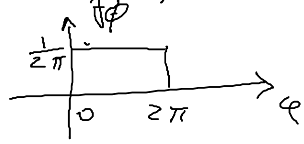
$$s(t) = A\cos(2\pi f_0 t + \varphi) \qquad P_s = ?$$

$$\varphi \propto U(0, 2\pi)$$

$$f_\Phi(\varphi)$$



$$P_s = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} E[s^2(t)]\,dt$$

$$\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$$

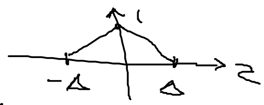
$$E_\Phi[s^2(t)] = \int_0^{2\pi} s^2(t,\varphi)\, f_\Phi(\varphi)\,d\varphi = \int_0^{2\pi} A^2\cos^2(2\pi f_0 t + \varphi)\frac{1}{2\pi}\,d\varphi$$

$$= \frac{A^2}{2\pi}\int_0^{2\pi}\left(\frac{1}{2} + \frac{1}{2}\cos(4\pi f_0 t + 2\varphi)\right)d\varphi$$

$$= \frac{A^2}{2\pi}\left(\frac{2\pi}{2} + \frac{1}{2}\cancel{\left[\frac{\sin(4\pi f_0 t + 2\varphi)}{2}\right]_0^{2\pi}}\right) = \frac{A^2}{2\pi}(\pi) = \frac{A^2}{2}$$

$$X(t) = B + s(t)\cos 2\pi f_0 t \qquad s(t) \text{ SSL} \qquad R_s(\tau) = \Lambda\left(\frac{\tau}{\Delta}\right)$$

$$E[s(t)] = 0$$



$$E[X(t)] = B + \underbrace{E[s(t)]}_{0}\cos 2\pi f_0 t = B$$

Staz. nella media

$$P_s(f) = \mathcal{F}[R_s(\tau)] = \Delta\,\mathrm{sinc}^2 \Delta f$$

$$R_X(t;\tau) = E[X(t)X(t-\tau)]$$

$$= E[(B + s(t)\cos 2\pi f_0 t)(B + s(t-\tau)\cos 2\pi f_0(t-\tau))]$$

$$= E[B^2 + s(t)s(t-\tau)\cos 2\pi f_0 t\cos 2\pi f_0(t-\tau) + B s(t)\cos 2\pi f_0 t + B s(t-\tau)\cos 2\pi f(t-\tau)]$$

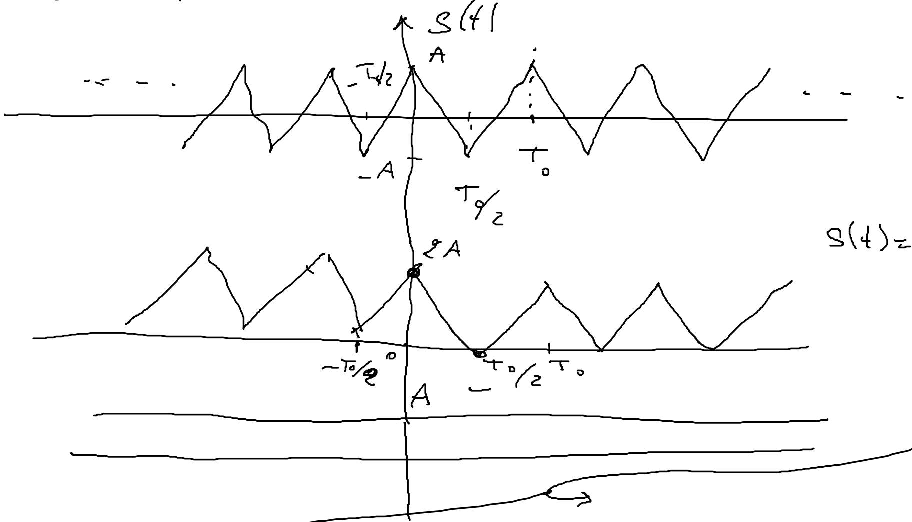
$$= B^2 + E[s(t)s(t-\tau)]\cos 2\pi f_0 t\cos 2\pi f_0(t-\tau) + B\underbrace{\cancel{E[s(t)]}}_{0}\cos 2\pi f_0 t + B\underbrace{\cancel{E[s(t-\tau)]}}_{0}\cos 2\pi f(t-\tau)$$

