SECONDA UNIVERSITA' DEGLI STUDI DI NAPOLI

DIPARTIMENTO DI INGEGNERIA INDUSTRIALE E DELL'INFORMAZIONE

Scuola Politecnica e delle Scienze di Base

Teoria dei Segnali/Telecomunicazioni 2

IIa PROVA INTRACORSO

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SOLUZIONI

(1) Si consideri il segnale

$$Y(t) = X(t)\cos^2 2\pi f_0 t + C, \qquad (1)$$

dove $X(t)$ é un processo aleatorio SSL avente spettro di potenza $P_X(f) = \Lambda\left(\frac{f}{2B}\right)$ e C una costante aleatoria indipendente da $X(t)$ e distribuita secondo una pdf gaussiana avente media $\mu_C = 1$ e varianza $\sigma_C^2 = 4$. (Si assuma $f_0 >> 2B$). Commentare sulla stazionarietá di $Y(t)$ e valutarne autocorrelazione e spettro di potenza.

(2) Un segnale aleatorio $x(t)$ SSL avente autocorrelazione $R_X(\tau) = \Lambda(\tau)$ é posto all'ingresso di un sistema lineare avente risposta impulsiva $h(t) = \delta(t-1) + \frac{1}{2}\delta(t-2)$. Valutare la mutua correlazione uscita-ingresso.

(3) Si consideri una cascata di 3 canali lineari rumorosi identici aventi funzione di trasferimento di energia $|H_i(f)|^2 = \Lambda\left(\frac{f}{B}\right), i = 1, 2, 3$ ($B = 10$ KHz) e rumore additivo all'uscita $n_1(t)$, $n_2(t)$ e $n_3(t)$ rispettivamente, di potenza pari a $9\ mV^2$ ognuno, e spettro piatto fino alla frequenza B. Si valuti il rapporto segnale(distorto)-rumore in uscita alla catena se all'ingresso é applicato un segnale a banda piatta fino a B con densitśpettrale pari a $8 \cdot 10^{-4}\ mV^2/Hz$.

(4) Un segnale $s(t)$ passa-banda a spettro piatto tra $b = 100$ e $B = 10000$ Hz di valore efficace $P_s^{RMS} = 10$ mV é inviato su un canale distorcente avente risposta armonica di ampiezza $|H_c(f)| = \Lambda\left(\frac{f}{12000}\right)$. Successivamente il segnale é contaminato da rumore additivo avente spettro di potenza piatto da 0 a 10000 Hz e valore efficace di 5 mV. (a) Valutare il rapporto segnale(distorto)/rumore in dB alla fine della catena; (b) Proporre filtri di enfasi e de-enfasi schizzandone l'andamento approssimativo.

(4) Si considerino due sistemi lineari aventi risposte impulsive $h_1(t)$ e $h_2(t)$, e agli ingressi i due segnali aleatori di potenza $X_1(t)$ e $X_2(t)$. I due segnali siano singolarmente e mutuamente stazionari almeno in senso lato. Dimostrare la formula per la mutua correlazione tra le due uscite $R_{y_1 y_2}(\tau)$.

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$$Y(t) = X(t)\cos^2 2\pi f_0 t + C = \frac{X(t)}{2} + \frac{X(t)}{2}\cos 4\pi f_0 t + C$$

$$E[Y(t)] = \overset{0}{E[\cancel{X(t)}]}\cos^2 2\pi f_0 t + E[C] = 1$$

$$R_Y(t,\tau) = E[Y(t)Y(t-\tau)] = E\left[\left(\frac{X(t)}{2} + \frac{X(t)}{2}\cos 4\pi f_0 t + C\right)\left(\frac{X(t-\tau)}{2} + \frac{X(t-\tau)}{2}\cos 4\pi f_0(t-\tau) + C\right)\right]$$

$$= \frac{1}{4}E[X(t)X(t-\tau)] + \frac{1}{4}E[X(t)X(t-\tau)]\cos 4\pi f_0(t-\tau) + \frac{1}{2}\overset{0}{E[\cancel{X(t)}]}E[C]$$

