Digital Communications

— Lecture 08 —

Comparison of Digital-Modulation Schemes

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Outline

Resources of Interest

- bit-rate
  related to transmission speed
  preferred to be high
- bandwidth
  related to frequency occupation
  preferred to be small
- energy
  related to power consumption
  preferred to be low
- error probability
  related to transmission quality
  preferred to be low

Spectral Efficiency (1/2)

The Spectral Efficiency (measured in b/s/Hz) of a modulation scheme with transmission rate R and bandwidth B is defined as

$$\rho = R/B$$

Consider an M-ary modulation scheme with baseband pulse $s(t)$ with duration T and null-to-null bandwidth $2/T$, then $R = (1/T)\log_2(M)$

Modulation Format	Bandwidth	Spectral Efficiency
ASK (DSB)	$2/T$	$(1/2)\log_2(M)$
ASK (SSB)	$1/T$	$\log_2(M)$
PSK	$2/T$	$(1/2)\log_2(M)$
QAM (M)	$2/T$	$(1/2)\log_2(M)$
QAM (M^2)	$2/T$	$\log_2(M)$
FSK ($\Delta_f = 1/2T$)	$(M+3)/2T$	$(2/(M+3))\log_2(M)$
FSK ($\Delta_f = 1/T$)	$(M+1)/T$	$(1/(M+1))\log_2(M)$

Spectral Efficiency (2/2)

A different bandwidth definition scales the spectral efficiency

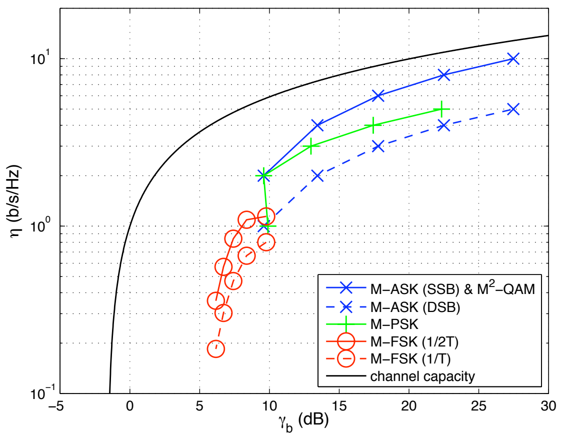
Halving null-to-null bandwidth (almost as 3dB bandwidth):

Modulation Format	Bandwidth	Spectral Efficiency
ASK (DSB)	$1/T$	$\log_2(M)$
ASK (SSB)	$1/2T$	$2\log_2(M)$
PSK	$1/T$	$\log_2(M)$
QAM (M)	$1/T$	$\log_2(M)$
QAM (M^2)	$1/T$	$2\log_2(M)$
FSK ($\Delta_f = 1/2T$)	$(M+3)/4T$	$(4/(M+3))\log_2(M)$
FSK ($\Delta_f = 1/T$)	$(M+1)/2T$	$(2/(M+1))\log_2(M)$

SER and BER

Modulation Format	SER	BER
ASK	$2(1-\frac{1}{M})Q\left(\sqrt{\frac{6\log_2(M)}{M^2-1}\gamma_b}\right)$	$\frac{1}{\log_2(M)}\mathrm{SER}$
PSK	$2Q\left(\sqrt{2\log_2(M)\gamma_b}\sin(\pi/M)\right)$	$\frac{1}{\log_2(M)}\mathrm{SER}$
QAM (M^2)	$4(1-\frac{1}{M})Q\left(\sqrt{\frac{6\log_2(M)}{M^2-1}\gamma_b}\right)$	$\frac{1}{2\log_2(M)}\mathrm{SER}$
FSK	$(M-1)Q\left(\sqrt{\log_2(M)\gamma_b}\right)$	$\frac{1}{2}\mathrm{SER}$

Comparison (1/2)



The curves are obtained for BER$= 10^{-5}$

Comparison (2/2)

- Bandwidth-limited region ($\rho > 1$)
  - spectrally-efficient and power-inefficient modulations
  - ASK, PSK, QAM, etc.
- Power-limited region ($\rho < 1$)
  - spectrally-inefficient and power-efficient modulations
  - FSK, Biorthogolan, Simplex, etc.

