TDS_IIPi_Dic2013_1

SECONDA UNIVERSITA' DEGLI STUDI DI NAPOLI
CORSO DI LAUREA IN INGEGNERIA ELETTRONICA E INFORMATICA

Teoria dei Segnali SOLUZIONI
IIa Prova Intracorso
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(1) Si consideri il seguente processo aleatorio

$$X(t) = A(t) + 1 + \cos(2\pi f_0 t + \Theta + \frac{\pi}{10}), \qquad (1)$$

dove $A(t)$ è un processo aleatorio SSL avente spettro di potenza $P_A(f) = \Lambda\left(\frac{f-f_0}{2B}\right) + \Lambda\left(\frac{f+f_0}{2B}\right)$ e Θ è una variabile aleatoria uniformemente distribuita in $[0, 2\pi]$. $A(t)$ e Θ sono indipendenti e $f_0 >> B$.
(a) Valutare e schizzare autocorrelazione e spettro di potenza di $X(t)$
(b) Valutare il rapporto tra la potenza di $A(t)$ e quella del resto del segnale.

(2) Un segnale $x(t)$, avente trasformata di Fourier $X(f) = 2\ \Pi\left(\frac{f}{4B}\right)$, è campionato con frequenza di campionamento $f_c = 3B$. Valutare il risultato della ricostruzione mediante la formula di interpolazione cardinale.

(3) Usando un polo e uno zero si progetti un filtro IIR passa-basso tale che la risposta armonica alla frequenza normalizzata $\nu = 1/2$ sia 10/19.
(a) Scrivere l'equazione alle differenze;
(b) Valutare la risposta impulsiva;
(c) Valutare la risposta del filtro all'ingresso $x[n] = u[n] - u[n-2]$ (graficamente, analiticamente o per simulazione).

(1) $$X(t) = A(t) + 1 + \cos\left(2\pi f_0 t + \theta + \frac{\pi}{10}\right)$$

(a)

$$R_X(t;\tau) \triangleq E[X(t)X(t-\tau)] = E\left\{\left[A(t) + 1 + \cos\left(2\pi f_0 t + \theta + \frac{\pi}{10}\right)\right] \cdot \left[A(t-\tau) + 1 + \cos\left(2\pi f_0 (t-\tau) + \theta + \frac{\pi}{10}\right)\right]\right\}$$

$$= E[A(t)A(t-\tau)] + \cancel{E[A(t)]} + \cancel{E[A(t)]}\,E\left[\cos\left(2\pi f_0(t-\tau) + \theta + \frac{\pi}{10}\right)\right]$$

$$+ \cancel{E[A(t-\tau)]} + 1 + E\left[\cos\left(2\pi f_0(t-\tau) + \theta + \frac{\pi}{10}\right)\right]$$

$$+ E\left[\cos\left(2\pi f_0 t + \theta + \frac{\pi}{10}\right)\right]\cancel{E[A(t-\tau)]} + E\left[\cos\left(2\pi f_0 t + \theta + \frac{\pi}{10}\right)\right]$$

$$+ E\left[\cos\left(2\pi f_0 t + \theta + \frac{\pi}{10}\right)\cos\left(2\pi f_0(t-\tau) + \theta + \frac{\pi}{10}\right)\right] \quad (*)$$

$\boxed{E[A(t)] = 0 \text{ altrimenti ci sarebbe una } \delta(f) \text{ in } P_A(f)}$

$$E\left[\cos\left(2\pi f_0(t-\tau) + \theta + \frac{\pi}{10}\right)\right] = \int_0^{2\pi} \cos\left(2\pi f_0(t-\tau) + \theta + \frac{\pi}{10}\right)\frac{1}{2\pi}\,d\theta$$

$$= \frac{1}{2\pi}\sin\left(2\pi f_0(t-\tau) + \theta + \frac{\pi}{10}\right)\Big|_0^{2\pi} = \frac{\sin\left(2\pi f_0(t-\tau) + 2\pi + \frac{\pi}{10}\right) - \sin\left(2\pi f_0(t-\tau) + \frac{\pi}{10}\right)}{2\pi} = 0$$

Lo stesso per $E\left[\cos\left(2\pi f_0 t + \theta + \frac{\pi}{10}\right)\right] = 0$

$$(*) = \frac{1}{2}\cancel{E\left[\cos\left(2\pi f_0(2t-\tau) + 2\theta + \frac{\pi}{5}\right)\right]} + \frac{1}{2}\cos 2\pi f_0 \tau$$

$$R_X(t;\tau) = R_A(\tau) + 1 + \frac{1}{2}\cos 2\pi f_0 \tau = R_X(\tau) \quad \text{SSL}$$

$$P_X(f) = P_A(f) + \delta(f) + \frac{1}{4}\delta(f-f_0) + \frac{1}{4}\delta(f+f_0)$$