$$+ \frac{1}{4}E[X(t)X(t-\tau)]\cos 4\pi f_0 t + \frac{1}{4}E[X(t)X(t-\tau)]\cos 4\pi f_0 t \cos 4\pi f_0(t-\tau)$$

$$+ \frac{1}{2}\overset{0}{E[\cancel{X(t)}]}E[C]$$

$$+ \frac{1}{2}E[C]\overset{0}{E[\cancel{X(t-\tau)}]} + \frac{1}{2}E[C]\overset{0}{E[\cancel{X(t-\tau)}]}\cos 4\pi f_0(t-\tau)$$

$$+ E[C^2]$$

$$= \frac{1}{4}R_X(\tau) + \frac{1}{4}R_X(\tau)\cos 4\pi f_0(t-\tau) + \frac{1}{4}R_X(\tau)\cos 4\pi f_0 t$$

$$+ \frac{1}{4}R_X(\tau)\left(\frac{1}{2}\cos 4\pi f_0(2t-\tau) + \frac{1}{2}\cos 4\pi f_0\tau\right) + E[C^2]$$

$$= \frac{1}{4}R_X(\tau) + \frac{1}{4}R_X(\tau)\cos 4\pi f_0(t-\tau) + \frac{1}{4}R_X(\tau)\cos 4\pi f_0 t$$

$$+ \frac{1}{8}R_X(\tau)\cos 4\pi f_0(2t-\tau) + \frac{1}{8}R_X(\tau)\cos 4\pi f_0\tau + E[C^2]$$

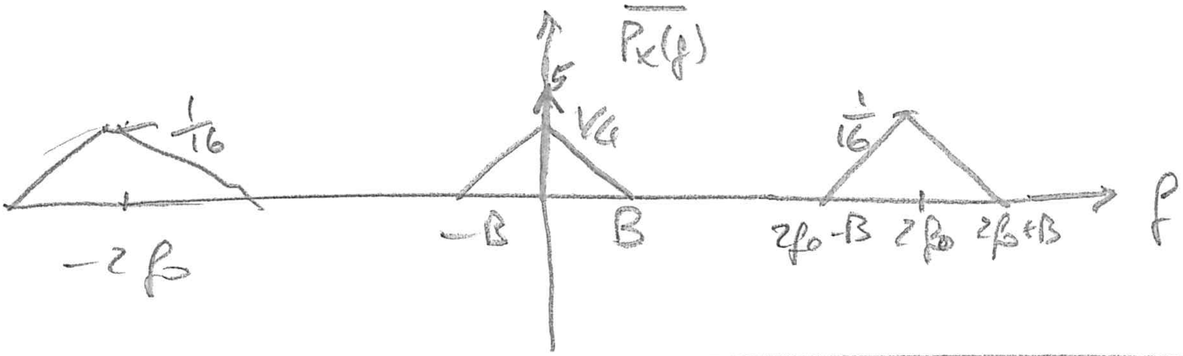
Processo ciclostazionario; mediando nel periodo

$$\overline{R_X(\tau)} = \langle R_X(t,\tau)\rangle_{per} = \frac{1}{4}R_X(\tau) + \frac{1}{8}R_X(\tau)\cos 4\pi f_0\tau + E[C^2]$$

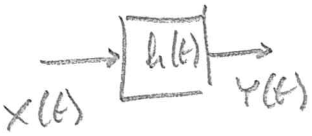
sol IIpi_3Dic2015_2

$\sigma_c^2 = 4 \qquad \sigma_c^2 = E[C^2] - \mu_c^2 \qquad E[C^2] = \sigma_c^2 + \mu_c^2 = 4 + 1 = 5$

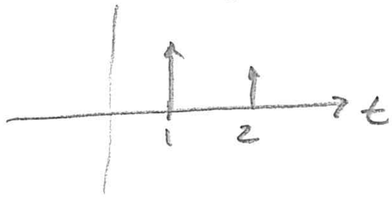
$$\overline{P_x(f)} = \frac{1}{4} P_x(f) + \frac{1}{16} P_x(f - 2f_0) + \frac{1}{16} P_x(f + 2f_0) + 5\,\delta(f)$$



