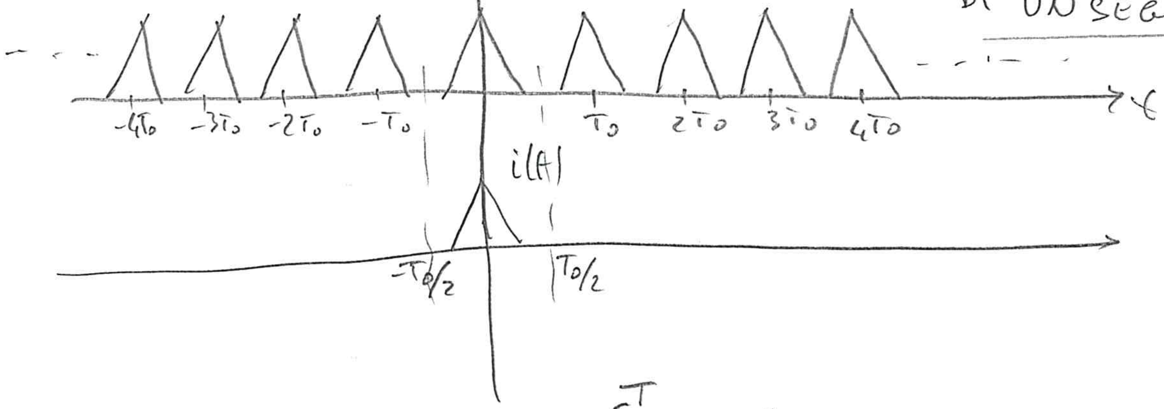
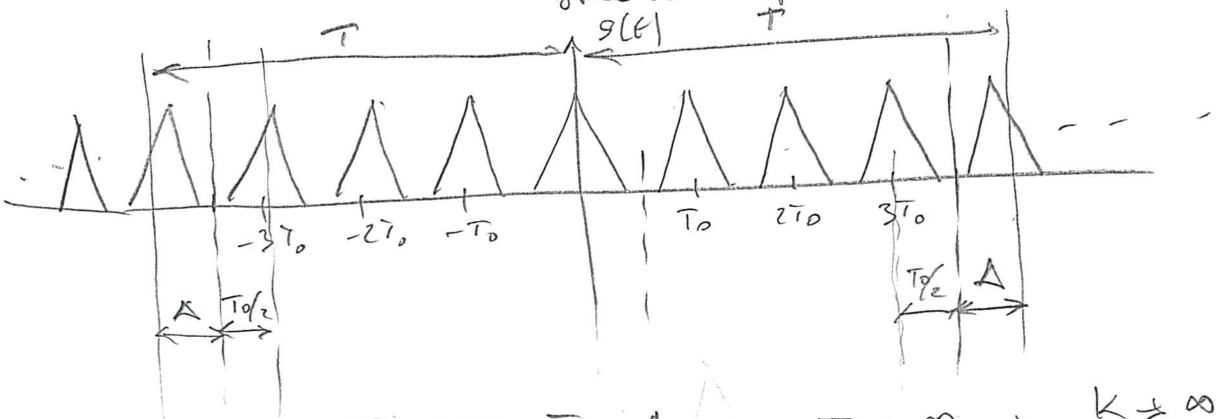
AUTOCORRELAZIONE E SPETTRO DI POTENZA DI UN SEGNALE PERIODICO

$$s(t)=\sum_{n=-\infty}^{+\infty} i(t-nT_0)$$



$$R_s(\tau) \triangleq \lim_{T\to\infty} \frac{1}{2T} \underbrace{\int_{-T}^{T} s(t)\, s^*(t-\tau)\, dt}_{\text{studiamo questo integrale graficamente}}$$



$$T = KT_0 + \frac{T_0}{2} + \Delta \qquad T\to\infty \Rightarrow K\to\infty$$

$$0 < \Delta < T_0$$

$$\int_{-KT_0-\frac{T_0}{2}-\Delta}^{+KT_0+\frac{T_0}{2}+\Delta} s(t)\, s^*(t-\tau)\, dt = \underbrace{\int_{-KT_0-\frac{T_0}{2}}^{KT_0+\frac{T_0}{2}} s(t)\, s^*(t-\tau)\, dt}_{\text{Consideriamo per ora solo questo.}} + \underbrace{\int_{-KT_0-\frac{T_0}{2}-\Delta}^{-KT_0-\frac{T_0}{2}} s(t) s^*(t-\tau) dt}_{a} + \underbrace{\int_{KT_0+\frac{T_0}{2}}^{KT_0+\frac{T_0}{2}+\Delta} s(t) s^*(t-\tau) dt}_{b}$$

