Digital Communications

— Lecture 11 —
Wireless Channels

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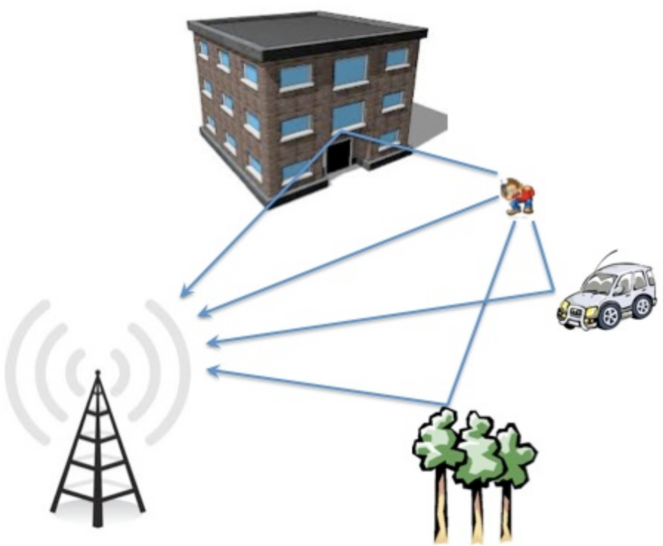
Outline

1. Path-Loss, Large-Scale-Fading, Small-Scale Fading
2. Simplified Models
3. Multipath Fading Channel
4. WSSUS Channel, Correlations and Power Spectra
5. Underspread and Overspread Channels

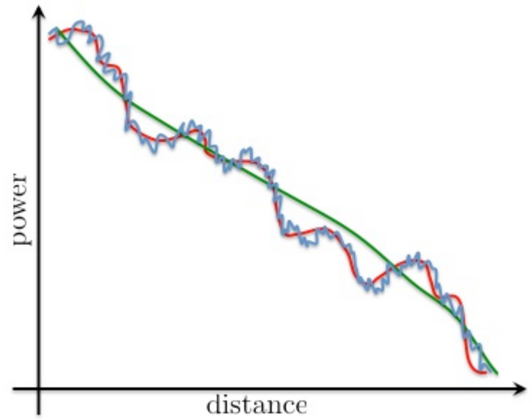
Wireless Channels



Main Phenomena (1/2)

- Path-Loss
  - due to dissipation of the radiated power
  - variations occur over very-large distances (100m-1000m)
  - modeled deterministically
- Shadowing or Large-Scale Fading
  - due to obstruction of obstacles between TX and RX
  - variations occur over obstacle size (1m-100m)
  - modeled stochastically
- Multipath Fading or Small-Scale Fading
  - due to constructive/destructive combination of signal replicas at the RX
  - variations occur over signal wavelength (1mm-1m)
  - modeled stochastically

Main Phenomena (2/2)



- Path-Loss (green)
- Shadowing (red)
- Multipath Fading (blue)

We will focus on Small-Scale Fading

One Path, Static TX, Static RX

TX in the origin transmits one single sinusoid

$$x(t) = \exp(j2\pi f t)$$

RX at point $\boldsymbol{p} = (r, \theta, \psi)$ receives

$$y(t) = \frac{\alpha(\theta, \psi, f)}{r} \exp\left(j2\pi f\left(t - \frac{r}{c}\right)\right)$$

$$H(f) = \frac{y(t)}{x(t)} = \frac{\alpha(\theta, \psi, f)}{r} \exp\left(-j2\pi f \frac{r}{c}\right)$$

Two Paths, Static TX, Static RX

TX in the origin transmits one single sinusoid

$$x(t) = \exp(j2\pi f t)$$

RX at point $\boldsymbol{p} = (r, \theta, \psi)$ receives one direct ray (LOS) and one reflected ray (overall distance $r_1 = r + \Delta r$)

$$y(t) = \frac{\alpha(\theta, \psi, f)}{r} \exp\left(j2\pi f\left(t - \frac{r}{c}\right)\right) + \frac{\alpha(\theta, \psi, f)}{r_1} \exp\left(j2\pi f\left(t - \frac{r_1}{c}\right)\right)$$

$$\begin{aligned} H(f) &= \frac{\alpha(\theta, \psi, f)}{r} \exp\left(-j2\pi f \frac{r}{c}\right) + \frac{\alpha(\theta, \psi, f)}{r + \Delta r} \exp\left(-j2\pi f \frac{r + \Delta r}{c}\right) \\ \Delta\theta &= 2\pi \frac{\Delta r}{\lambda} \end{aligned}$$