(2)



$h(t) = \delta(t-1) + \frac{1}{2}\delta(t-2)$

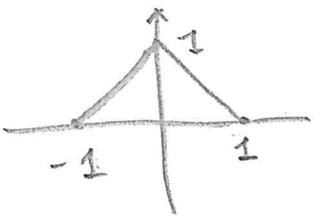


$R_{yx}(\tau) = E[Y(t)X(t-\tau)] =$

$$= E\left[\int_{-\infty}^{+\infty} h(\eta)\, x(t-\eta)\, d\eta \; x(t-\tau)\right] = \int_{-\infty}^{+\infty} h(\eta)\, \underbrace{E[X(t-\eta)X(t-\tau)]}_{R_x(\cancel{t}-\eta-\cancel{t}+\tau)}\, d\eta$$

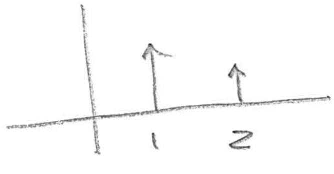
$$= \int_{-\infty}^{+\infty} h(\eta)\, R_x(\tau - \eta)\, d\eta = (h * R_x)(\tau)$$

$R_x(\tau)$

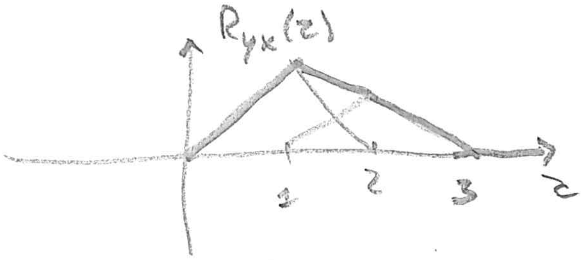


$*$

$\delta(t-1) + \frac{1}{2}\delta(t-2)$



$= R_x(\tau - 1) + \frac{1}{2} R_x(\tau - 2)$



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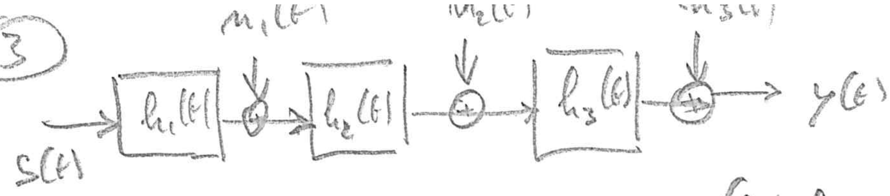
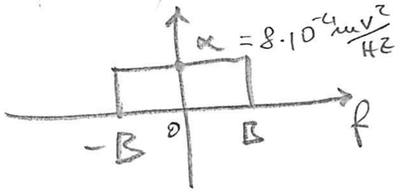


$$y(t) = \underbrace{(h_1 * h_2 * h_3 * s)(t)}_{s_R(t)} + \underbrace{(h_3 * h_2 * n_1)(t) + (h_3 * n_2)(t) + n_3(t)}_{n_R(t)}$$

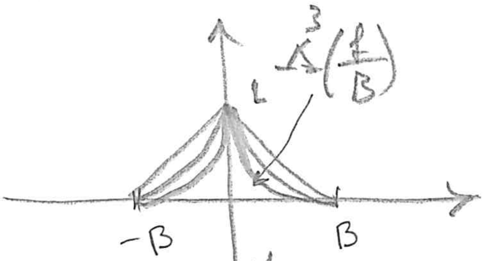
$$P_y(f) = \underbrace{|H_1(f)|^2 |H_2(f)|^2 |H_3(f)|^2 P_s(f)}_{P_{s_R}(f)} + \underbrace{|H_3(f)|^2 |H_2(f)|^2 P_{n_1}(f) + |H_3(f)|^2 P_{n_2}(f) + P_{n_3}(f)}_{P_{n_R}(f)}$$

