Analog Communications

— Lecture 04 —
Performance of Linear Modulation over AWGN Channel

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Outline

Transmitted Signal

The transmitted signal in the case of linear modulation is

$$z(t) = z_I(t)\cos(2\pi f_0 t) + z_Q(t)\sin(2\pi f_0 t)$$

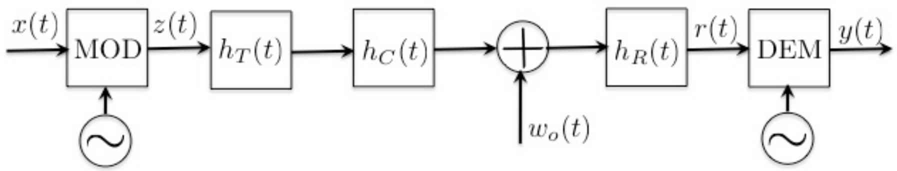
where

- DSB: $z_I(t) = Ax(t)$ and $z_Q(t) = 0$
- AM: $z_I(t) = A(1 + kx(t))$ and $z_Q(t) = 0$
- SSB: $z_I(t) = Ax(t)$ and $z_Q(t) = \mp A\hat{x}(t)$
- QAM: $z_I(t) = Ax_1(t)$ and $z_Q(t) = Ax_2(t)$

Denote

- B_x the bandwidth of the information signal
- B_z the band of the transmitted signal

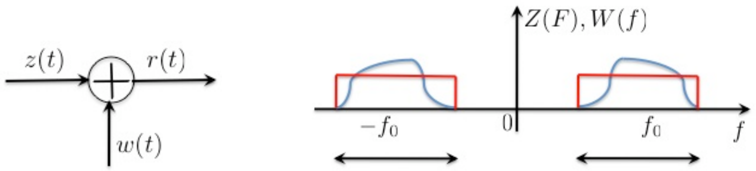
AWGN Channel



Assumptions:

- $H_T(f)H_C(f)H_R(f)$ is flat over B_z
- $w_o(t)$ is white and Gaussian

Received Signal over AWGN



The received signal in the case of AWGN channel is

$$r(t) = z(t) + w(t)$$

where

$$P_w(f) = \begin{cases} \frac{\eta_o}{2} & |f| \in B_z \\ 0 & |f| \notin B_z \end{cases}$$

and

- DSB, AM, QAM: $B_z = (f_0 - B_x, f_0 + B_x)$
- SSB: $B_z = (f_0 - B_x, f_0)$ or $B_z = (f_0, f_0 + B_x)$

Coherent Receiver

The received signal is a band-pass signal
(being the sum of two band-pass signals)

$$\begin{aligned} r(t) &= z(t) + w(t) \\ &= r_I(t)\cos(2\pi f_0 t) + r_Q(t)\sin(2\pi f_0 t) \end{aligned}$$

where

$$\begin{aligned} r_I(t) &= z_I(t) + w_I(t) \\ r_Q(t) &= z_Q(t) + w_Q(t) \end{aligned}$$

A coherent receiver (in case of perfect synchronization) provides

$$\begin{aligned} y(t) &= \frac{B}{2} r_I(t) \\ &= \frac{B}{2} z_I(t) + \frac{B}{2} w_I(t) \end{aligned}$$

Performance (1/3)

The Signal-to-Noise Ratio (SNR) at the input of the receiver is

$$\mathrm{SNR}_{in} = \frac{P_z}{P_w}$$

$$\mathrm{DSB}: \quad P_z = \frac{A^2}{2} P_x\,, \; P_w = 2\eta_o B_x\,, \quad \mathrm{SNR}_{in} = \frac{A^2 P_x}{4\eta_o B_x}$$

$$\mathrm{AM}: \quad P_z = \frac{A^2}{2}(1 + k^2 P_x)\,, \; P_w = 2\eta_o B_x\,, \quad \mathrm{SNR}_{in} = \frac{A^2(1 + k^2 P_x)}{4\eta_o B_x}$$

$$\mathrm{SSB}: \quad P_z = A^2 P_x\,, \; P_w = \eta_o B_x\,, \quad \mathrm{SNR}_{in} = \frac{A^2 P_x}{\eta_o B_x}$$

$$\mathrm{QAM}: \quad P_z = \frac{A^2}{2}(P_{x_1} + P_{x_2})\,, \; P_w = 2\eta_o B_x\,, \quad \mathrm{SNR}_{in} = \frac{A^2(P_{x_1} + P_{x_2})}{4\eta_o B_x}$$

Performance (2/3)

