Analog Communications

— Lecture 03 —
Linear Modulation

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Outline

Linear Modulation

- $x(t)$ is the information low-pass signal or modulating signal
- $p(t) = A\cos(2\pi f_0 t + \vartheta)$ is the carrier signal
- the transmitted signal or modulated signal has the form

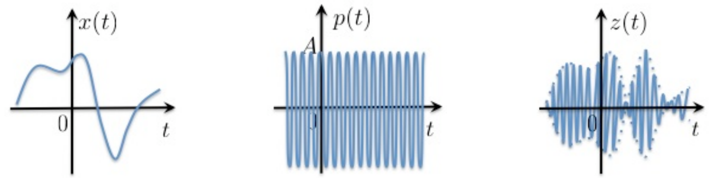
$$z(t) = z_I(t)\cos(2\pi f_0 t) + z_Q(t)\sin(2\pi f_0 t)$$

  where $z_I(t)$ and $z_Q(t)$ depend on $x(t)$ through affine transformations
- The power efficiency η_p is defined as the ratio between the transmitted power of the information signal component and the whole transmitted power
- The spectral efficiency η_s is defined as the ratio between the bandwidth of the information signal (B_x) and the bandwidth of the transmitted signal (B_z)

Double Side Band (DSB) Modulation

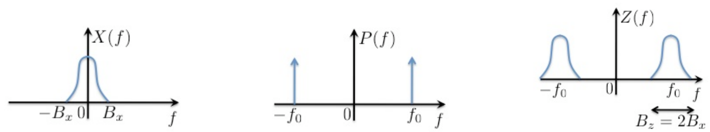
The time-domain transmitted signal is

$$z(t) = A\ x(t)\ \cos(2\pi f_0 t)$$

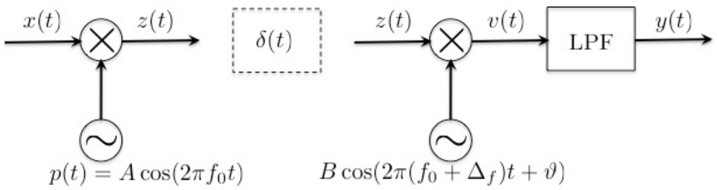


The frequency-domain transmitted signal is

$$Z(f) = \frac{A}{2}\left(X(f - f_0) + X(f + f_0)\right)$$



DSB Demodulation (1/2)



$$\begin{aligned} v(t) &= ABx(t)\cos(2\pi f_0 t)\cos(2\pi(f_0+\Delta_f)t+\vartheta) \\ &= \frac{AB}{2}x(t)\cos(2\pi\Delta_f t+\vartheta) + \frac{AB}{2}x(t)\cos(2\pi(2f_0+\Delta_f)t+\vartheta) \end{aligned}$$

$$\begin{aligned} V(f) &= \frac{AB}{4}\left(e^{j\vartheta}X(f-\Delta_f)+e^{-j\vartheta}X(f+\Delta_f)\right) \\ &\quad + \frac{AB}{4}\left(e^{j\vartheta}X(f-(2f_0+\Delta_f))+e^{-j\vartheta}X(f+(2f_0+\Delta_f))\right) \end{aligned}$$

DSB Demodulation (2/2)

Assume an ideal LPF with unitary gain and cutoff frequency $f_t = B_x$

$$H(f) = \text{rect}\left(\frac{f}{2B_x}\right)$$

then

$$y(t) = \frac{AB}{2}x(t)\cos(2\pi\Delta_f t+\vartheta)$$

$$Y(f) = \frac{AB}{4}\left(e^{j\vartheta}X(f-\Delta_f)+e^{-j\vartheta}X(f+\Delta_f)\right)$$

Underlying assumption

$$f_0 > B_x$$

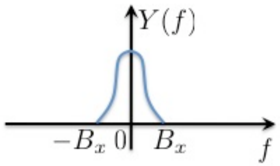
DSB Case 1: Perfect Synchronism

If carrier frequency and phase are perfectly known at the receiver

$$\Delta_f = 0 \quad \text{and} \quad \vartheta = 0$$

then

$$\begin{aligned} y(t) &= \frac{AB}{2}x(t) \\ Y(f) &= \frac{AB}{2}X(f) \end{aligned}$$