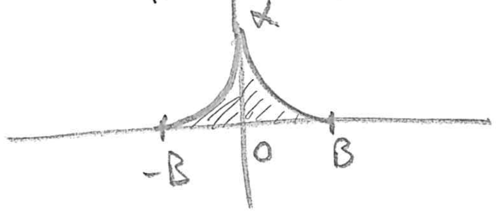
$$P_s(f) = \alpha\, \Pi\left(\frac{f}{2B}\right)$$

$$P_s = 2B\alpha = 2\cdot\cancel{10000}\cdot 8\cdot\cancel{10^{-4}}\ mV^2 = 16\ mV^2$$



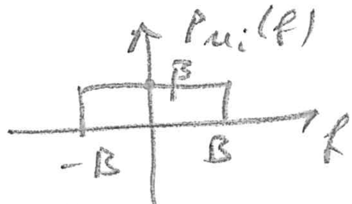
$$P_{s_R}(f) = \Delta^3\left(\frac{f}{2B}\right)\alpha\, \Pi\left(\frac{f}{2B}\right)$$



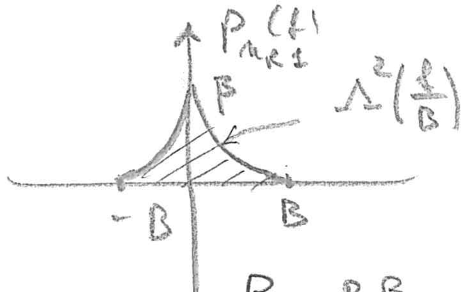


$$P_{sR} = \frac{B\alpha}{\cancel{4}\,2}\cdot\cancel{2} = \frac{10000\cdot 8\cdot 10^{-4}}{2} = 4\ mV^2$$

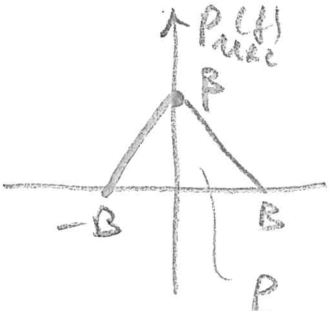
$$P_{n_1}(f) = P_{n_2}(f) = P_{n_3}(f) = \beta\, \Pi\left(\frac{f}{2B}\right);$$



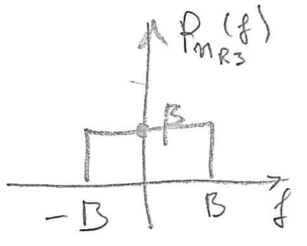
$$P_{n_i} = 2B\beta \; ; \; \beta = \frac{P_{n_i}}{2B} = \frac{9\ mV^2}{20000\ Hz} = 4.5\ 10^{-4}\ \frac{mV^2}{Hz}$$



$$P_{n_{R1}} = \frac{B\beta}{3}\cdot 2$$



$$P_{n_{R2}} = \frac{B\beta}{\cancel{2}}\,\cancel{2}$$



$$P_{n_{R3}} = 2B\beta$$

$$\left(\frac{S}{N}\right) = \frac{P_{SR}}{P_{MR}} = \frac{\frac{\cancel{B}\alpha}{2}}{\frac{\cancel{B}\beta}{3}\cdot 2 + \cancel{B}\beta + 2\cancel{B}\beta} = \frac{\alpha}{\beta}\,\frac{1}{\frac{4}{3}+2+4}$$

$$= \frac{\alpha}{\beta}\,\frac{3}{4+6+12} = \frac{\alpha}{\beta}\,\frac{3}{22} = \frac{8\cdot\cancel{10^{-4}}}{4.5\cdot\cancel{10^{-4}}}\,\frac{3}{22} = 0.24$$

