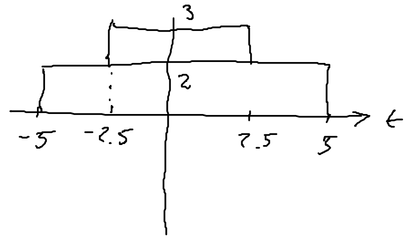
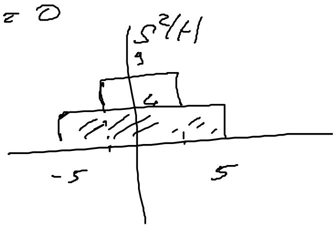
Lezione 10 Esercitazione 21 Ott 2021

giovedì 21 ottobre 2021 08.54

$$s(t) = \Pi\left(\frac{t}{5}\right) + 2\,\Pi\left(\frac{t}{10}\right)$$



$P_s = 0$

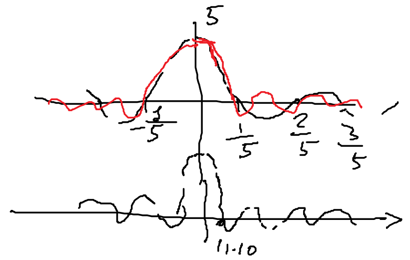


$$\mathcal{E}_s = \int_{-\infty}^{+\infty} s^2(t)\,dt = 10 \cdot 4 + 4 \cdot 9 = 76$$

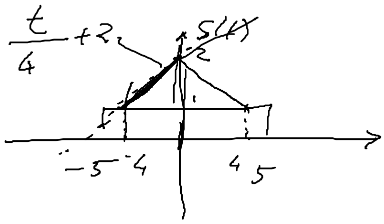
$$S(f) = 5\,\mathrm{sinc}\,5f + 2 \cdot 10 \cdot \mathrm{sinc}\,10f$$

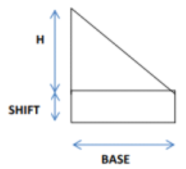
$5f = k$

$f = \frac{k}{5}$



$$s(t) = \Pi\left(\frac{t}{10}\right) + \Lambda\left(\frac{t}{4}\right)$$



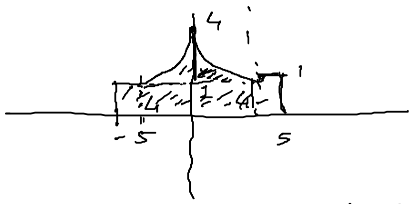


$$\int_0^b |y(x)|^2 dx = \frac{H^2 * BASE}{3} + (H + SHIFT) * BASE * SHIFT$$

$$\mathcal{E}_s = 2 + 2\left(\frac{1^2 \cdot 4}{3} + (1+1) \cdot 4 \cdot 1\right)$$

$$= 2 + 2\left(\frac{4}{3} + 8\right) = 2 + 2\,\frac{4+24}{3} = 2 + 2\,\frac{28}{3}$$

$$= 2\left(\frac{3+28}{3}\right) = \frac{62}{3}$$



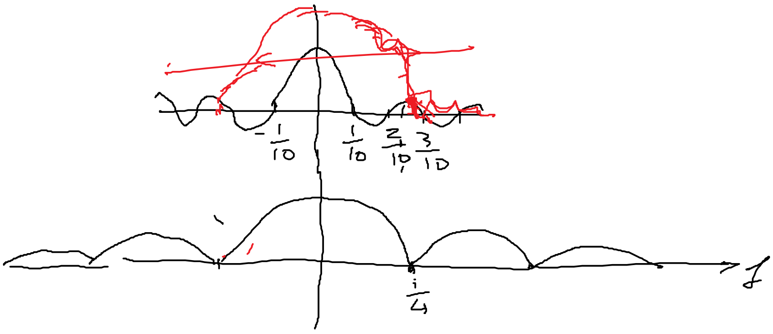
$$\mathcal{E}_s = \left[\int_{-5}^{-4} 1^2\,dt + \int_{-4}^{0}\left(\frac{t}{4}+2\right)^2 dt\right] \cdot 2$$

$$= 2\left[1 + \int_{-4}^{0}\left(\frac{1}{16}t^2 + 4 + t\right) dt\right] = 2\left[1 + \frac{1}{16}\frac{t^3}{3}\Big]_{-4}^{0} + 4t\Big]_{-4}^{0} + \frac{t^2}{2}\Big]_{-4}^{0}\right]$$

$$= 2\left[1 + \frac{1}{\cancel{16}}\left(\frac{\cancel{64}^{\,4}}{3}\right) + 4 \cdot 4 + \frac{1}{2}(-16)\right]$$