$$R_A(\tau) = \mathcal{F}^{-1}\left[\Lambda\left(\frac{f-f_0}{2B}\right) + \Lambda\left(\frac{f+f_0}{2B}\right)\right]$$

$$= e^{j2\pi f_0 \tau}\, 2B\,\mathrm{sinc}^2 2B\tau + e^{-j2\pi f_0 \tau}\, 2B\,\mathrm{sinc}^2 2B\tau$$

$$= 4B\,\mathrm{sinc}^2 2B\tau \cos 2\pi f_0 \tau$$

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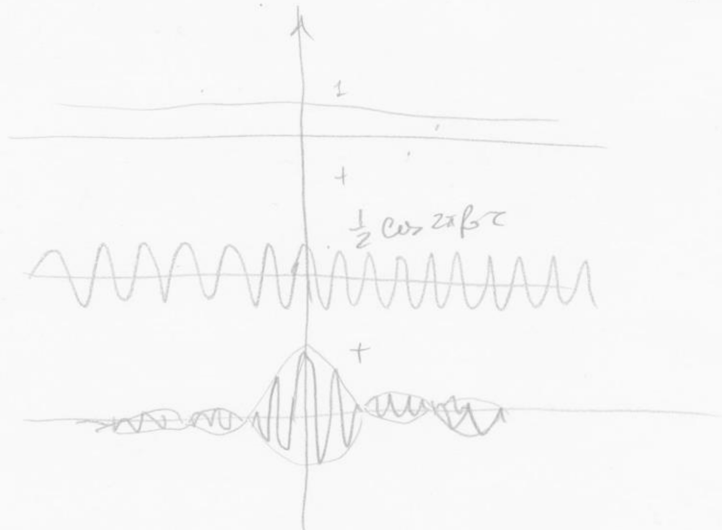
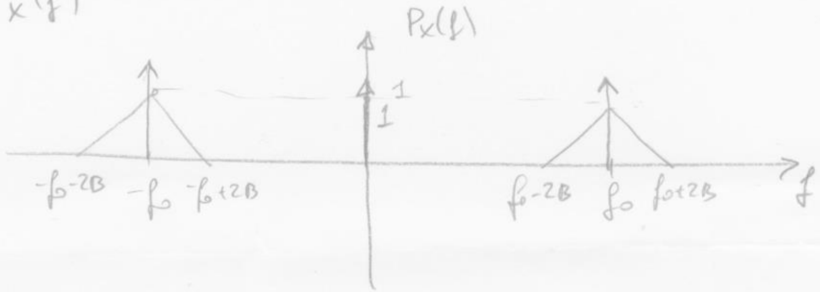
$R_x(\tau)$



$P_x(f)$



(b)

$$P_A = R_A(0) = 4B$$

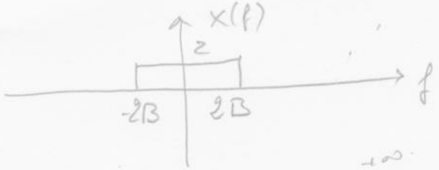
Potenza di $\delta(f) = \int_{-\infty}^{+\infty} \delta(f)\,df = 1$

Potenza di $\frac{1}{4}\delta(f-f_0) = \int_{-\infty}^{+\infty} \frac{1}{4}\delta(f-f_0)\,df = \frac{1}{4}$

" $\frac{1}{4}\delta(f+f_0) = \frac{1}{4}$

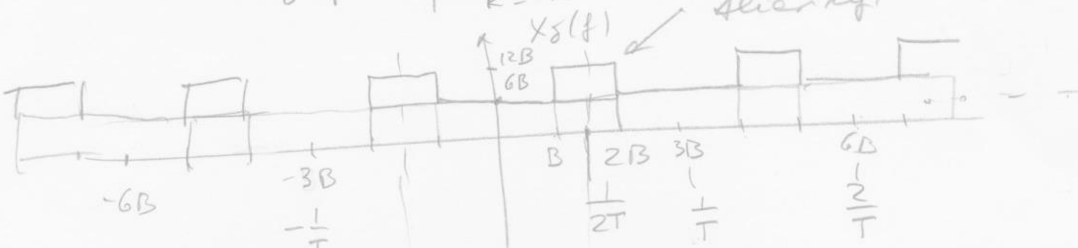
$$\frac{P_A}{P_{resto}} = \frac{4B}{1+\frac{1}{2}} = \frac{8B}{3}$$

(2) $x(t) \longleftrightarrow X(f) = 2\,\Pi\left(\frac{f}{2B}\right)$ $\qquad x(t) = 4B\,\mathrm{sinc}\,2Bt$