$$\left(\frac{S}{N}\right)_{dB} = 10\lg_{10} 0.24 = -6.15\ dB$$

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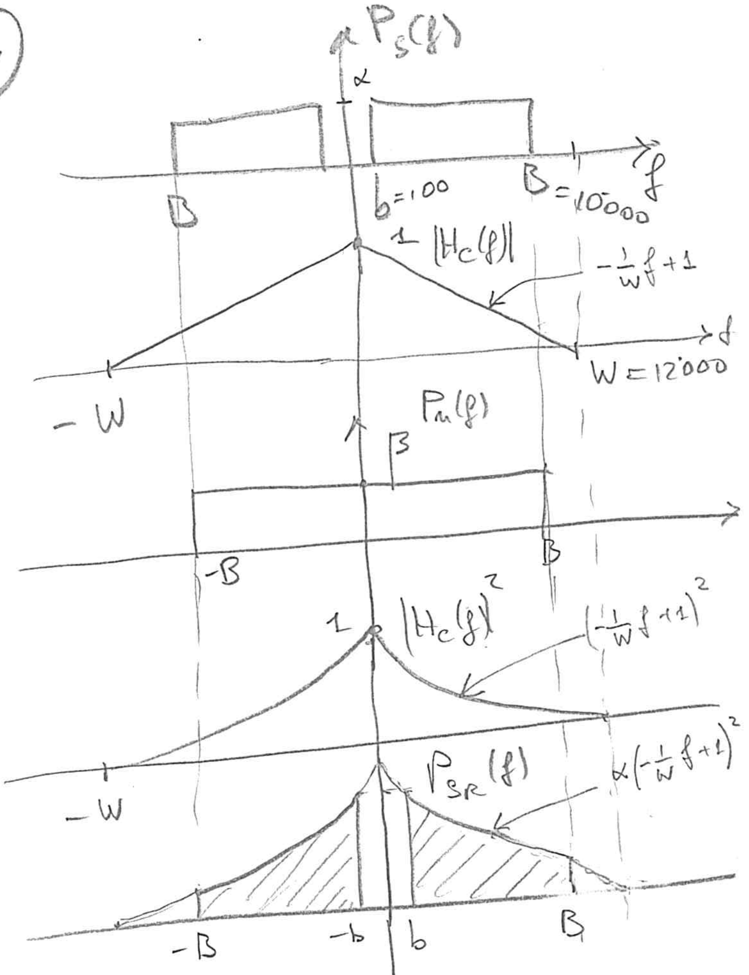
(4)



$P_s^{RMS} = 10\ mV$

$P_s = 100\ mV^2$

$P_s = (B-b)\alpha\cdot 2$

$$\alpha = \frac{P_s}{2(B-b)} = \frac{100}{2\cdot 9900} = 5.1\cdot 10^{-3}\ \frac{mV^2}{Hz}$$

$P_n^{RMS} = 5\ mV$

$P_n = 25\ mV^2$

$P_n = B\cdot\beta\cdot 2$

$$\beta = \frac{P_n}{B\cdot 2} = \frac{25}{10000\cdot 2} = 1.3\cdot 10^{-3}\ \frac{mV^2}{Hz}$$

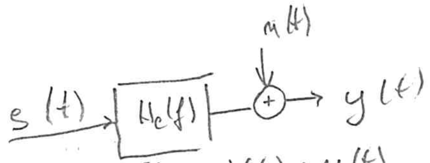
$$P_{SR} = 2\int_b^B \alpha\left(-\frac{1}{W}f+1\right)^2 df = 2\alpha\int_b^B\left(\frac{1}{W^2}f^2+1-\frac{2}{W}f\right)df$$

$$= 2\alpha\left[\frac{1}{W^2}\frac{f^3}{3}\Big|_b^B + (B-b) - \frac{\cancel{2}}{W}\frac{f^2}{\cancel{2}}\Big|_b^B\right] =$$

$$= 2\alpha\left[\frac{B^3-b^3}{3W^2} + (B-b) - \frac{B^2-b^2}{W}\right] =$$

$$= 2\cdot 5.1\cdot 10^{-3}\left[\frac{(10000)^3-(100)^3}{3(12000)^2} + 9900 - \frac{(10000)^2-(100)^2}{12000}\right] = 10.2\cdot 10^{-3}\left(2.32\cdot 10^3 + 9900 - 8.33\cdot 10^3\right)$$