- the received signal is a scaled version of the transmitted signal
- the whole chain is non-distorting

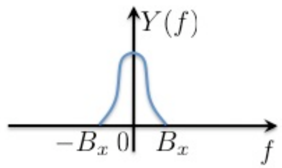
DSB Case 2: Phase Error

If carrier frequency only is perfectly known at the receiver

$$\Delta_f = 0 \quad \text{and} \quad \vartheta \neq 0$$

then

$$\begin{aligned} y(t) &= \frac{AB}{2}\cos(\vartheta)x(t) \\ Y(f) &= \frac{AB}{2}\cos(\vartheta)X(f) \end{aligned}$$



- the received signal is a scaled version of the transmitted signal
- the whole chain is non-distorting
- critical only if the two oscillators are in quadrature ($\vartheta \approx \pi/2$) in this case $y(t) \approx 0$

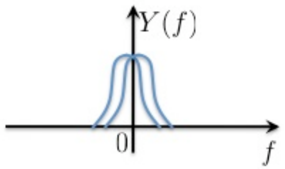
DSB Case 3: Frequency Error

If carrier phase only is perfectly known at the receiver

$$\Delta_f \neq 0 \qquad \text{and} \qquad \vartheta = 0$$

then

$$\begin{aligned} y(t) &= \frac{AB}{2} x(t) \cos(2\pi \Delta_f t) \\ Y(f) &= \frac{AB}{4} (X(f - \Delta_f) + X(f + \Delta_f)) \end{aligned}$$



- the received signal is a distorted version of the transmitted signal
- the whole chain is distorting
- non-linear distortion (residual modulation)

Correlation and PSD of DSB Signals

If $x(t)$ is a WSS signal, then $z(t)$ is cyclostationary with period $1/2f_0$

$$\begin{aligned} R_z(t, \tau) &= \frac{A^2}{2} R_x(\tau) \cos(2\pi f_0 \tau) + \frac{A^2}{2} R_x(\tau) \cos(2\pi f_0 (2t - \tau)) \\ R_z(\tau) &= \frac{A^2}{2} R_x(\tau) \cos(2\pi f_0 \tau) \end{aligned}$$

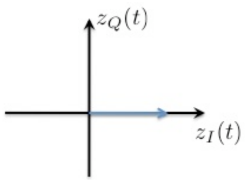
$$\begin{aligned} P_z(\tau) &= \frac{A^2}{4} (P_x(f - f_0) + P_x(f + f_0)) \\ P_z &= \frac{A^2}{2} P_x \end{aligned}$$

Complex Envelope of DSB Signals

Analytic signal and complex envelope are

$$\begin{aligned} \mathring{z}(t) &= A x(t) \exp(j 2\pi f_0 t) \\ \tilde{z}(t) &= A x(t) \end{aligned}$$

- The complex envelope is real-valued
- The complex envelope assumes both positive and negative values



Power and Spectral Efficiencies for DSB

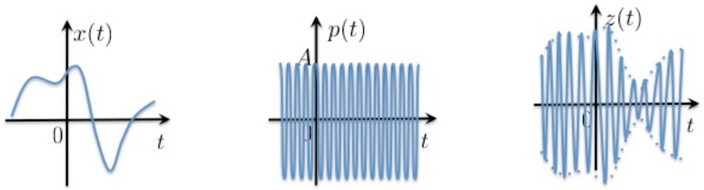
$$\begin{aligned} \eta_p &= \frac{\frac{A^2}{2} P_x}{\frac{A^2}{2} P_x} \\ &= 1 \end{aligned}$$

$$\begin{aligned} \eta_s &= \frac{B_x}{2 B_x} \\ &= 1/2 \end{aligned}$$

Amplitude Modulation (AM)