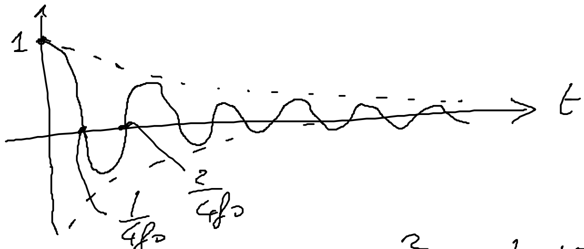
$$= 2\left[1 + \frac{4}{3} + 16 - 8\right] = 2\left[9 + \frac{4}{3}\right] = 2\,\frac{27+4}{3} = 2\,\frac{31}{3} = \frac{62}{3}$$

$$S(f) = 10\,\mathrm{sinc}\,10f + 4\,\mathrm{sinc}^2 4f$$



$$s(t) = e^{-2t}\cos 2\pi f_0 t \; u(t)$$

$$2\pi f_0 t = \frac{\pi}{2} k$$
$$t = \frac{1}{4f_0} k$$



$$P_s = 0$$

$$\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$$

$$\mathcal{E}_s = \int_0^\infty s^2(t)\,dt = \int_0^\infty e^{-4t}\cos^2 2\pi f_0 t \; dt$$

$$= \int_0^\infty e^{-4t}\left(\frac{1}{2} + \frac{1}{2}\cos 4\pi f_0 t\right) dt$$

$$= \frac{1}{2}\int_0^\infty e^{-4t}\,dt + \frac{1}{2}\int_0^\infty e^{-4t}\cos 4\pi f_0 t \; dt$$

$$\int_0^\infty e^{-4t}\,dt = \left.\frac{e^{-4t}}{-4}\right]_0^\infty = \frac{1}{4}$$

$$\int_0^\infty e^{-4t}\left(\frac{1}{2}e^{j4\pi f_0 t} + \frac{1}{2}e^{-j4\pi f_0 t}\right) dt$$

$$= \frac{1}{2}\int_0^\infty e^{-(4-j4\pi f_0)t}\,dt + \frac{1}{2}\int_0^\infty e^{-(4+j4\pi f_0)t}\,dt$$

$$= \frac{1}{2}\left.\frac{e^{-(4-j4\pi f_0)t}}{-(4-j4\pi f_0)}\right]_0^\infty + \frac{1}{2}\left.\frac{e^{-(4+j4\pi f_0)t}}{-(4+j4\pi f_0)}\right]_0^\infty$$

$$= \frac{1}{2}\,\frac{1}{4-j4\pi f_0} + \frac{1}{2}\,\frac{1}{4+j4\pi f_0} = \frac{1}{2}\,\frac{4+\cancel{j4\pi f_0}+4-\cancel{j4\pi f_0}}{4^2+4^2\pi^2 f_0^2}$$

$$= \frac{1}{2}\,\frac{1}{4-j4\pi f_0} + \frac{1}{2}\,\frac{1}{4+j4\pi f_0} = \frac{1}{2}\,\frac{4+j4\pi f_0 \cdots}{4^2+4^2\pi^2 f_0^2}$$

$$= \frac{4}{16+16\pi^2 f_0^2}$$

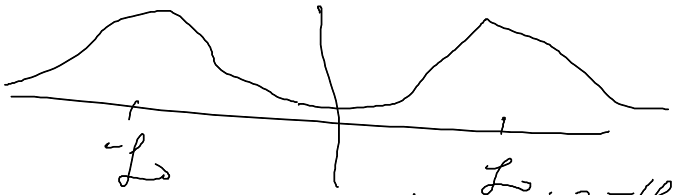
$$\mathcal{E}_s = \frac{1}{2}\,\frac{1}{4} + \frac{1}{2}\,\frac{4}{16+16\pi^2 f_0^2}$$

$$\mathcal{F}[s(t)] = S(f) = \int_0^{+\infty} e^{-2t}\cos 2\pi f_0 t\, e^{-j2\pi f t}\,dt = \int_0^{+\infty} e^{-2t}\,\frac{1}{2}\left(e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}\right) e^{-j2\pi f t}\,dt$$