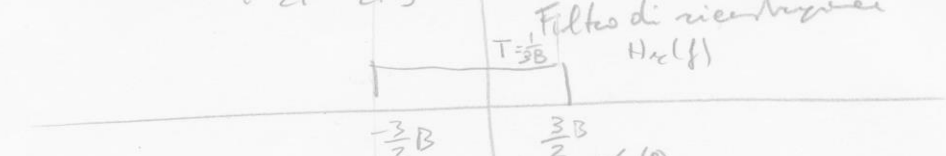


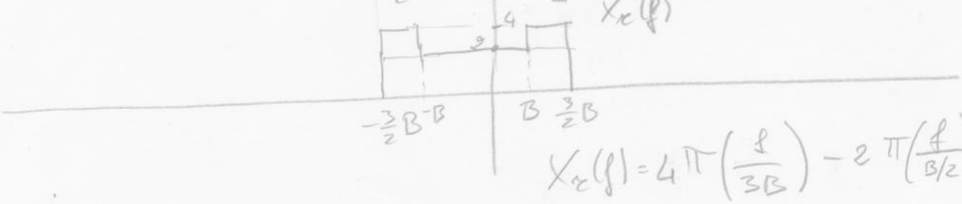
$\underline{x(t)} \not\times \underline{x_\delta(t)}$ $\qquad x_\delta(t) = \sum_{n=-\infty}^{+\infty} x(nT)\,\delta(t-nT)$

$$X_\delta(f) = \frac{1}{T}\sum_{k=-\infty}^{+\infty} X\left(f - \frac{k}{T}\right) \qquad \frac{1}{T} = 3B$$

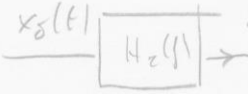


La ricostruzione cardinale corrisponde a un taglio ideale a $\left[-\frac{1}{2T}, \frac{1}{2T}\right]$ con guadagno T





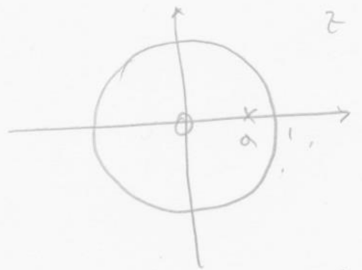
$$X_r(f) = 4\,\Pi\left(\frac{f}{3B}\right) - 2\,\Pi\left(\frac{f}{B/2}\right)$$



$x_r(t) = 12B\,\mathrm{sinc}\,3Bt - B\,\mathrm{sinc}\frac{B}{2}t$

$\qquad = \sum_{n=-\infty}^{+\infty} x[n]\,\mathrm{sinc}\frac{(t-nT)}{T}$

(3)



$$H(z) = \frac{z}{z-a} = \frac{1}{1-az^{-1}}$$

$$H(\nu) = H(z)\Big|_{z=e^{j2\pi\nu}} = \frac{1}{1-a e^{-j2\pi\nu}}$$

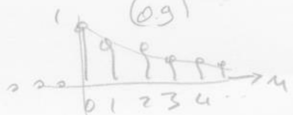
$$H\left(\tfrac{1}{2}\right) = \frac{1}{1-a \underbrace{e^{-j2\pi\frac{1}{2}}}_{-1}} = \frac{1}{1+a} = \frac{10}{19} = \frac{1}{1.9} \qquad a = 0.9$$

(a)

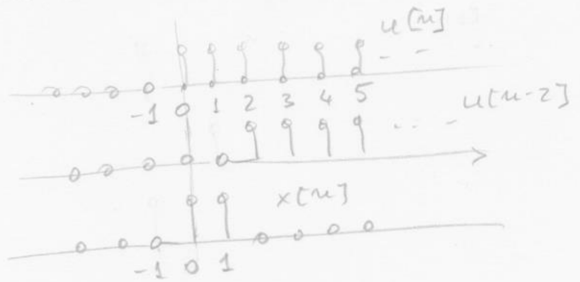
$$y[n] = a\,y[n-1] + x[n]$$

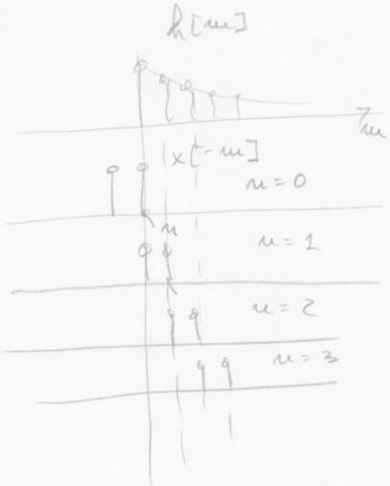
(b)

$$h[n] = a^n u[n]$$



(c) $x[n] = u[n] - u[n-2]$





$$y[n] = \sum_{m=-\infty}^{+\infty} h[m]\,x[n-m]$$

$y[n] = 0 \quad n < 0$

$y[0] = 1$

$y[1] = a^0 + a^1 = 1 + 0.9 = 1.9$

$y[2] = a^1 + a^2$

$\vdots$

$y[n] = a^{n-1} + a^n = (a+1)\,a^{n-1} \quad n > 0$