- Channel coding aims at improving performance without sacrificing (too much) the rate or the bandwidth
- The price is complexity

Channel Capacity (1/2)

Shannon's formula for channel capacity:

$$C = B\log_2(1+\mathrm{SNR})$$

where

$$\mathrm{SNR} = \frac{P_{\mathrm{av}}}{B\eta_0} = \frac{C\mathcal{E}_b}{B\eta_0} = \frac{C}{B}\gamma_b$$

thus

$$\begin{aligned}(C/B) &= \log_2(1+(C/B)\gamma_b)\\ \gamma_b &= \frac{2^{C/B}-1}{C/B}\end{aligned}$$

Channel Capacity (2/2)

$$\gamma_b = \frac{2^\rho - 1}{\rho}$$

The SNR increases exponentially with the spectral efficiency

$$\lim_{\rho\to\infty}\frac{2^\rho-1}{\rho} = \infty$$

$$\lim_{\rho\to 0}\frac{2^\rho-1}{\rho} = \ln(2) = -1.6\ \mathrm{dB}$$

Derivation: Mutual Information and Capacity

A discrete memoryless channel with input $X \in \{x_1,\dots,x_N\}$ and output $Y \in \{y_1,\dots,y_N\}$ has capacity $C = \max_{\Pr(X)} I(X,Y)$ where

$$I(X,Y) = \sum_{n=1}^{N}\sum_{m=1}^{N}\Pr(x_n)\Pr(y_m|x_n)\log_2\left(\frac{\Pr(y_m|x_n)}{\Pr(x_n)}\right)$$

while in the case of real-valued input and output

$$I(X,Y) = \int_{\mathbb{R}} dx \int_{\mathbb{R}} dy\, f_X(x) f_{Y|X}(y|x)\log_2\left(\frac{f_{Y|X}(y|x)}{f_X(x)}\right)$$

Derivation: Bandlimited Waveform Channel with AWGN

AWGN channel

$$y(t) = x(t) + w(t)$$

with symbol duration T and bandwidth B

Sampling theory provides the following discrete-time model

$$y_n = x_n + w_n\ , \qquad n = 1,\dots,N$$

where $y(t) = \sum_{n=1}^{N} y_n\psi_n(t)$ and $x(t) = \sum_{n=1}^{N} x_n\psi_n(t)$, with

$$\psi_n(t) = \mathrm{sinc}\left(\frac{t}{2B} - n\right) \qquad \text{and} \qquad N = 2BT$$

$$f(\boldsymbol{y}|\boldsymbol{x}) = \prod_{n=1}^{N} f(y_n|x_n) = \prod_{n=1}^{N}\frac{1}{\sqrt{\pi\eta_0}}\exp\left(-\frac{(y_n-x_n)^2}{\eta_0}\right)$$

Derivation: Capacity Expression

The maximum of

$$I(\boldsymbol{x}, \boldsymbol{y}) = \sum_{n=1}^{N} \int_{\mathbb{R}} dx_n \int_{\mathbb{R}} dy_n f(x_n) f(y_n|x_n) \log_2 \left(\frac{f(y_n|x_n)}{f(x_n)} \right)$$

is achieved when $x_n \sim \mathcal{N}(0, \sigma_x^2)$, then

$$\max(I) = \sum_{n=1}^{N} \frac{1}{2} \log_2 \left(1 + \frac{2\sigma_x^2}{\eta_0} \right) = \frac{N}{2} \log_2 \left(1 + \frac{2\sigma_x^2}{\eta_0} \right)$$

Also, $P_{\text{av}} = \frac{1}{T} \int_0^T \mathbb{E}\{x^2(t)\} dt = \frac{1}{T} \sum_{n=1}^{N} \mathbb{E}\{x_n^2\} = \frac{N\sigma_x^2}{T}$, then

$$\max(I) = BT \log_2 \left(1 + \frac{P_{\text{av}}}{B\eta_0} \right)$$

and finally, $C = \max(I)/T$