$$= \frac{1}{2}\int_0^{+\infty} e^{-(2-j2\pi f_0+j2\pi f)t}\,dt + \frac{1}{2}\int_0^{+\infty} e^{-(2+j2\pi f_0+j2\pi f)t}\,dt$$

$$= \frac{1}{2}\left.\frac{e^{-(2-j2\pi f_0+j2\pi f)t}}{-(2+j2\pi(f-f_0))}\right]_0^{\infty} + \frac{1}{2}\left.\frac{e^{-(2+j2\pi f_0+j2\pi f)t}}{-(2+j2\pi(f+f_0))}\right]_0^{\infty}$$

$$= \frac{1}{2}\left(\underbrace{\frac{1}{2+j2\pi(f-f_0)}} + \underbrace{\frac{1}{(2+j2\pi(f+f_0))}}\right)$$



$$S(f) = \frac{1}{2}\,\frac{2+j2\pi(f+f_0)+2+j2\pi(f-f_0)}{4-4\pi^2(f-f_0)(f+f_0)+j2\pi(f-f_0)+j2\pi(f+f_0)}$$

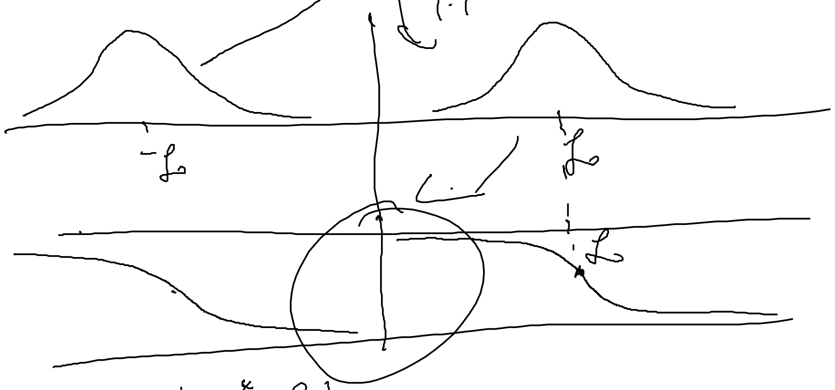
$$S(f) = \frac{1}{2}S_1(f) + \frac{1}{2}S_2(f)$$

$$S_1(f) = \frac{1}{2+j2\pi(f-f_0)} \qquad S_2(f) = \frac{1}{2+j2\pi(f+f_0)}$$

$$\begin{cases} |S_1(f)|^2 = \dfrac{1}{4+4\pi^2(f-f_0)^2} \\ \angle S_1(f) = -tg^{-1}\dfrac{2\pi(f-f_0)}{2} \end{cases} \qquad \begin{cases} |S_2(f)|^2 = \dfrac{1}{4+4\pi^2(f+f_0)^2} \\ \angle S_2(f) = -tg^{-1}\dfrac{2\pi(f+f_0)}{2} \end{cases}$$

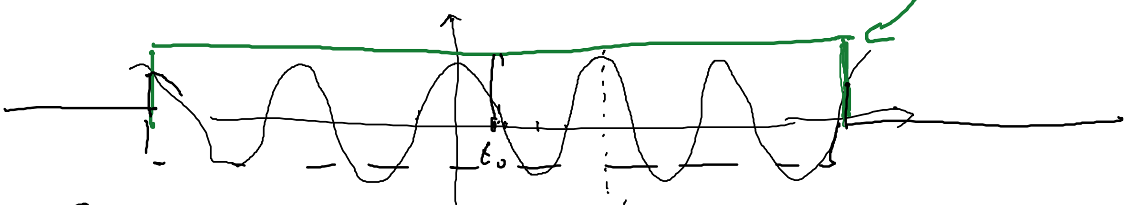
$$\angle S_1(f) = -\mathrm{tg}^{-1}\frac{2\pi(f-f_0)}{2} \qquad \angle S_2(f) = -\mathrm{tg}^{-1}\frac{2\pi(f+f_0)}{2}$$

