Analog Communications

— Lecture 06 —
Performance of Angle Modulation over AWGN Channel

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Outline

1. Transmitted and Received Signals

2. Input and Output SNR

Transmitted Signal

The transmitted signal in the case of angle modulation is

$$z(t) = A\cos\left(2\pi f_0 t + \psi_x(t)\right)$$

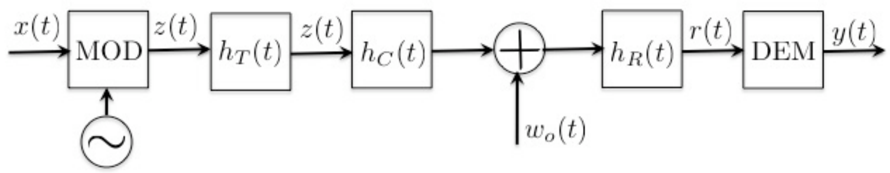
where

- PM: $\psi_x(t) = s x(t)$
- FM: $\psi_x(t) = s\int_{t_0}^{t} x(\tau)d\tau$

Denote

- B_x the bandwidth of the information signal
- B_z the band of the transmitted signal

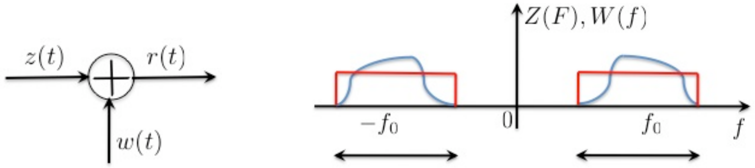
AWGN Channel



Assumptions:

- $H_T(f)H_C(f)H_R(f)$ is flat over B_z
- $w_o(t)$ is white and Gaussian

Received Signal over AWGN



The received signal in the case of AWGN channel is

$$r(t) = z(t) + w(t)$$

where

$$P_w(f) = \begin{cases} \frac{\eta_0}{2} & |f| \in B_z \\ 0 & |f| \notin B_z \end{cases}$$

and assuming wideband modulation ($\beta \gg 1$)

$$B_z = 2(1+\beta)B_x \approx 2\beta B_x = 2\Delta f_{max}$$

Baseband Representation

The received signal is a band-pass signal
(being the sum of two band-pass signals)

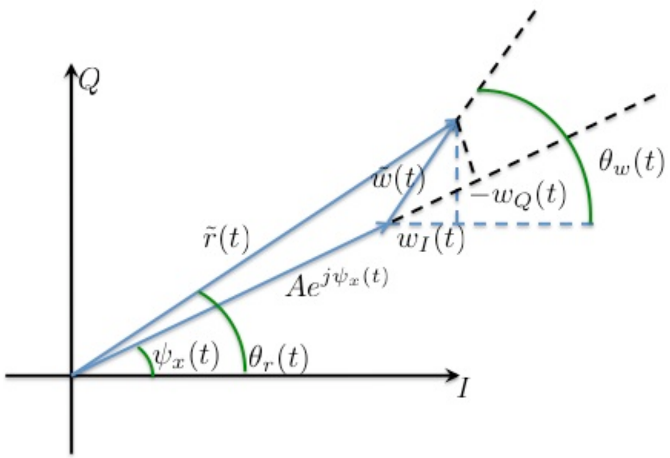
$$r(t) = A\cos\left(2\pi f_0 t + \psi_x(t)\right) + w_I(t)\cos(2\pi f_0 t) + w_Q(t)\sin(2\pi f_0 t)$$

thus

$$\begin{aligned} \tilde{w}(t) &= w_I(t) - jw_Q(t) = \rho_w(t)\exp(j\theta_w(t)) \\ \mathring{r}(t) &= (A\exp(j\psi_x(t)) + \tilde{w}(t))\exp(j2\pi f_0 t) \\ \tilde{r}(t) &= A\exp(j\psi_x(t)) + \tilde{w}(t) = \rho_r(t)\exp(j\theta_r(t)) \end{aligned}$$

Again we are assuming that $f_0 \gg B_z$

Graphical Representation of the Received Signal



Ideal Angle Receiver

It is easy to show that

$$\theta_r(t) - \psi_x(t) = \arctan\left(\frac{\rho_w(t)\sin(\theta_w(t) - \psi_x(t))}{A + \rho_w(t)\cos(\theta_w(t) - \psi_x(t))}\right)$$