$$= B^2 + R_s(\tau)\cos 2\pi f_0 t\cos 2\pi f_0(t-\tau)$$

non è stazionario nella autocorrelazione $\rightarrow$ no SSL!

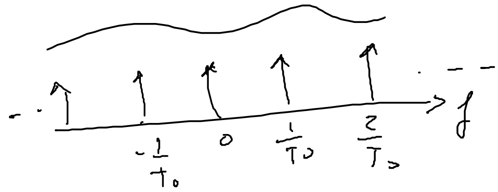
$$\mathcal{E} = \int_{-\infty}^{+\infty} s^2(t)\,dt < \infty \longrightarrow = \int_{-\infty}^{+\infty} |S(f)|^2\,df$$

Segnale periodico



$$\boxed{s(t) = -A + 2A \sum_{k=-\infty}^{+\infty} \Lambda\left(\frac{t-kT_0}{T_0/2}\right)}$$

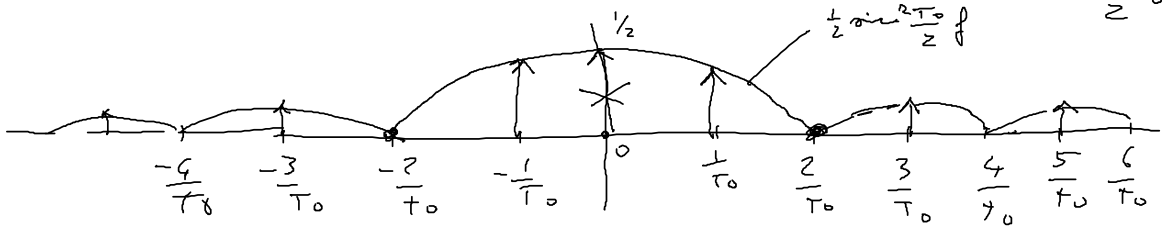
$$\Lambda\left(\frac{t}{T_0/2}\right) * \sum_{k=-\infty}^{+\infty} \delta(t-kT_0)$$



$$\overset{\mathcal{F}}{\Longleftrightarrow} \cancel{\frac{T_0}{2}} \, \mathrm{sinc}^2\left(\frac{T_0}{2} f\right) \cdot \frac{1}{\cancel{T_0}} \sum_{n=-\infty}^{+\infty} \delta\left(f - \frac{n}{T_0}\right)$$

$$= \frac{1}{2} \sum_{n=-\infty}^{+\infty} \mathrm{sinc}^2\left(\frac{\cancel{T_0}}{2} \frac{n}{\cancel{T_0}}\right) \delta\left(f - \frac{n}{T_0}\right)$$

$$\frac{T_0}{2} f = K \qquad f = \frac{2K}{T_0}$$



$$S(f) = -A\delta(f) + \frac{2A}{2} \sum_{n=-\infty}^{+\infty} \mathrm{sinc}^2 \frac{n}{2} \, \delta\left(f - \frac{n}{T_0}\right)$$