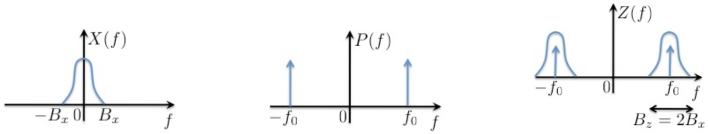
The time-domain transmitted signal is

$$z(t) = A\,(1 + kx(t))\cos(2\pi f_0 t)$$

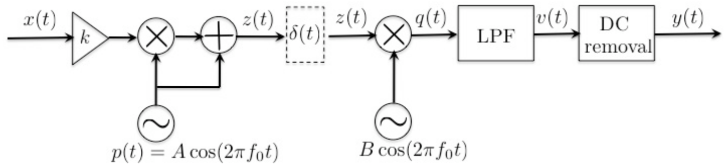


The frequency-domain transmitted signal is

$$Z(f) = \frac{A}{2}\left(\delta(f-f_0)+\delta(f+f_0)\right) + \frac{kA}{2}\left(X(f-f_0)+X(f+f_0)\right)$$



AM Coherent Demodulation (1/2)



The same scheme used for DSB would recover the information signal assuming that the DC component is absent (or irrelevant) and adding a system for DC removal

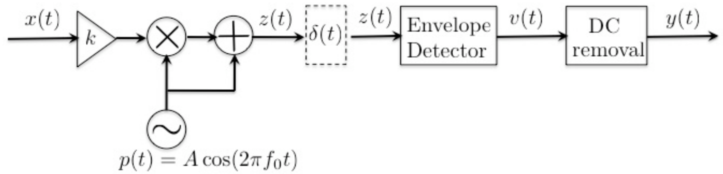
$$v(t) = \frac{AB}{2}(1 + kx(t)) \quad \text{and} \quad V(f) = \frac{AB}{2}\delta(f) + \frac{kAB}{2}X(f)$$

$$y(t) = \frac{kAB}{2}x(t) \quad \text{and} \quad Y(f) = \frac{kAB}{2}X(f)$$

AM Coherent Demodulation (2/2)

- It is worth noticing that AM use part of the available power to transmit the unmodulated carrier
- That may appear as a waste of resources
- Using a coherent receiver DOES make that a waste of resources
- The main point of AM (i.e. of wasting transmission power) is the possibility to use an extremely-simple receiver

AM Incoherent Demodulation (1/2)



$$\begin{aligned} v(t) &= A\,|1 + kx(t)| \\ &= A\,(1 + kx(t)) \end{aligned}$$

Last equality does not always hold, in the positive case it is apparent that

$$y(t) = kAx(t)$$

AM Incoherent Demodulation (2/2)

The condition

$$|1 + kx(t)| = (1 + kx(t))$$

is equivalent to

$$1 + kx(t) \geq 0 \qquad \forall\ t$$

i.e.

$$k|x(t)| \leq 1 \qquad \forall\ t$$

Finally, denoting $x_m = \max_t |x(t)|$ we get

$$k \leq \frac{1}{x_m}$$

Correlation and PSD of AM Signals

If $x(t)$ is a WSS signal with null expected value, then $z(t)$ is cyclostationary with period $1/2f_0$

$$\begin{aligned} R_z(t,\tau) &= \frac{A^2}{2}\cos(2\pi f_0 \tau) + \frac{A^2}{2}\cos(2\pi f_0(2t-\tau)) \\ &\quad + \frac{k^2A^2}{2}R_x(\tau)\cos(2\pi f_0\tau) + \frac{k^2A^2}{2}R_x(\tau)\cos(2\pi f_0(2t-\tau)) \\ R_z(\tau) &= \frac{A^2}{2}\cos(2\pi f_0\tau) + \frac{k^2A^2}{2}R_x(\tau)\cos(2\pi f_0\tau) \end{aligned}$$