$$\int_{-KT_0+\frac{T_0}{2}}^{KT_0+\frac{T_0}{2}} s(t)\, s^*(t-\tau)\, dt = \int_{-KT_0+\frac{T_0}{2}}^{KT_0+\frac{T_0}{2}} \underbrace{\sum_{n_1=-K}^{K} i(t-n_1T_0)}_{\text{solo } 2K+1 \text{ termini}} \sum_{n_2=-\infty}^{+\infty} i^*(t-n_2T_0-\tau)\, dt$$

$$= \sum_{n_1=-K}^{K} \sum_{n_2=-\infty}^{+\infty} \int_{-KT_0+\frac{T_0}{2}}^{KT_0+\frac{T_0}{2}} i(t-n_1T_0)\, i^*(t-n_2T_0-\tau)\, dt$$

$$= \sum_{n_1=-K}^{K} \sum_{n_2=-\infty}^{+\infty} \int_{-\infty}^{+\infty} i(t-n_1T_0)\, i^*(t-n_2T_0-\tau)\,dt$$

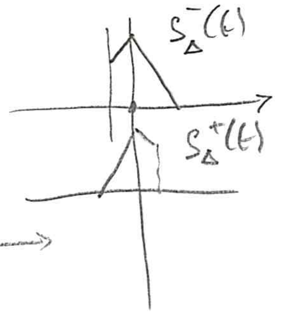
$$= \sum_{n_1=-K}^{K} \sum_{n_2=-\infty}^{+\infty} r_i(\cancel{t}-n_1T_0-\cancel{t}+n_2T_0+\tau)$$

$$= \sum_{n_1=-K}^{K} \underbrace{\sum_{n_2=-\infty}^{+\infty} r_i(\tau-(n_1-n_2)T_0)}_{\text{periodica}}$$

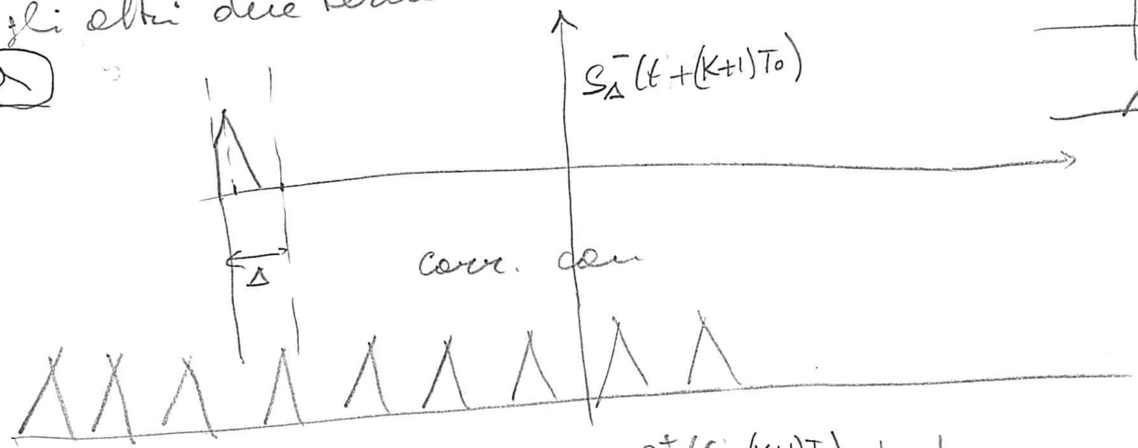
$$n_1-n_2=\ell$$

$$= \sum_{n_1=-K}^{K} \underbrace{\sum_{\ell=-\infty}^{+\infty} r_i(\tau-\ell T_0)}_{\text{non dipende da } n_1} = (2K+1)\sum_{\ell=-\infty}^{+\infty} r_i(\tau-\ell T_0)$$