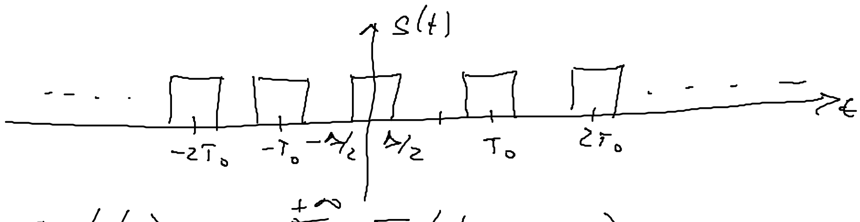
$$\Big\downarrow n = 0$$

$$= 1$$

$$= A \sum_{\substack{n=-\infty \\ n \neq 0}}^{+\infty} \mathrm{sinc}^2 \frac{n}{2} \, \delta\left(f - \frac{n}{T_0}\right)$$

$$\Delta \ll T_0$$

$$s(t) = \sum_{k=-\infty}^{+\infty} \Pi\left(\frac{t-kT_0}{\Delta}\right)$$



$$s(t) = \Pi\left(\frac{t}{\Delta}\right) * \sum_{k=-\infty}^{+\infty} \delta(t-kT_0)$$

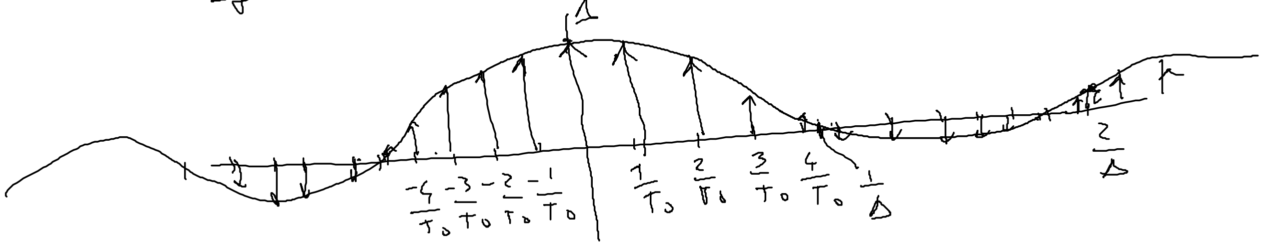
$$s(t) = \Pi\left(\frac{t}{\Delta}\right) * \sum_{k=-\infty}^{+\infty} \delta(t - kT_0)$$

$$S(f) = \Delta \,\mathrm{sinc}\, \Delta f \cdot \frac{1}{T_0} \sum_{n=-\infty}^{+\infty} \delta\left(f - \frac{n}{T_0}\right)$$

$\Delta << T_0$

$\frac{1}{\Delta} >> \frac{1}{T_0}$

$\Delta f = k \Rightarrow f = \frac{k}{\Delta}$



$$= \frac{\Delta}{T_0} \sum_{n=-\infty}^{+\infty} \mathrm{sinc}\, \frac{\Delta n}{T_0}\, \delta\left(f - \frac{n}{T_0}\right)$$

$$s(t) = 3 \cos\left(2\pi f_0 t + \frac{1}{2}\right) + \sin 2\pi f_1 t$$

$f_0 = 3\,\mathrm{KHz}$

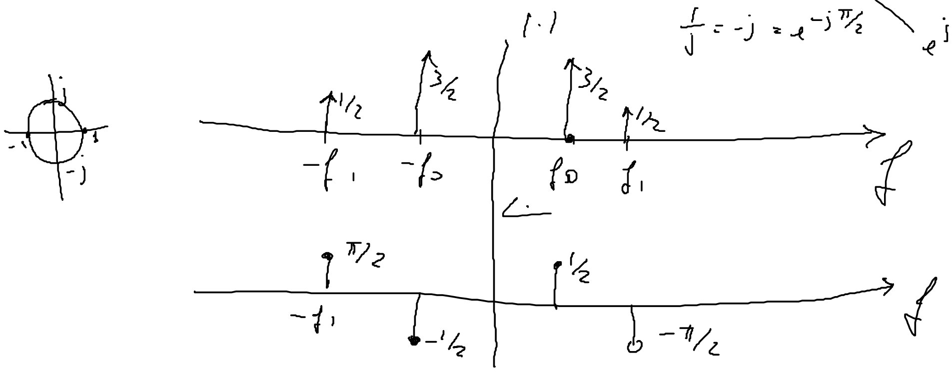
$f_1 = 5\,\mathrm{KHz}$

$$S(f) = \frac{3}{2}\delta(f - f_0)e^{j\frac{1}{2}} + \frac{3}{2}\delta(f + f_0)e^{-j\frac{1}{2}} + \frac{1}{2j}\delta(f - f_1) - \frac{1}{2j}\delta(f + f_1)$$