$|\cdot|$



$$|S(f)| = S(f)\,S^*(f) = \dots$$

$$s(t) = x(t)\cos 2\pi f_0 t \qquad x(t) = \Pi\left(\frac{t-t_0}{T}\right) \qquad T \gg \frac{1}{f_0}$$



$$\cos^2 x = \frac{1}{2} + \frac{1}{2}$$

$P_s = 0$

$$E_s = \int_{t_0-T/2}^{t_0+T/2} \cos^2 2\pi f_0 t \, dt = \int_{t_0-T/2}^{t_0+T/2} \frac{1}{2}\,dt + \frac{1}{2}\int_{t_0-T/2}^{t_0+T/2} \cos 4\pi f_0 t \, dt$$

$$= \frac{1}{2}\, t\Big]_{t_0-T/2}^{t_0+T/2} + \frac{1}{2}\,\frac{\sin 4\pi f_0 t}{4\pi f_0}\Big]_{t_0-T/2}^{t_0+T/2}$$

$$= \frac{1}{2}\left(\cancel{t_0} + \frac{T}{2} - \left(\cancel{t_0} - \frac{T}{2}\right)\right) + \frac{\sin 4\pi f_0(t_0+T/2) - \sin 4\pi f_0(t_0+T/2)}{8\pi f_0}$$

$$= \frac{T}{2} +$$

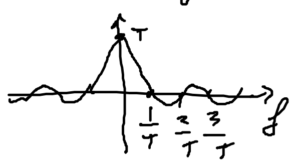
$t_0 = 0$

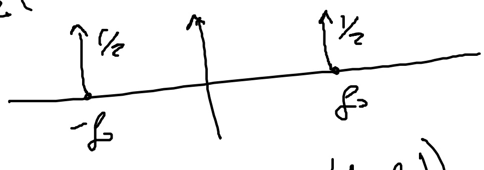
$t_0 = 0 \quad T = \frac{K}{f_0} \Rightarrow \; = 0$

$$s(t) = \Pi\left(\frac{t}{T}\right)\cos 2\pi f_0 t$$

$T\,\mathrm{sinc}\,Tf$

$$\frac{1}{2}\left(\delta(f-f_0) + \delta(f+f_0)\right)$$





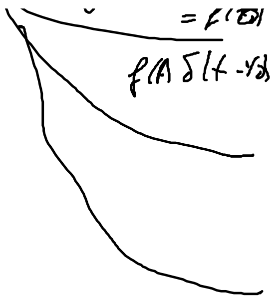
$\delta(t)$

$f(t)\delta(t)$

$\int f(t)\delta(t)\,dt = f(0)$

$f(t)\delta(t-t_0)$

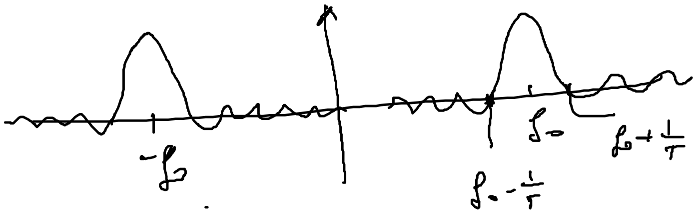
$(\delta(f-f_0)+\delta(f+f_0))$



$$S(f) = T\,\mathrm{sinc}\,Tf * \frac{1}{2}\left(\delta(f-f_0)+\delta(f+f_0)\right)$$

$$= \frac{T}{2}\mathrm{sinc}\,Tf * \delta(f-f_0) + \frac{T}{2}\mathrm{sinc}\,Tf * \delta(f+f_0)$$

$$= \frac{T}{2}\mathrm{sinc}\,T(f-f_0) + \frac{T}{2}\mathrm{sinc}\,T(f+f_0)$$

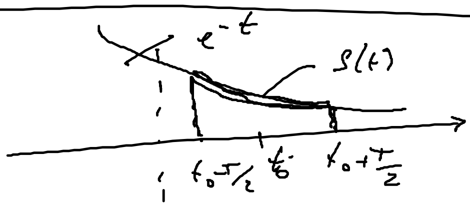


$$s(t) = \Pi\left(\frac{t-t_0}{T}\right)\cos 2\pi f_0 t$$

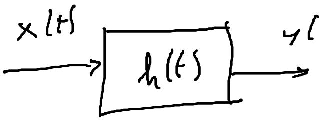
$$e^{-j2\pi f t_0}\,T\,\mathrm{sinc}\,Tf \qquad \frac{1}{2}\left(\delta(f-f_0)+\delta(f+f_0)\right)$$

$$S(f) = \frac{T}{2}e^{-j2\pi(f-f_0)t_0}\mathrm{sinc}\,T(f-f_0) + \frac{T}{2}e^{-j2\pi(f+f_0)t_0}\mathrm{sinc}\,T(f+f_0)$$