An ideal frequency discriminator would provide

$$y(t) = \frac{\dot{\theta}_r(t)}{2\pi} = \frac{\dot{\psi}_x(t)}{2\pi} + \frac{1}{2\pi}\frac{\partial}{\partial t}\left(\arctan\left(\frac{\rho_w(t)\sin(\theta_w(t) - \psi_x(t))}{A + \rho_w(t)\cos(\theta_w(t) - \psi_x(t))}\right)\right)$$

SNR at the Input of the Receiver

The SNR at the input of the receiver is

$$\mathrm{SNR}_{in} = \frac{A^2/2}{2\Delta f_0 \eta_o}$$

Two cases are typically considered

- high SNR, i.e. $\mathrm{SNR}_{in} \gg 1$
- low SNR, i.e. $\mathrm{SNR}_{in} \ll 1$

High SNR (1/2)

If $\mathrm{SNR}_{in} \gg 1$ then we can assume that $\rho_w(t) \ll A$ and

$$\begin{aligned}
\theta_r(t) - \psi_x(t) &\approx \arctan\left(\frac{\rho_w(t)\sin(\theta_w(t) - \psi_x(t))}{A}\right) \\
&\approx \arctan\left(\frac{\rho_w(t)\sin(\theta_w(t))}{A}\right) \\
&\approx -\arctan\left(\frac{w_Q(t)}{A}\right) \\
&\approx -\frac{w_Q(t)}{A}
\end{aligned}$$

then

$$y(t) \approx \frac{\dot{\psi}_x(t)}{2\pi} - \frac{\dot{w}_Q(t)}{2\pi A}$$

High SNR (2/2)

Focus on FM case

$$y(t) \approx \frac{s}{2\pi} x(t) - \frac{\dot{w}_Q(t)}{2\pi A}$$

- the bandwidth of $z(t)$, thus of $\dot{w}_Q(t)$, is much larger than the bandwidth of $x(t)$
- an LPF with unitary gain and cutoff frequency B_x, after the ideal angle receiver, enhances the performance

$$y_{LP}(t) \approx \frac{s}{2\pi} x(t) - \frac{\dot{w}_Q(t)}{2\pi A} \star h_{LP}(t)$$

PDF at the Output of the Receiver

The PSD of the output is:

$$P_{y_{LP}}(f) \approx \left(\frac{s}{2\pi}\right)^2 P_x(f) - \left|\frac{j2\pi f}{2\pi A}\right|^2 |H_{LP}(f)|^2 P_{w_Q}(f)$$

Assuming an ideal LPF, i.e. $H_{LP}(f) = \mathrm{rect}\left(\frac{f}{2B_x}\right)$, we get

$$P_{y_{LP}}(f) \approx \frac{s^2}{4\pi^2} P_x(f) - \frac{f^2}{A^2} P_{w_Q}(f) \mathrm{rect}\left(\frac{f}{2B_x}\right)$$

It is easy to identify a signal contribution and a noise contribution

SNR at the Output of the Receiver (1/2)

The SNR at the output of the receiver is

$$
\begin{aligned}
\mathrm{SNR}_{out} &= \frac{\int_{\mathbb{R}} \frac{s^2}{4\pi^2} P_x(f) df}{\int_{\mathbb{R}} \frac{f^2}{A^2} P_{w_Q}(f) \mathrm{rect}\left(\frac{f}{2B_x}\right) df} \\
&= \frac{\frac{s^2}{4\pi^2} P_x}{\frac{\eta_o}{A^2} \int_{-B_x}^{B_x} f^2 df} \\
&= \frac{3A^2 s^2 P_x}{8\pi^2 \eta_o B_x^3} \\
&= 3 \left(\frac{A^2/2}{\eta_o B_x}\right) \left(\frac{P_x}{x_m^2}\right) \left(\frac{\Delta f_{max}}{B_x}\right)^2
\end{aligned}
$$

SNR at the Output of the Receiver (2/2)

Define

$$
m_p = \sqrt{\frac{P_x}{x_m^2}}
$$

and recall

$$
\begin{aligned}
\gamma &= \frac{P_z}{\eta_o B_x} \\
m_f &= \frac{\Delta f_{max}}{B_x}
\end{aligned}
$$

then

$$
\mathrm{SNR}_{out} = 3 m_p^2 \beta^2 \gamma
$$

Also, the spectral efficiency is

$$
\eta_s = \frac{B_x}{B_z} = \frac{1}{2\beta}
$$