$\frac{1}{j} = -j = e^{-j\pi/2}$ $\quad e^{j\pi/2}$

$$\mathcal{F}\left[\cos(2\pi f_0 t + \theta)\right] = \frac{1}{2}\left(\delta(f - f_0)e^{j\theta} + \delta(f + f_0)e^{-j\theta}\right)$$

$$\mathcal{F}\left[\sin(2\pi f_0 t + \theta)\right] = \frac{1}{2j}\left(\delta(f - f_0)e^{j\theta} - \delta(f + f_0)e^{-j\theta}\right)$$



$-\frac{1}{j} = j$

$$x(t) = A_1 \cos(2\pi f_1 t + \varphi_1) + A_2 \cos(2\pi f_2 t + \varphi_2)$$

$$P_x = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x^2(t)\, dt$$

$$\int_{-T}^{T}\left(A_1 \cos(2\pi f_1 t + \varphi_1) + A_2 \cos(2\pi f_2 t + \varphi_2)\right)^2 dt = \int_{-T}^{T} A_1^2 \cos^2(2\pi f_1 t + \varphi_1)\, dt + \int_{-T}^{T} A_2^2 \cos^2 2\pi f_2 t + \varphi_2\, dt$$

$$+ 2A_1A_2 \int_{-T}^{T} \cos(2\pi f_1 t + \varphi_1)\cos(2\pi f_2 t + \varphi_2)$$

$$\cos\alpha \cos\beta = \frac{1}{2}\cos(\alpha + \beta) + \frac{1}{2}\cos(\alpha - \beta)$$

$$\int^{T} \frac{1}{2}\cos(2\pi(f_1 + f_2)t + \varphi_1 + \varphi_2)\, dt + \int \frac{1}{2}\cos(2\pi(f_1 - f_2)t + \varphi_1 - \varphi_2)\, dt$$

$$\hookrightarrow \int_{-T}^{T} \frac{1}{2}\cos\left(2\pi(f_1+f_2)t+\varphi_1+\varphi_2\right)dt + \int_{-T}^{T} \frac{1}{2}\cos\left(2\pi(f_1-f_2)t+\varphi_1-\varphi_2\right)dt \qquad +\frac{1}{2}\cos(\alpha-\beta)$$

$$= \frac{1}{2}\left.\frac{\sin\left(2\pi(f_1+f_2)t+\varphi_1+\varphi_2\right)}{2\pi(f_1+f_2)}\right]_{-T}^{T} + \frac{1}{2}\left.\frac{\sin\left(2\pi(f_1-f_2)t+\varphi_1-\varphi_2\right)}{2\pi(f_1-f_2)}\right]_{-T}^{T}$$

$$\downarrow \qquad\qquad\qquad\qquad \downarrow$$

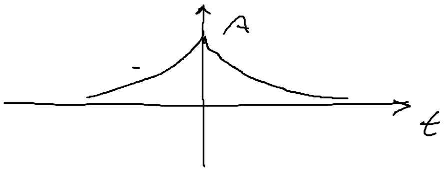
$$\frac{1}{2T} \qquad\qquad\qquad\qquad 1/2T$$