Small received power for $\Delta r = n\lambda/2$ and $n \in \mathbb{Z}$

One Path, Static TX, Mobile RX

TX in the origin transmits one single sinusoid

$$x(t) = \exp(j2\pi f t)$$

RX at point $\boldsymbol{p}(t) = (r(t), \theta, \psi)$, with $r(t) = r_0 + vt$, receives

$$\begin{aligned} y(t) &= \frac{\alpha(\theta, \psi, f)}{r(t)} \exp\left(j2\pi f\left(t - \frac{r(t)}{c}\right)\right) \\ &= \frac{\alpha(\theta, \psi, f)}{r_0 + vt} \exp\left(-j2\pi f \frac{r_0}{c}\right) \exp\left(-j2\pi f\left(1 - \frac{v}{c}\right) t\right) \end{aligned}$$

$$\frac{y(t)}{x(t)} = \frac{\alpha(\theta, \psi, f)}{r_0 + vt} \exp\left(-j2\pi f \frac{r_0}{c}\right) \exp\left(-j2\pi f \frac{v}{c} t\right)$$

Doppler shift of fv/c

Multipath Fading Channel (1/2)

- Assume no AWGN for simplicity
- Multiple paths from TX to RX

$$y_{BP}(t) = \sum_{\ell=1}^{N(t)} \alpha_\ell(t) x_{BP}(t - \tau_\ell(t))$$

- $N(t)$ is the number of resolvable scatterers at time t
- $\alpha_\ell(t)$ is the attenuation introduced by the ℓth scatterer at time t
- $\tau_\ell(t)$ is the delay introduced by the ℓth scatterer at time t

- Baseband representation:

$$s_{BP}(t) = \Re\{s(t) \exp(j2\pi f_c t)\}$$

Multipath Fading Channel (2/2)

- Baseband I/O relation

$$y(t) = \sum_{\ell=1}^{N(t)} \underbrace{\alpha_\ell(t) \exp(-j2\pi f_c \tau_\ell(t))}_{c_\ell(t)=\alpha_\ell(t)\exp(-j\vartheta_\ell(t))} x(\tau - \tau_\ell(t))$$

- Stochastic LTV channel:

$$\begin{aligned} y(t) &= \int_{\mathbb{R}} h(t,\tau) x(t-\tau) d\tau \\ h(t,\tau) &= \sum_{\ell=1}^{N(t)} c_\ell(t) \delta(\tau - \tau_\ell(t)) \end{aligned}$$

- Generalization for Continuum of Multipath Components

$$h(t,\tau) = \alpha(t,\tau) \exp(-j2\pi f_c \tau)$$

Stochastic Impulse Response (1/2)

- Large variations of $\alpha_\ell(t)$ are needed to impact the received signal
- Small variations of $\vartheta_\ell(t)$ do impact the received signal

For a given t_0, we assume that $h(t_0, \tau)$ is

- deterministic
- summable

For a given τ_0, we assume that $h(t, \tau_0)$ is

- stochastic
- not summable

The (time-variant) $\mathcal{F}$-transform of the channel response is

$$H(t,f) = \int_{\mathbb{R}} h(t,\tau) \exp(-j2\pi f \tau) d\tau$$

Stochastic Impulse Response (2/2)

- The channel response is modeled as a complex-valued Gaussian random process (Central Limit Theorem)

- Components may add up constructively or destructively causing fluctuations of the received signal power, i.e. **fading**

- Rayleigh fading
  - $h(t,\tau)$ is a zero-mean complex-valued Gaussain process
  - suitable for NLOS channel
- Ricean fading
  - $h(t,\tau)$ is a nonzero-mean complex-valued Gaussain process
  - suitable for LOS channel
- Nakagami fading
  - matching empirical measurements

WSSUS Channel

- Wide Sense Stationary (WSS):

$$\mathbb{E}\{h(t_1,\tau_1\}h^*(t_2,\tau_2)\} = \mathbb{E}\{h(0,\tau_1\}h^*(t_2-t_1,\tau_2)\}$$

- Uncorrelated Scattering (US):

$$\mathbb{E}\{h(t_1,\tau_1\}h^*(t_2,\tau_2)\} = \begin{cases} \mathbb{E}\{h(t_1,\tau_1\}h^*(t_2,\tau_1)\} & \tau_1 = \tau_2 \\ 0 & \tau_1 \neq \tau_2 \end{cases}$$

WSSUS Channel

$$\mathbb{E}\{h(t,\tau\}h^*(t+\Delta t,\tau')\} = \phi_h(\Delta t,\tau)\delta(\tau-\tau')$$