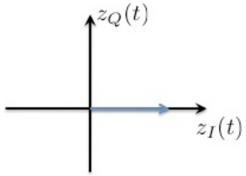
$$\begin{aligned} P_z(\tau) &= \frac{A^2}{4}(\delta(f-f_0)+\delta(f+f_0)) \\ &\quad + \frac{k^2A^2}{4}(P_x(f-f_0)+P_x(f+f_0)) \\ P_z &= \frac{A^2}{2} + \frac{k^2A^2}{2}P_x \end{aligned}$$

Complex Envelope of AM Signals

Analytic signal and complex envelope are

$$\begin{aligned} \mathring{z}(t) &= A(1+kx(t))\exp(j2\pi f_0 t) \\ \tilde{z}(t) &= A(1+kx(t)) \end{aligned}$$

- The complex envelope is real-valued
- The complex envelope assumes positive values only



Power and Spectral Efficiencies for AM

$$\begin{aligned} \eta_p &= \frac{\frac{k^2A^2}{2}P_x}{\frac{A^2}{2}+\frac{k^2A^2}{2}P_x} \\ &= \frac{1}{1+\frac{1}{k^2P_x}} \end{aligned}$$

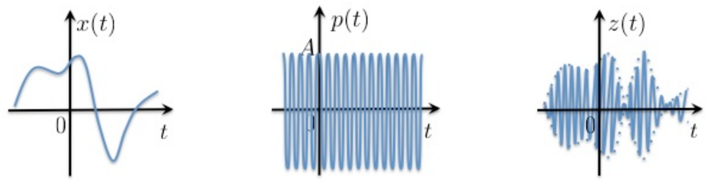
Also remember that $k^2P_x \leq 1$ thus $\eta_p \leq 1/2$

$$\begin{aligned} \eta_s &= \frac{B_x}{2B_x} \\ &= 1/2 \end{aligned}$$

Single Side Band (SSB) Modulation (1/2)

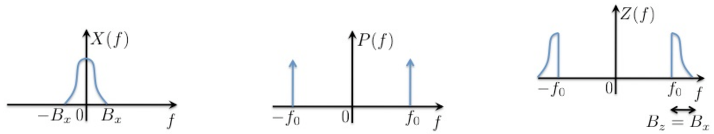
The time-domain transmitted signal is

$$z(t) = A\ x(t)\ \cos(2\pi f_0 t) \mp A\ \hat{x}(t)\ \sin(2\pi f_0 t)$$



The frequency-domain transmitted signal is

$$Z(f) = \frac{A}{2} X(f - f_0)\,(1 \pm \mathrm{sign}(f - f_0)) + \frac{A}{2} X(f + f_0)\,(1 \mp \mathrm{sign}(f + f_0))$$



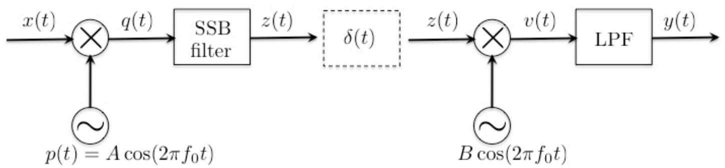
SSB Modulation (2/2)

The main idea is to halve the bandwidth of the transmitted signal

- SSB-upper (SSB-U) selects the external frequencies and removes the internal frequencies
- SSB-lower (SSB-L) selects the internal frequencies and removes the external frequencies

$$\begin{aligned}
Q(f) &= \frac{A}{2}(X(f - f_0) + X(f + f_0)) \\
H_u(f) &= 2\ \mathrm{rect}\left(\frac{f}{2f_0}\right) \\
Z_u(f) &= Q(f)H_u(f) \\
H_l(f) &= 2\left(1 - \mathrm{rect}\left(\frac{f}{2f_0}\right)\right) \\
Z_l(f) &= Q(f)H_l(f)
\end{aligned}$$