$$= 39.60\ mV^2$$



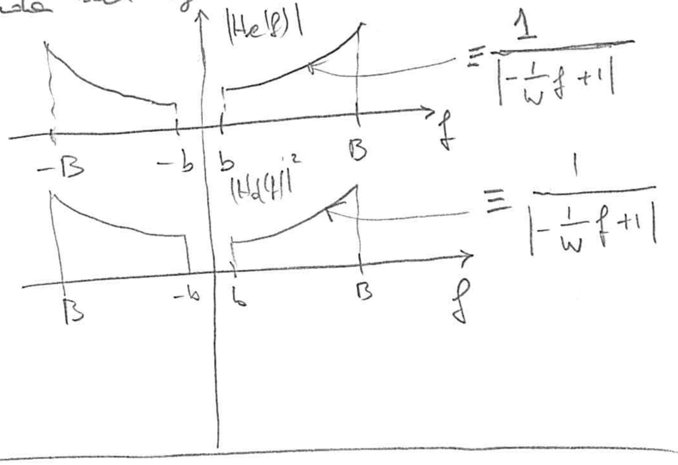
$$y(t) = \underbrace{(h_c * s)(t)}_{s_R(t)} + \underbrace{n(t)}_{n_R(t)}$$

$$\left(\frac{S}{N}\right) = \frac{39.60}{25.00} = 1.58 \qquad \left(\frac{S}{N}\right)_{dB} = 10 \log_{10} 1.58 = 2.00 \text{ dB}$$

$$s(t) \longrightarrow \boxed{H_e(f)} \longrightarrow \boxed{H_c(f)} \longrightarrow \underset{\uparrow\, n(t)}{\oplus} \longrightarrow \boxed{H_d(f)} \longrightarrow y(t)$$

Limitiamo l'attenzione alla banda del segnale $[b, B]$

$$\begin{cases} |H_e(f)|^2 = \dfrac{\alpha K}{|H_c(f)|} \dfrac{P_n^{1/2}(f)}{P_s^{1/2}(f)} & \text{cost} \\[2ex] |H_d(f)|^2 = \dfrac{K}{\alpha |H_c(f)|} \dfrac{P_s^{1/2}(f)}{P_n^{1/2}(f)} & \text{cost.} \end{cases}$$



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(5)

$$x_1(t) \longrightarrow \boxed{h_1(t)} \longrightarrow y_1(t)$$

$$x_2(t) \longrightarrow \boxed{h_2(t)} \longrightarrow y_2(t)$$

$$\boxed{R_{y_1 y_2}(\tau) = (r_{h_1 h_2} * R_{x_1 x_2})(\tau)}$$

$$R_{y_1 y_2}(\tau) = E\left[y_1(t) y_2^*(t-\tau)\right] = E\left[\int_{-\infty}^{+\infty} x_1(\eta) h_1(t-\eta) d\eta \int_{-\infty}^{+\infty} x_2^*(\xi) h_2^*(t-\tau-\xi) d\xi\right]$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \underbrace{E\left[x_1(\eta) x_2^*(\xi)\right]}_{R_{x_1 x_2}(\eta-\xi)} h_1(t-\eta) h_2^*(t-\tau-\xi) d\eta\, d\xi$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} R_{x_1 x_2}(\eta-\xi) h_1(t-\eta) h_2^*(t-\tau-\xi) d\eta\, d\xi$$

$$\alpha = \eta - \xi$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} R_{x_1 x_2}(\alpha) h_1(t-\eta) h_2^*(t-\tau-\eta+\alpha) d\alpha\, d\eta$$

$$= \int_{-\infty}^{+\infty} R_{x_1 x_2}(\alpha) \int_{-\infty}^{+\infty} h_1(t-\eta) h_2^*(t-\tau-\eta+\alpha) d\eta \quad d\alpha$$

$$\beta = t - \eta$$

$$= \int_{-\infty}^{+\infty} R_{x_1 x_2}(\alpha) \underbrace{\int_{-\infty}^{+\infty} h_1(\beta) h_2^*(\beta-\tau+\alpha) d\beta}_{r_{h_1 h_2}(\tau-\alpha)} d\alpha$$

$$= \int_{-\infty}^{+\infty} R_{x_1 x_2}(\alpha)\, r_{h_1 h_2}(\tau-\alpha) d\alpha = (R_{x_1 x_2} * r_{h_1 h_2})(\tau)$$