Multipath Intensity Profile

$\phi_h(0,\tau)$ is called Multipath Intensity Profile or Delay Power Spectrum and provides the average power output as a function of delay

$$\begin{aligned} y(t) &= \int_{\mathbb{R}} h(t,\tau)x(t-\tau)d\tau \\ K_y(t,t-\Delta t) &= \mathbb{E}\{y(t)y^*(t-\Delta t)\} \\ &= \int_{\mathbb{R}} \phi_h(\Delta t,\tau)x(t-\tau)x^*(t-\Delta t-\tau)d\tau \\ P_y(t) &= \mathbb{E}\{|y(t)|^2\} = K_y(t,t) \\ &= \int_{\mathbb{R}} \phi_h(0,\tau)|x(t-\tau)|^2 d\tau \\ P_y &= \overline{P_y(t)} \\ &= P_x \int_{\mathbb{R}} \phi_h(0,\tau)d\tau \end{aligned}$$

Spaced-Time Spaced-Frequency Correlation Function

$\Phi_h(\Delta t,\Delta f)$ is called spaced-time spaced-frequency correlation function

$$\begin{aligned} \Phi_h(\Delta t,\Delta f) &= \mathbb{E}\{H(t,f)H^*(t-\Delta t,f-\Delta f)\} \\ &= \int_{\mathbb{R}} \phi_h(\Delta t,\tau)\exp(-j2\pi\Delta f\ \tau)d\tau \\ &= \mathcal{F}\{\phi_h(\Delta t,\tau)\}(\tau\to\Delta f) \end{aligned}$$

$$y(t) = \int_{\mathbb{R}} h(t,\tau)x(t-\tau)d\tau = \int_{\mathbb{R}} H(t,f)X(f)\exp(j2\pi ft)df$$
$$x_n(t) = e^{j2\pi f_n t} \to X_n(f) = \delta(f-f_n) \to y_n(t) = H(t,f_n)e^{j2\pi f_n t}$$

$$\begin{aligned} \mathbb{E}\{y_n(t)y_m^*(t)\} &= \mathbb{E}\{H(t,f_n)H^*(t,f_m)\}\exp(j2\pi(f_n-f_m)t) \\ &= \Phi_h(0,f_n-f_m)\exp(j2\pi(f_n-f_m)t) \end{aligned}$$

Multipath Delay Spread and Coherence Bandwidth (1/2)

- $\phi_h(\Delta t,\tau)$ as a function of τ is a power spectrum
- The (essential) duration T_m of the function $\phi_h(0,\tau)$ is denoted Multipath Delay Spread
- $\Phi_h(\Delta t,\Delta f)$ as a function of Δf is an autocorrelation
- The (essential) duration B_c of the function $\Phi_h(0,\Delta f)$ is denoted Coherence Bandwidth

$$T_m B_c \approx 1$$

Multipath Delay Spread and Coherence Bandwidth (2/2)

- Two spectral components (of the transmitted signal) whose separation is larger than B_c are uncorrelated at the receiver
- The linear distortion is due to frequency selectivity
  - if $B_x \gg B_c$, then the signal undergoes linear distortion
  - if $B_x \ll B_c$, then the signal does not undergo linear distortion
- The transmitted signal is spread in time (due to the convolution with the channel response) and its duration is increased of T_m
- The linear distortion is due to inter-symbol interference
  - if $T_x \ll T_m$, then the signal undergoes linear distortion
  - if $T_x \gg T_m$, then the signal does not undergo linear distortion

Doppler Power Spectrum

$s_h(\lambda, 0)$ is called Doppler Power Spectrum and describes the time varying effects in terms of Doppler shifts

$$\begin{aligned} s_h(\lambda, \Delta f) &= \int_{\mathbb{R}} \Phi_h(\Delta t, \Delta f) \exp(-j2\pi\lambda\Delta t) d\Delta t \\ &= \mathcal{F}\left\{\Phi_h(\Delta t, \Delta f)\right\}(\Delta t \to \lambda) \end{aligned}$$

$$x_n(t) = \mathrm{e}^{j2\pi f_n t} \to X_n(f) = \delta(f - f_n) \to y_n(t) = H(t, f_n)\mathrm{e}^{j2\pi f_n t}$$

$$\begin{aligned} \mathbb{E}\{y(t)y^*(t-\Delta t)\} &= \mathbb{E}\{H(t, f_n)H^*(t-\Delta t, f_n)\} \exp(j2\pi f_n \Delta t) \\ &= \Phi_h(\Delta t, 0) \exp(j2\pi f_n \Delta t) \end{aligned}$$

Coherence Time and Doppler Spread (1/2)

