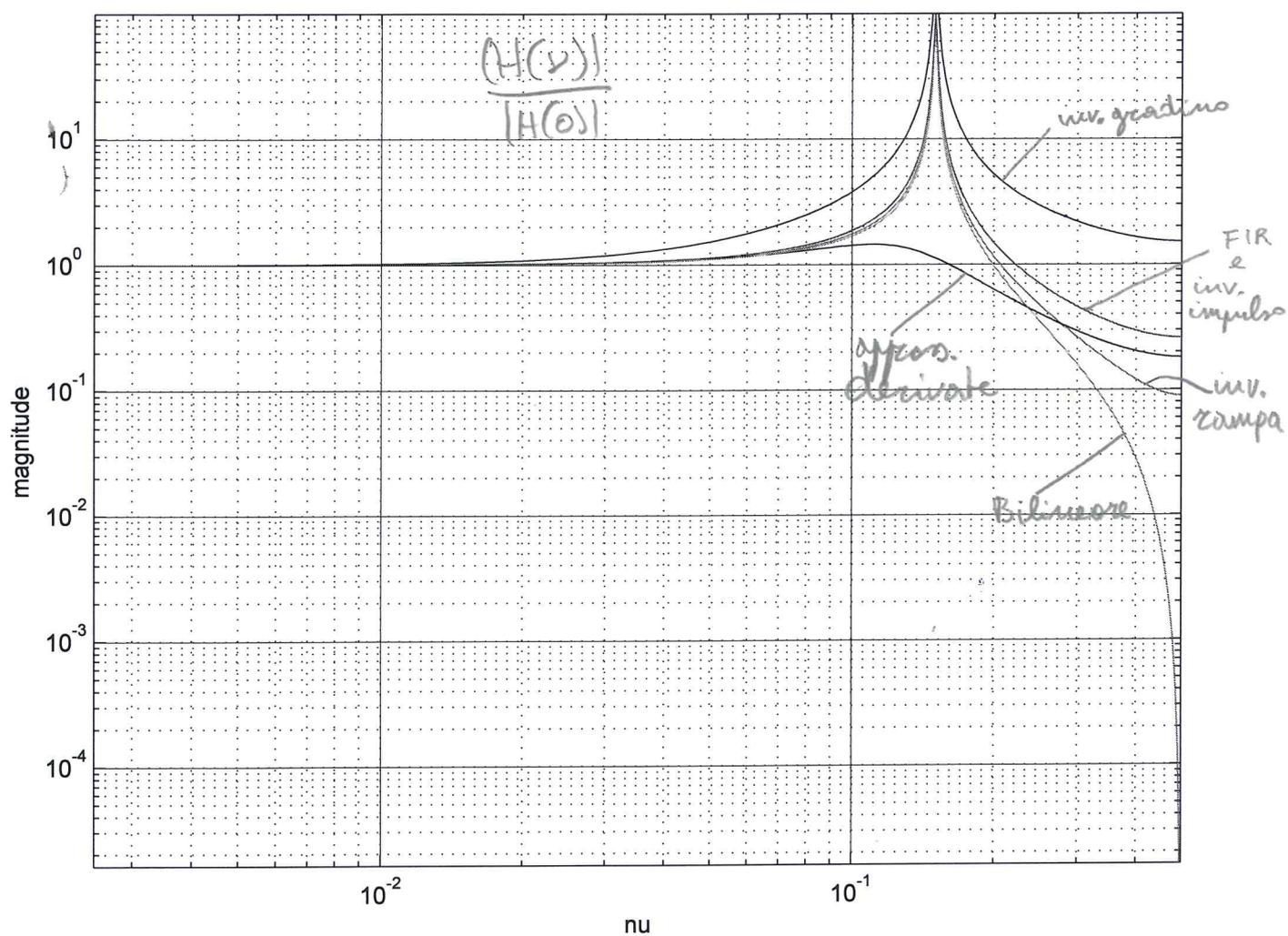
tmax = 0.0200
N = 400



PROGETTO FILTRO FIR
dal campionamento e troncamento a $N=400$
della risposta impulsiva.



CONFRONTO DELLA RISPOSTA DI AMPIEZZA DELLE VARIE TECNICHE



Senza nome

%%%%%%%%CONFRONTO%%%%%%%%

%Metodo dell'invarianza all'impulso

```
T=1/20000; %sampling interval
BII=[0 exp(-sigma0*T)*sin(omega0*T)]
AII=[1 -2*exp(-sigma0*T)*cos(omega0*T) exp(-2*sigma0*T)]
```

%medodo dell'invarianza al gradino

```
theta=atan(- sigma0 / omega0)
alpha=exp(-sigma0*T)*sec(theta)*cos(omega0*T - theta)
beta=exp(-sigma0*T)*cos(omega0*T)
gamma=exp(-2*sigma0*T)
AIG=[1 -2*beta gamma]
BIG=[0 alpha+1-2*beta -alpha]
```

%medodo dell'invarianza alla rampa

```
theta=atan((omega0^2 - sigma0^2) / (2*omega0*sigma0))
alpha=exp(-sigma0*T)*sec(theta)*cos(omega0*T - theta)
beta=exp(-sigma0*T)*cos(omega0*T)
gamma=exp(-2*sigma0*T)
delta=(2*sigma0)/(T*(sigma0^2+omega0^2))
AIR=[1 -2*beta gamma]
BIR=[1-delta-alpha*delta -2*beta-delta*gamma+delta+2*alpha*delta (1+delta)*gamma-alpha*delta]
```

%Metodo della approssimazione delle derivate 1

```
AAD=[1 -2*(1+sigma0*T)/(1+2*sigma0*T+rho^2*T^2) 1/(1+2*sigma0*T+rho^2*T^2) ]
BAD=[(rho^2*T^2)/(1+2*sigma0*T+rho^2*T^2)]
```

%metodo della trasformazione bilineare

```
omega0w=(2/T)*tan(pi*nu0)
rho=sqrt(sigma0^2+omega0w^2)
k=4+4*sigma0*T+rho^2*T^2
ATB=[1 (-8+2*rho^2*T^2)/k (4-4*sigma0*T+rho^2*T^2)/k]
BTB=[(rho^2*T^2)/k (2*rho^2*T^2)/k (rho^2*T^2)/k]
```

%metodo del FIR dal campionamento e troncamento della risposta impulsiva

```
tmax=2/sigma0
N=floor(tmax/T)
BFI=[]
for n=0:N
BFI=[BFI exp(-sigma0*T*n)*sin(omega0*T*n)]
end
AFI=[1]
```

figure(5)

```
[HII W]=freqz(BII,AII,200);
loglog(W/(2*pi),abs(HII)/abs(HII(1)),'b')
ylabel('magnitudine')
xlabel('nu')
axis([0 0.5 0 max(abs(HII))])
grid on
hold on
[HIG W]=freqz(BIG,AIG,200);
loglog(W/(2*pi),abs(HIG)/abs(HIG(1)),'r')
hold on
[HIR W]=freqz(BIR,AIR,200);
loglog(W/(2*pi),abs(HIR)/abs(HIR(1)),'g')
hold on
[HAD W]=freqz(BAD,AAD,200);
loglog(W/(2*pi),abs(HAD)/abs(HAD(1)),'k')
hold on
[HTB W]=freqz(BTB,ATB,200);
loglog(W/(2*pi),abs(HTB)/abs(HTB(1)),'c')
hold on
[HFI W]=freqz(BFI,AFI,200);
loglog(W/(2*pi),abs(HFI)/abs(HFI(1)),'k')
```