$$\downarrow \lim_{T\to\infty} \qquad\qquad\qquad \downarrow \lim_{T\to\infty}$$

$$0 \qquad\qquad\qquad\qquad 0$$

Sistema lineare $\quad h(t) = A e^{-\alpha|t|} \qquad \alpha > 0$

$$H(f) = \mathcal{F}[h(t)]$$



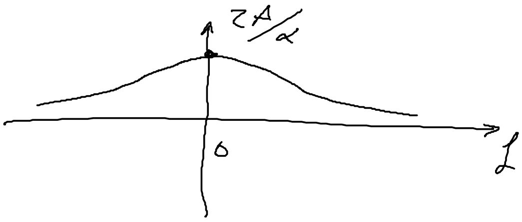
$$= \int_{-\infty}^{+\infty} A e^{-\alpha|t|} e^{-j2\pi f t}\,dt = A\int_{-\infty}^{0} e^{\alpha t} e^{-j2\pi f t}\,dt + A\int_{0}^{\infty} e^{-\alpha t} e^{-j2\pi f t}\,dt$$

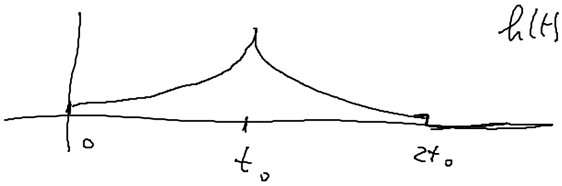
$$= A\int_{-\infty}^{0} e^{(\alpha-j2\pi f)t}\,dt + A\int_{0}^{\infty} e^{-(\alpha+j2\pi f)t}\,dt$$

$$= A\left.\frac{e^{(\alpha-j2\pi f)t}}{\alpha-j2\pi f}\right]_{-\infty}^{0} + A\left.\frac{e^{-(\alpha+j2\pi f)t}}{-(\alpha+j2\pi f)}\right]_{0}^{\infty}$$

$$= A\frac{1}{\alpha-j2\pi f} + A\frac{1}{+(\alpha+j2\pi f)} = A\frac{\alpha\cancel{+j2\pi f}+\alpha\cancel{-j2\pi f}}{\alpha^2+4\pi^2 f^2}$$

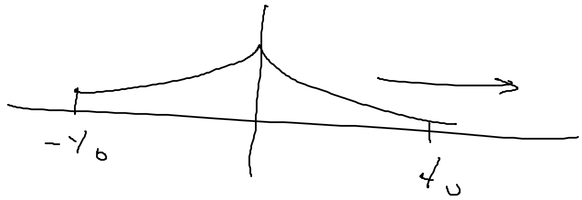
$$= \frac{2\alpha A}{\alpha^2+4\pi^2 f^2}$$

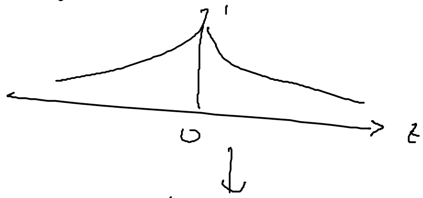




$$h(t) = e^{-\alpha|t-t_0|}\,\Pi\left(\frac{t-t_0}{2t_0}\right)$$

$$h_0(t) = e^{-\alpha|t|}$$





$$H_0(f) = \frac{2\alpha}{\alpha^2+4\pi^2 f^2}$$

$$h(t) = h_0(t-t_0)\,\Pi\left(\frac{t-t_0}{2t_0}\right)$$

$$H(f) = e^{-j2\pi f t_0}\,\mathcal{F}\left[h_0(t)\,\Pi\left(\frac{t}{2t_0}\right)\right]$$

$$= e^{-j2\pi f t_0}\left(\frac{2\alpha}{\alpha^2+4\pi^2 f^2} * 2t_0 \operatorname{sinc} 2t_0 f\right)$$

$$H(f) = \int_0^{t_0} e^{\alpha(t-t_0)} e^{-j2\pi f t}\,dt + \int_{t_0}^{2t_0} e^{-\alpha(t-t_0)} e^{-j2\pi f t}\,dt$$

$$= e^{-\alpha t_0}\int_0^{t_0} e^{(\alpha-j2\pi f)t}\,dt + e^{\alpha t_0}\int_{t_0}^{2t_0} e^{-(\alpha+j2\pi f)t}\,dt$$