SSB Demodulation (1/2)



$$\begin{aligned}
v(t) &= ABx(t)\cos^2(2\pi f_0 t) \mp AB\hat{x}(t)\sin(2\pi f_0 t)\cos(2\pi f_0 t) \\
&= \frac{AB}{2}x(t) + \frac{AB}{2}x(t)\cos(2\pi 2 f_0 t) \mp \frac{AB}{2}\hat{x}(t)\sin(2\pi 2 f_0 t)
\end{aligned}$$

$$\begin{aligned}
V(f) &= \frac{AB}{2}X(f) + \frac{AB}{4}(X(f - 2f_0) + X(f + 2f_0)) \\
&\quad \pm \frac{AB}{4}(X(f - 2f_0)\mathrm{sign}(f - 2f_0) - X(f + 2f_0)\mathrm{sign}(f + 2f_0))
\end{aligned}$$

SSB Demodulation (2/2)

Assume an ideal LPF with unitary gain and cutoff frequency $f_t = B_x$

$$H(f) = \mathrm{rect}\left(\frac{f}{2B_x}\right)$$

then

$$y(t) = \frac{AB}{2}x(t)$$

$$Y(f) = \frac{AB}{2}X(f)$$

Underlying assumption

$$f_0 > \frac{B_x}{2}$$

Correlation and PSD of SSB Signals

If $x(t)$ is a WSS signal, then $z(t)$ is WSS

$$R_z(\tau) = A^2 R_x(\tau)\cos(2\pi f_0 \tau) \mp A^2 \hat{R}_x(\tau)\sin(2\pi f_0 \tau)$$

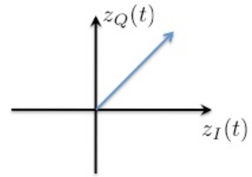
$$\begin{aligned} P_z(\tau) &= \frac{A^2}{2} P_x(f - f_0)\,(1 \pm \mathrm{sign}(f - f_0)) \\ &\quad + \frac{A^2}{2} P_x(f + f_0)\,(1 \mp \mathrm{sign}(f + f_0)) \\ P_z &= A^2 P_x \end{aligned}$$

Complex Envelope of SSB Signals

Analytic signal and complex envelope are

$$\begin{aligned} \mathring{z}(t) &= A\,(x(t) \pm \hat{x}(t))\exp(j2\pi f_0 t) \\ \tilde{z}(t) &= A\,(x(t) \pm \hat{x}(t)) \end{aligned}$$

- The complex envelope is complex-valued
- Real and imaginary components have the same absolute value



Power and Spectral Efficiencies for SSB

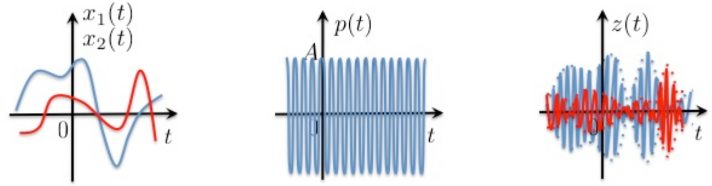
$$\begin{aligned} \eta_p &= \frac{A^2 P_x}{A^2 P_x} \\ &= 1 \end{aligned}$$

$$\begin{aligned} \eta_s &= \frac{B_x}{B_x} \\ &= 1 \end{aligned}$$

Quadrature Amplitude Modulation (QAM)

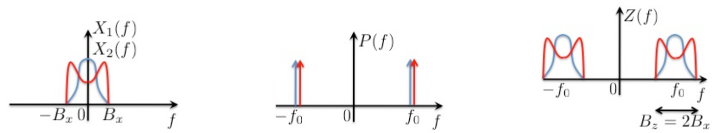
The time-domain transmitted signal is

$$z(t) = A\ x_1(t)\ \cos(2\pi f_0 t) + A\ x_2(t)\ \sin(2\pi f_0 t)$$

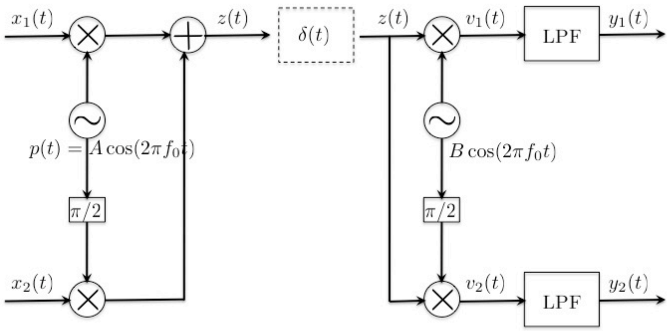


The frequency-domain transmitted signal is

$$Z(f) = \frac{A}{2}\,(X_1(f - f_0) + X_1(f + f_0)) + \frac{A}{2j}\,(X_2(f - f_0) - X_2(f + f_0))$$