- $\Phi_h(\Delta t, \Delta f)$ as a function of Δt is an autocorrelation
- The (essential) duration T_c of the function $\Phi_h(\Delta t, 0)$ is denoted Coherence Time
- $s_h(\lambda, \Delta f)$ as a function of λ is a power spectrum
- The (essential) duration B_d of the function $s_h(\lambda, 0)$ is denoted Doppler Spread

$$T_c B_d \approx 1$$

Coherence Time and Doppler Spread (2/2)

- Two delayed versions (of the transmitted signal) whose separation is larger than T_c are uncorrelated at the receiver
- The non-linear distortion is due to time-varying effects
  - if $T_x \gg T_c$, then the signal undergoes non-linear distortion
  - if $T_x \ll T_c$, then the signal does not undergo non-linear distortion
- The transmitted signal is spread in Doppler domain and its spectrum is increased of B_d
- The non-linear distortion is due to Doppler spreading
  - if $B_x \ll B_d$, then the signal undergoes non-linear distortion
  - if $B_x \gg B_d$, then the signal does not undergo non linear distortion

Scattering Function

$S_h(\lambda,\tau)$ is called scattering function and is related to the previous functions as follows

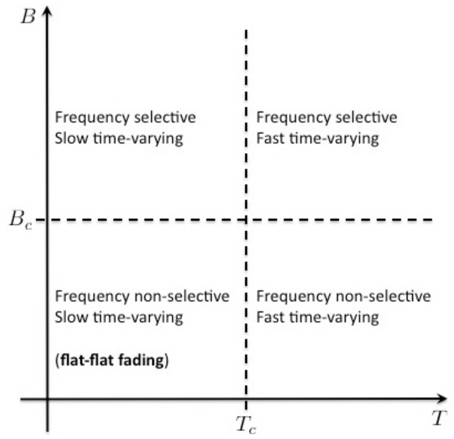
$$\begin{aligned} S_h(\lambda,\tau) &= \mathcal{F}\left\{\phi_h(\Delta t,\tau)\right\}(\Delta t \to \lambda) \\ &= \int_{\mathbb{R}} \phi_h(\Delta t,\tau)\exp(-j2\pi\lambda\Delta t)d\Delta t \end{aligned}$$

and

$$\begin{aligned} S_h(\lambda,\tau) &= \mathcal{F}^{-1}\left\{s_h(\lambda,\Delta f)\right\}(\Delta f \to \tau) \\ &= \int_{\mathbb{R}} s_h(\lambda,\Delta f)\exp(+j2\pi\tau\Delta f)d\Delta f \end{aligned}$$

Four Cases

- Digital-modulation scheme with symbol duration T and bandwidth B
- Wireless channel with coherence time T_c and coherence bandwidth B_c (or delay spread $T_m = 1/B_c$ and Doppler spread $B_d = 1/T_c$)



Frequency Non-Selective Channel

In general

$$r(t) = \int_{\mathbb{R}} h(t,\tau)s(t-\tau)d\tau = \int_{\mathbb{R}} H(t,f)S(f)\exp(j2\pi ft)df$$

however, $B \ll B_c$ means that all spectral components of $s(t)$ undergo the same attenuation and phase shift, i.e.

$$H(t,f) \approx H(t,0) \qquad \forall f \in [-B,B]$$

or equivalently that the ISI is negligible, and then

$$r(t) = \underbrace{\alpha(t)\exp(-j\vartheta(t))}_{H(t,0)}s(t)$$

Slow Time-Varying Channel

In general

$$r(t) = \int_{\mathbb{R}} h(t,\tau)s(t-\tau)d\tau = \int_{\mathbb{R}} H(t,f)S(f)\exp(j2\pi ft)df$$

however, $T \ll T_c$ means that nonlinear distortion introduced by time-varying effects is negligible, i.e.

$$h(t,\tau) \approx h(0,\tau) \qquad \forall t \in [0,T]$$

or equivalently there is no Doppler shift, and then

$$\begin{aligned} r(t) &= \underbrace{\left(\sum_{\ell=1}^{N} c_\ell \delta(t-\tau_\ell)\right)}_{h(0,t)} \star s(t) \\ &= \sum_{\ell=1}^{N} c_\ell s(t-\tau_\ell) \end{aligned}$$

Flat-Flat Fading Channel (with AWGN)

Most signals have $TB \approx 1$, then

- if $T_m B_d < 1$ (or equiv. $B_c T_c > 1$) the channel is said underspread
- if $T_m B_d > 1$ (or equiv. $B_c T_c < 1$) the channel is said overspread

Underspread channels allow to select $s(t)$ such that

- $T < T_c$
- $B < B_c$

i.e. transmitting over a frequency non-selective slow time-varying channel (often denoted flat-flat fading)

The signal model is

$$r(t) = Hs(t) + w(t)$$