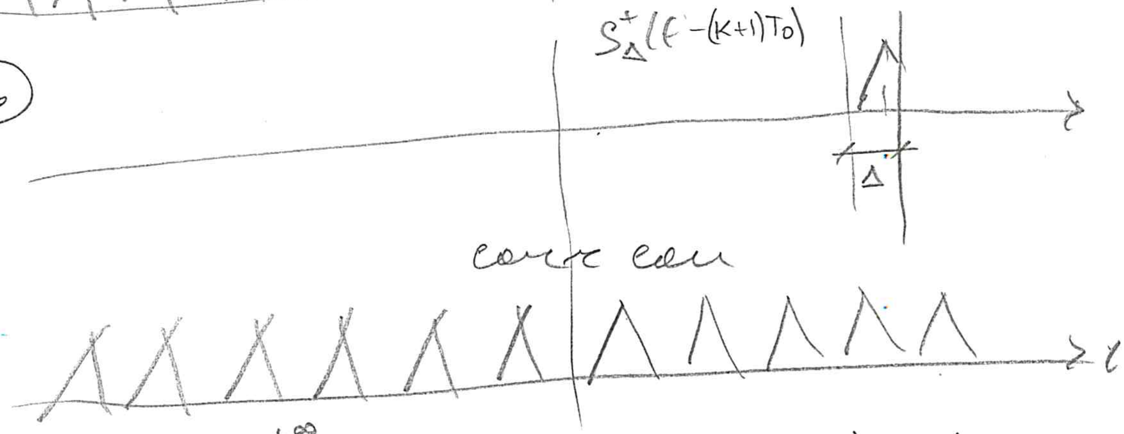
Per gli altri due termini:



a



b



$$(a) = \sum_{n_2=-\infty}^{+\infty} \int_{-\infty}^{+\infty} s_\Delta^-(t+(K+1)T_0)\, i(t-\tau-n_2T_0)\,dt$$

$$= \underbrace{\sum_{n_2=-\infty}^{+\infty} r_{s_\Delta^- i}(\cancel{t}+(K+1)T_0-\cancel{t}+\tau+n_2T_0)}_{\text{periodica}} \qquad \ell = K+1+n_2$$

$$= \sum_{\ell=-\infty}^{+\infty} r_{s_\Delta^- i}(\tau+\ell T_0)$$

Stesso per ⑤

$$⑥ = \sum_{\ell=-\infty}^{+\infty} r_{s_\Delta^+ i}(\tau - \ell T_0)$$

Mettendo i tre termini insieme

$$\frac{1}{2T}\int_{-T}^{T} s(t)\, s^*(t-\tau)\, dt = \frac{1}{2KT_0 + T_0 + 2\Delta}\Bigg[(2K+1)\sum_{\ell=-\infty}^{+\infty} r_i(\tau-\ell T_0)$$

$$+ \underbrace{\sum_{\ell=-\infty}^{+\infty} r_{s_\Delta^- i}(\tau+\ell T_0) + \sum_{\ell=-\infty}^{+\infty} r_{s_\Delta^+ i}(\tau-\ell T_0)}_{\text{valori costanti al crescere di } K}\Bigg]$$

$K \to \infty$

$$\boxed{r_s(\tau) = \frac{1}{T_0}\sum_{\ell=-\infty}^{+\infty} r_i(\tau-\ell T_0)}$$

AUTOCORRELAZIONE PERIODICA

$$P_s(f) = \mathcal{F}\left[r_s(\tau)\right] = \mathcal{F}\left[\frac{1}{T_0} r_i(\tau) * \sum_{\ell=-\infty}^{+\infty}\delta(\tau-\ell T_0)\right]$$

$$= \frac{1}{T_0}\left|I(f)\right|^2 \frac{1}{T_0}\sum_{n=-\infty}^{+\infty}\delta\left(f-\frac{n}{T_0}\right) = \frac{1}{T_0^2}\sum_{n=-\infty}^{+\infty}\left|I\left(\frac{n}{T_0}\right)\right|^2\delta\left(f-\frac{n}{T_0}\right)$$

$$\boxed{P_s(f) = \sum_{n=-\infty}^{+\infty}\frac{1}{T_0^2}\left|I\left(\frac{n}{T_0}\right)\right|^2\delta\left(f-\frac{n}{T_0}\right)}$$