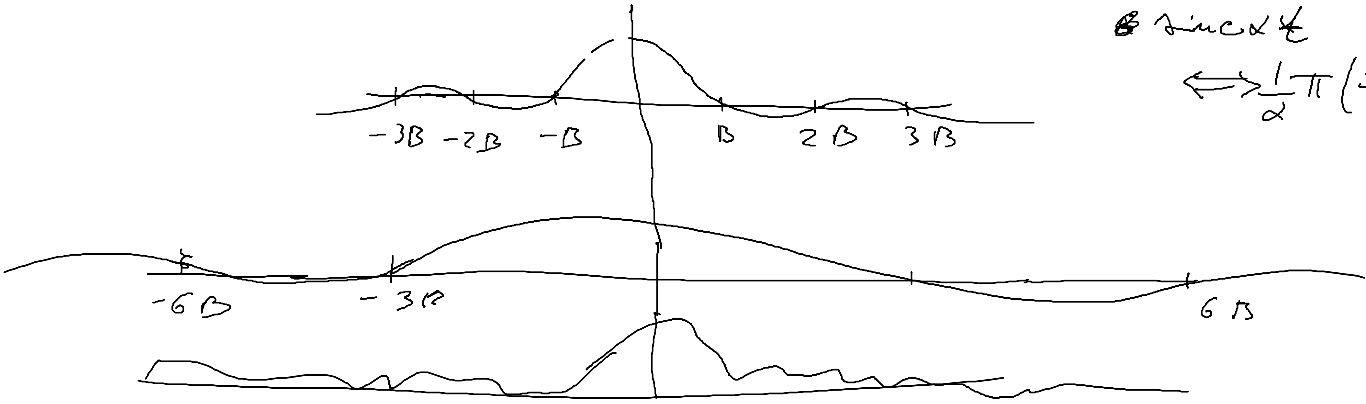
$$= e^{-\alpha t_0}\left.\frac{e^{(\alpha-j2\pi f)t}}{\alpha-j2\pi f}\right]_0^{t_0} + e^{\alpha t_0}\left.\frac{e^{-(\alpha+j2\pi f)t}}{-(\alpha+j2\pi f)}\right]_{t_0}^{2t_0}$$

$$= e^{-\alpha t_0}\,\frac{e^{(\alpha-j2\pi f)t_0}-1}{\alpha-j2\pi f} + e^{\alpha t_0}\,\frac{e^{-(\alpha+j2\pi f)2t_0}-e^{-(\alpha+j2\pi f)t_0}}{-(\alpha+j2\pi f)}$$

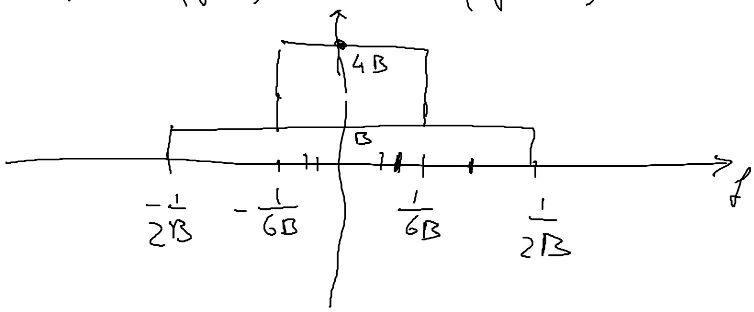
$$= \frac{e^{-j2\pi f t_0}-e^{-\alpha t_0}}{\alpha-j2\pi f} - \frac{e^{-\alpha t_0}e^{-j2\pi f 2t_0}-e^{-j2\pi f t_0}}{\alpha+j2\pi f}$$