$$s(t) = e^{-t}\,\Pi\left(\frac{t-t_0}{T}\right)$$



$$S(f) = \int_{t_0-T/2}^{t_0+T/2} e^{-t}e^{-j2\pi f t}\,dt = \dots\dots\dots$$

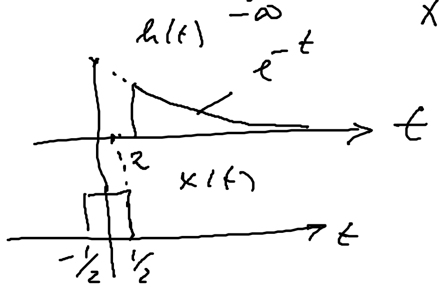


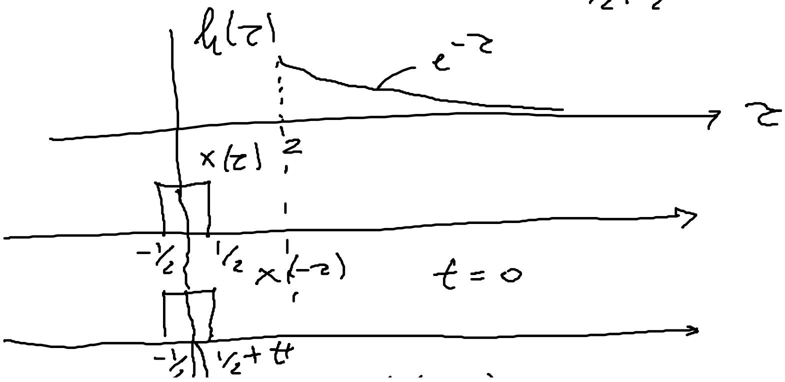
$$y(t) = (h*x)(t) = \int_{-\infty}^{+\infty} h(\tau)x(t-\tau)\,d\tau$$

$$x(-(\tau-t))$$

$$h(t) = e^{-t}u(t-2)$$

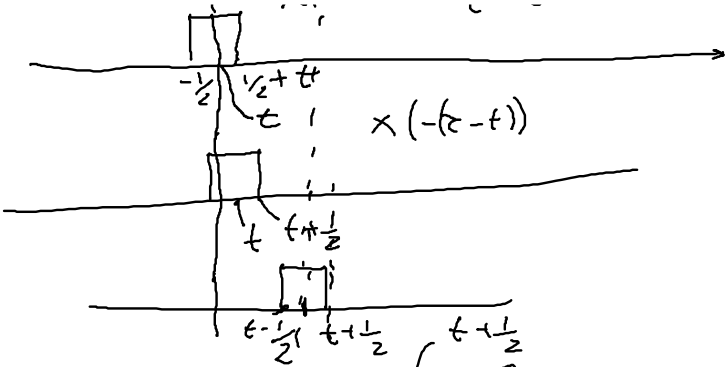
$$x(t) = \Pi(t)$$





$$t=0$$

$$t+1/2 \dots$$



$t+\frac{1}{2}<2$

$t<\frac{3}{2} \quad y=0$

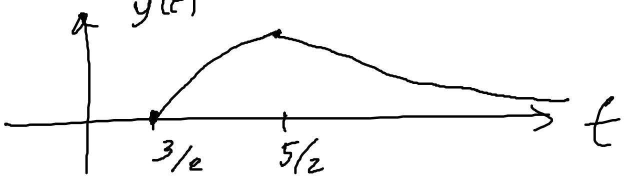
$t+\frac{1}{2}>2 \quad t>\frac{3}{2}$

$t-\frac{1}{2}<2 \quad t<\frac{5}{2}$

$$y(t)=\int_{2}^{t+\frac{1}{2}} e^{-\tau}\,d\tau = \left.\frac{e^{-\tau}}{-1}\right]_{2}^{t+\frac{1}{2}} = -\left(e^{-(t+\frac{1}{2})}-e^{-2}\right)$$

$t-\frac{1}{2}>2 \quad t>\frac{5}{2}$

$$y(t)=\int_{t-1/2}^{t+1/2} e^{-\tau}\,d\tau = \left.\frac{e^{-\tau}}{-1}\right]_{t-1/2}^{t+1/2} = -\left(e^{-(t+\frac{1}{2})}-e^{-(t-1/2)}\right) = e^{-t}\left(e^{1/2}-e^{-1/2}\right)$$



$s(t)=A\cos(2\pi f_0 t+\varphi) \qquad P_s=\frac{A^2}{2}$

$$P_s=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} s^2(t)\,dt = \dots$$