The Signal-to-Noise Ratio (SNR) at the output of the receiver is

$$\mathrm{SNR}_{out} = \frac{\frac{B^2}{4} P_{z_I}}{\frac{B^2}{4} P_{w_I}} = \frac{P_{z_I}}{P_{w_I}}$$

$$\mathrm{DSB}: \quad P_{z_I} = A^2 P_x\,, \; P_{w_I} = 2\eta_o B_x\,, \quad \mathrm{SNR}_{out} = \frac{A^2 P_x}{2\eta_o B_x}$$

$$\mathrm{AM}: \quad P_{z_I} = A^2 k^2 P_x\,, \; P_{w_I} = 2\eta_o B_x\,, \quad \mathrm{SNR}_{out} = \frac{A^2 k^2 P_x}{2\eta_o B_x}$$

$$\mathrm{SSB}: \quad P_{z_I} = A^2 P_x\,, \; P_{w_I} = \eta_o B_x\,, \quad \mathrm{SNR}_{out} = \frac{A^2 P_x}{\eta_o B_x}$$

$$\mathrm{QAM}: \quad P_{z_{I,i}} = A^2 P_{x_i}\,, \; P_{w_I} = 2\eta_o B_x\,, \quad \mathrm{SNR}_{out,i} = \frac{A^2 P_{x_i}}{2\eta_o B_x}$$

Performance (3/3)

Common perf. indicators are $\text{SNR}_{out}(\text{SNR}_{in})$ and $\text{SNR}_{out}(\gamma)$ where

$$\gamma = \frac{P_z}{\eta_o B_x}$$

represents the ratio of two powers

- the power of the transmitted signal at the numerator
- the product between the noise PSD and the information bandwidth

$$\begin{aligned}
\text{DSB}: &\quad \text{SNR}_{out} = 2\ \text{SNR}_{in}\ , \quad \text{SNR}_{out} = \gamma \\
\text{AM}: &\quad \text{SNR}_{out} = \frac{2}{1+\frac{1}{k^2 P_x}}\ \text{SNR}_{in}\ , \quad \text{SNR}_{out} = \frac{1}{1+\frac{1}{k^2 P_x}}\gamma \\
\text{SSB}: &\quad \text{SNR}_{out} = \text{SNR}_{in}\ , \quad \text{SNR}_{out} = \gamma \\
\text{QAM}: &\quad \text{SNR}_{out,i} = \text{SNR}_{in,i}\ , \quad \text{SNR}_{out} = \gamma_i
\end{aligned}$$

AM reception with Envelope Detector

The received signal is

$$r(t) = (A(1+kx(t)) + w_I(t))\cos(2\pi f_o t) + w_Q(t)\sin(2\pi f_o t)$$

then the envelope detector provides

$$\begin{aligned}
y(t) &= \sqrt{(A(1+kx(t)) + w_I(t))^2 + w_Q^2(t)} \\
&= \sqrt{A^2(1+kx(t))^2 + 2A(1+kx(t))w_I(t) + w_I^2(t) + w_Q^2(t)}
\end{aligned}$$

Not possible to point out signal and noise components, thus consider

- low-SNR approximation ($\text{SNR}_{in} \ll 1$)
- high-SNR approximation ($\text{SNR}_{in} \gg 1$)

and apply $\sqrt{1+x} \approx 1 + x/2$ if $x \ll 1$

AM reception with Envelope Detector - low SNR

$w_I^2(t) + w_Q^2(t)$ is the dominant term within the square root, then

$$\begin{aligned}
y(t) &= \sqrt{w_I^2(t) + w_Q^2(t)}\sqrt{\frac{A^2(1+kx(t))^2}{w_I^2(t)+w_Q^2(t)} + 2\frac{A(1+kx(t))w_I(t)}{w_I^2(t)+w_Q^2(t)} + 1} \\
&\approx \sqrt{w_I^2(t) + w_Q^2(t)}\sqrt{1 + 2\frac{A(1+kx(t))w_I(t)}{w_I^2(t)+w_Q^2(t)}} \\
&\approx \sqrt{w_I^2(t) + w_Q^2(t)}\left(1 + \frac{A(1+kx(t))w_I(t)}{w_I^2(t)+w_Q^2(t)}\right) \\
&= \rho_w(t) + A(1+kx(t))\cos(\theta_w(t))
\end{aligned}$$

