Digital Communications

— Lecture 10 —
Communications over Bandlimited Channels

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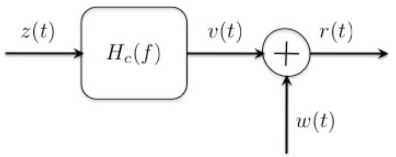
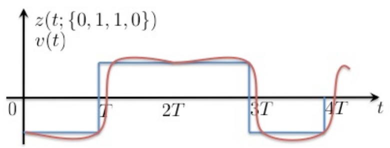
Outline

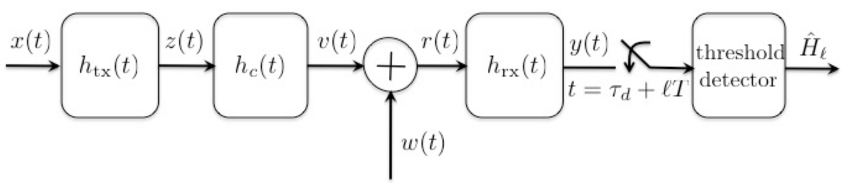
Inter Symbol Interference





- In addition to the AWGN, the channel is modeled through an LTI system with impulse response $h_c(t)$
- The convolution of the transmitted pulse with the channel impulse response spreads the pulse duration at receiver location outside the generic symbol interval
- The signal received within the kth symbol interval depends on the kth transmitted symbols and also on other transmitted symbols
- This phenomenon is denoted Inter Symbol Interference (ISI)

ISI in ASK Transmission (1/2)



$$
\begin{aligned}
x(t) &= \sum_{k=-\infty}^{+\infty} L_k \delta(t-kT), \qquad L_k \in \{A_1,\dots,A_M\} \\
z(t) &= x(t) \star h_{\rm tx}(t) = \sum_{k=-\infty}^{+\infty} L_k h_{\rm tx}(t-kT) \\
v(t) &= z(t) \star h_c(t), \qquad r(t) = z(t) \star h_c(t) + w(t)
\end{aligned}
$$

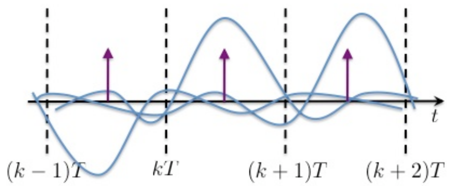
Denote

$$
\begin{aligned}
p(t) &= h_{\rm tx}(t) \star h_c(t) \star h_{\rm rx}(t) \\
w_o(t) &= w(t) \star h_{\rm rx}(t)
\end{aligned}
$$

ISI in ASK Transmission (2/2)

τ_d takes into account for transmission delay

$$
\begin{aligned}
y(t) &= \sum_{k=-\infty}^{+\infty} L_k p(t-kT) + w_o(t) \\
y_\ell &= y(\tau_d + \ell T) = \sum_{k=-\infty}^{+\infty} L_k p(\tau_d + \ell T - kT) + w_o(\tau_d + \ell T) \\
&= L_\ell p(\tau_d) + \underbrace{\sum_{k \neq \ell} L_k p(\tau_d + \ell T - kT)}_{\text{ISI contribution}} + w_o(\tau_d + \ell T)
\end{aligned}
$$



Nyquist Criterion for Zero ISI (1/2)

Transmit and receive filters must be designed such that the equivalent pulse $p(t)$ satisfies

$$
\begin{cases} p(\tau_d) = 1 \\ p(\tau_d + kT) = 0 & \forall k \neq 0 \end{cases}
$$

or else, denoting $p_d(t) = p(t + \tau_d)$,

$$
p_d(kT) = \begin{cases} 1 & k = 0 \\ 0 & \forall k \neq 0 \end{cases}
$$

or else

$$
p_d(t) \cdot \sum_{k=-\infty}^{+\infty} \delta(t - kT) = \delta(t)
$$

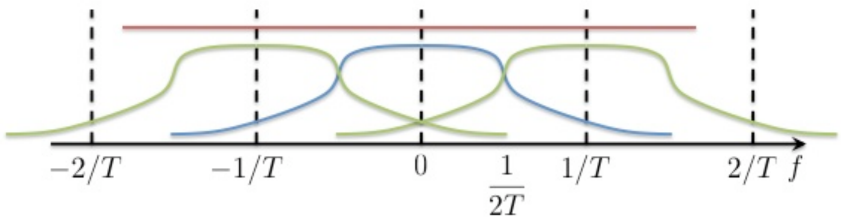
Nyquist Criterion for Zero ISI (2/2)

In frequency domain, last equality provides

$$
P_d(f) \star \frac{1}{T} \sum_{k=-\infty}^{+\infty} \delta\left(f - \frac{k}{T}\right) = 1
$$

thus, in frequency domain, Nyquist criterion for Zero ISI is

$$
\sum_{k=-\infty}^{+\infty} P_d\left(f - \frac{k}{T}\right) = T
$$

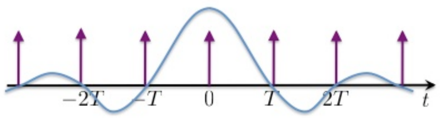


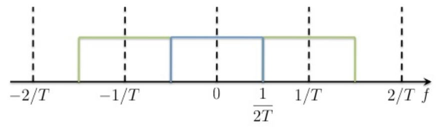
Nyquist Pulses

Denoting W the bandwidth of the channel, it should be clear that

- if $T < \frac{1}{2W}$ there is no possibility to have zero ISI
- if $T = \frac{1}{2W}$ there is one possibility to have zero ISI
  i.e. the sinc pulse
- if $T > \frac{1}{2W}$ there are many possibilities to have zero ISI
  one popular choice is the raised cosine pulse

Nyquist Pulses: sinc pulse

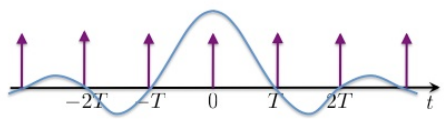


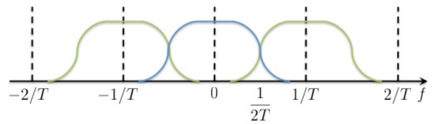


$$
\begin{aligned}
p_d(t) &= \operatorname{sinc}\left(\frac{t}{T}\right) \\
P_d(f) &= T\operatorname{rect}\left(Tf\right)
\end{aligned}
$$

- $W = \frac{1}{2T}$
- ideal pulse
- non-causal
- discontinuous in frequency domain

Nyquist Pulses: raised cosine pulse





$$
p_d(t) = \operatorname{sinc}\left(\frac{t}{T}\right)\frac{\cos\left(\frac{\pi\beta t}{T}\right)}{1-\left(\frac{2\beta t}{T}\right)^2}
$$

$$
P_d(f) = \begin{cases} T & 0 \le |f| \le \frac{1-\beta}{2T} \\ \frac{T}{2}\left(1+\cos\left(\frac{\pi T}{\beta}\left(|f|-\frac{1-\beta}{2T}\right)\right)\right) & \frac{1-\beta}{2T} < |f| \le \frac{1+\beta}{2T} \\ 0 & |f| \ge \frac{1+\beta}{2T} \end{cases}
$$

- $W > \frac{1}{2T}$
- non-causal
- $\beta \in [0,1]$ is called roloff factor and represents a measure for the excess bandwidth w.r.t. $\frac{1}{2T}$