QAM Demodulation (1/2)



$$\begin{aligned}
v_1(t) &= ABx_1(t)\cos^2(2\pi f_0 t) + ABx_2(t)\sin(2\pi f_0 t)\cos(2\pi f_0 t) \\
&= \frac{AB}{2}x_1(t) + \frac{AB}{2}x_1(t)\cos(2\pi 2 f_0 t) + \frac{AB}{2}x_2(t)\sin(2\pi 2 f_0 t) \\
V_1(f) &= \frac{AB}{2}X_1(f) + \frac{AB}{4}\left(X_1(f-2f_0) + X_1(f+2f_0)\right) \\
& \quad +\frac{AB}{4j}\left(X_2(f-2f_0) - X_2(f+2f_0)\right)
\end{aligned}$$

QAM Demodulation (2/2)

Assume an ideal LPF with unitary gain and cutoff frequency $f_t = B_x$

$$H(f) = \text{rect}\left(\frac{f}{2B_x}\right)$$

then

$$\begin{aligned}
y_1(t) &= \frac{AB}{2}x_1(t) \\
y_2(t) &= \frac{AB}{2}x_2(t)
\end{aligned}$$

$$\begin{aligned}
Y_1(f) &= \frac{AB}{2}X_1(f) \\
Y_2(f) &= \frac{AB}{2}X_2(f)
\end{aligned}$$

Underlying assumption

$$f_0 > B_x$$

Correlation and PSD of QAM Signals

If $x(t)$ is a WSS signal, then $z(t)$ is cyclostationary with period $1/2f_0$

$$\begin{aligned}
R_z(\tau) &= \frac{A^2}{2}\left(R_{x_1}(\tau) + R_{x_2}(\tau)\right)(\tau)\cos(2\pi f_0 \tau) \\
& \quad -\frac{A^2}{2}\left(R_{x_1x_2}(\tau) - R_{x_2x_1}(\tau)\right)(\tau)\sin(2\pi f_0 \tau)
\end{aligned}$$

Usually $R_{x_1x_2}(\tau) = 0$, thus

$$R_z(\tau) = \frac{A^2}{2}\left(R_{x_1}(\tau) + R_{x_2}(\tau)\right)(\tau)\cos(2\pi f_0 \tau)$$

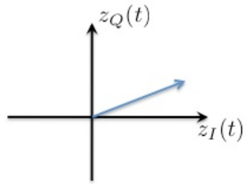
$$\begin{aligned}
P_z(\tau) &= \frac{A^2}{4}\left(P_{x_1}(f-f_0) + P_{x_1}(f+f_0) + P_{x_2}(f-f_0) + P_{x_2}(f+f_0)\right) \\
P_z &= \frac{A^2}{2}P_{x_1} + \frac{A^2}{2}P_{x_2}
\end{aligned}$$

Complex Envelope of QAM Signals

Analytic signal and complex envelope are

$$\begin{aligned}
\mathring{z}(t) &= A\left(x_1(t) - x_2(t)\right)\exp(j2\pi f_0 t) \\
\tilde{z}(t) &= A\left(x_1(t) - x_2(t)\right)
\end{aligned}$$

- The complex envelope is complex-valued
- Real and imaginary components have arbitrary values



Power and Spectral Efficiencies for QAM

$$\eta_p = \frac{\frac{A^2}{2}P_{x_1} + \frac{A^2}{2}P_{x_2}}{\frac{A^2}{2}P_{x_1} + \frac{A^2}{2}P_{x_2}} = 1$$

$$\eta_s = \frac{2B_x}{2B_x} = 1$$