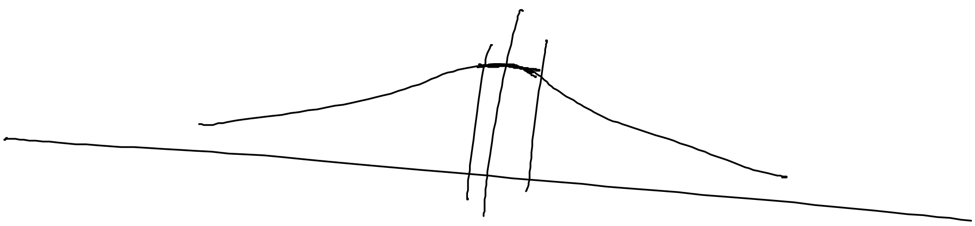
$$h(t) = \operatorname{sinc}\frac{t}{B} + \operatorname{sinc}\frac{t}{3B} \qquad \frac{t}{B} = k$$

$$\mathrm{sinc}\,\alpha t \Longleftrightarrow \frac{1}{\alpha}\,\Pi\left(\frac{f}{\alpha}\right)$$



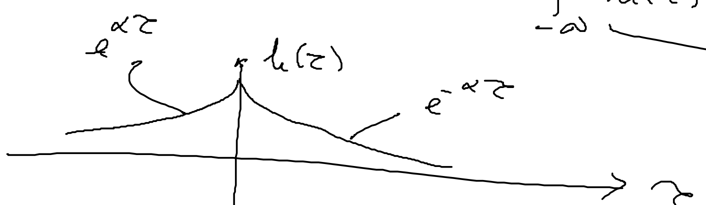
$$H(f) = B\,\Pi(fB) + 3B\,\Pi(f3B)$$





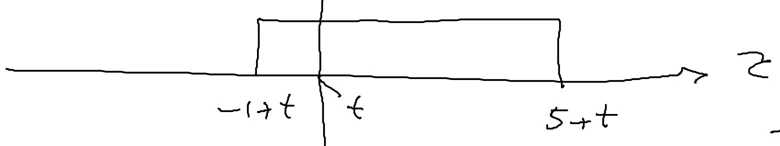
$$h(t) = e^{-\alpha|t|} \qquad x(t) = u(t+5) - u(t-1)$$

$$y(t) = (h * x)(t) = \int_{-\infty}^{+\infty} h(\tau)\,x(t-\tau)\,d\tau$$



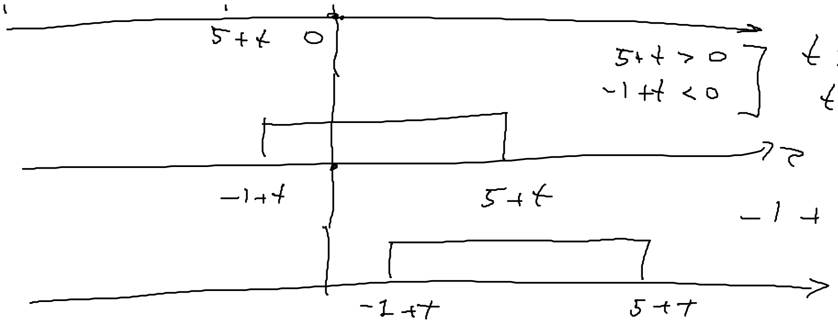
$x(\tau)$

$x(-\tau) \qquad t=0$



$$5+t < 0 \Rightarrow \boxed{t < -5}$$

$$\left.\begin{matrix} 5+t > 0 \\ -1+t < 0 \end{matrix}\right\} \quad \begin{matrix} t > -5 \\ t < 1 \end{matrix} \quad \boxed{-5 < t < 1}$$



$$\left.\begin{array}{l} 5+t>0 \\ -1+t<0 \end{array}\right] \quad \begin{array}{l} t>-5 \\ t<1 \end{array} \quad \boxed{-5<t<1}$$

$$-1+t>0 \quad \boxed{t>1}$$