The term $\cos(\theta_w(t))$ affects irremediably the useful signal

The whole system does NOT work

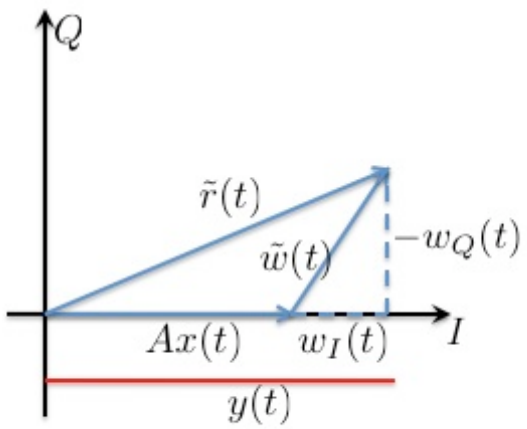
AM reception with Envelope Detector - high SNR

$A(1+kx(t))$ is the dominant term within the square root, then

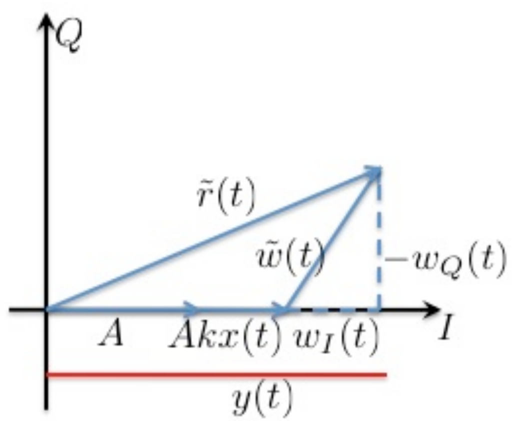
$$\begin{aligned}
y(t) &= A(1+kx(t))\sqrt{1 + 2\frac{w_I(t)}{A(1+kx(t))} + \frac{w_I^2(t)+w_Q^2(t)}{A^2(1+kx(t))^2}} \\
&\approx A(1+kx(t))\sqrt{1 + 2\frac{w_I(t)}{A(1+kx(t))}} \\
&\approx A(1+kx(t))\left(1 + \frac{w_I(t)}{A(1+kx(t))}\right) \\
&= A(1+kx(t)) + w_I(t)
\end{aligned}$$

Same performance as for the coherent receiver

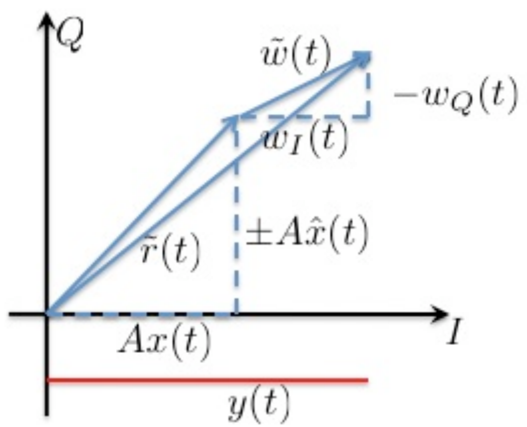
Graphical Representation for DSB



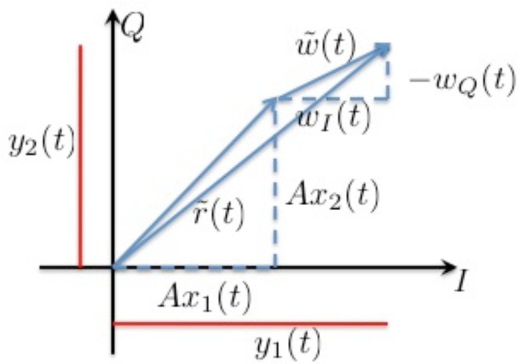
Graphical Representation for AM (coherent receiver)



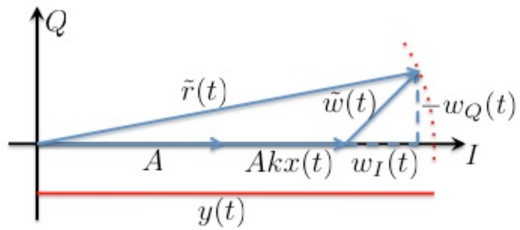
Graphical Representation for SSB

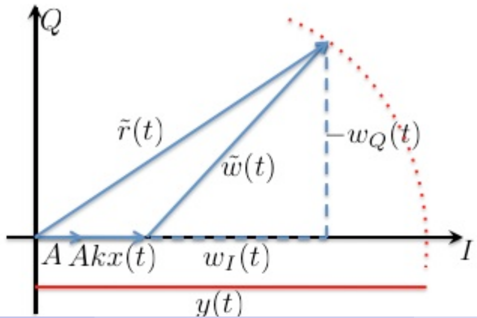


Graphical Representation for QAM



Graphical Representation for AM (envelope detector)





Graphical Representation for Phase Error