SECONDA UNIVERSITA' DEGLI STUDI DI NAPOLI
FACOLTA' DI INGEGNERIA

Teoria dei Segnali
Ia Prova Intracorso
Prof. Francesco A. N. Palmieri
mercoledi 23 ottobre, 2013

SOLUZIONI

Schizzare i seguenti segnali e valutarne l'energia, la potenza e la trasformata di Fourier.

(1) $s(t) = e^{-|t-2|}$;

(2) $s(t) = 2\ \cos^2 \pi t$;

(3) $s(t) = 2\Lambda\left(t+4\right) + \Lambda\left(\frac{t}{3}\right) + 2\Lambda\cancel{2}(t-4)$.

(4) Valutare una espressione del modulo e della fase della risposta armonica per in sistema lineare tempo-invariante con risposta impulsiva

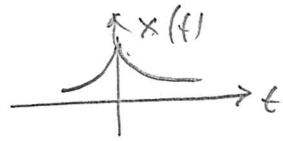
$$h(t) = (1 - e^{-t})u(t) - (1 - e^{-(t-7)})u(t-7). \qquad (1)$$

(5) Usando il metodo grafico valutare la risposta nel dominio del tempo all'ingresso $s(t) = u\left(t-3\right)$ per la risposta impulsiva di (4).

(6) valutare la migliore approssimazione al segnale $s(t) = sin\frac{4\pi}{T}t$ nell'intervallo $[0, T]$ in termini della funzioni di base $\Psi_1(t) = -u(t) + u(t-T)$ e $\Psi_2(t) = u(t) - 2u(t - \frac{T}{2}) + u(t-T)$

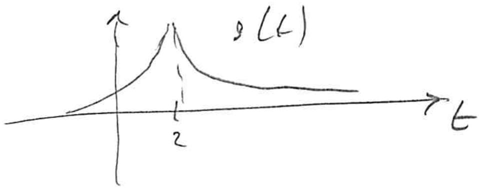
(7) Valutare autocorrelazione e spettro di energia del segnale

$$s(t) = \text{sinc}\ 3t. \qquad (2)$$



(1) $s(t) = e^{-|t-2|} = x(t-2)$; $x(t) = e^{-|t|}$

$$X(f) = \int_{-\infty}^{0} e^{t} e^{-j2\pi f t}\, dt + \int_0^\infty e^{-t} e^{-j2\pi f t}\, dt$$



$$= \int_{-\infty}^0 e^{(1-j2\pi f)t}\,dt + \int_0^\infty e^{-(1+j2\pi f)t}\,dt$$

$$= \left.\frac{e^{(1-j2\pi f)t}}{1-j2\pi f}\right|_{-\infty}^0 + \left.\frac{e^{-(1+j2\pi f)t}}{-(1+j2\pi f)}\right|_0^\infty = \frac{1}{1-j2\pi f}+\frac{1}{(1+j2\pi f)}$$

$$= \frac{1+\cancel{j2\pi f}+1-\cancel{j2\pi f}}{1+4\pi^2f^2} = \frac{2}{1+4\pi^2 f^2}$$

$$S(f) = \frac{2e^{-j2\pi f 2}}{1+4\pi^2 f^2}$$

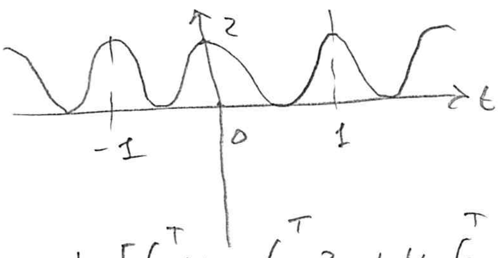
Segnale di energia $P_s = 0$

$$\mathcal{E}_s = \mathcal{E}_x = 2\int_0^\infty (e^{-t})^2 dt = 2\int_0^\infty e^{-2t}\,dt = \cancel{2}\left.\frac{e^{-2t}}{-\cancel{2}}\right|_0^\infty = 1$$

(stessa energia di s(t))

(2) $s(t) = 2\cos^2\pi t = 2\left(\frac12 + \frac12\cos 2\pi t\right) = 1+\cos 2\pi t$

$S(f) = \delta(f) + \frac12\delta(f-1)+\frac12\delta(f+1)$



Segnale di potenza $\mathcal{E}_s=\infty$

$$P_s = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^T (1+\cos 2\pi t)^2\, dt = \lim_{T\to\infty} \frac{1}{2T}\left[\int_{-T}^T dt + \int_{-T}^T\cos^2 2\pi t\, dt + \int_{-T}^T 2\cos 2\pi t\, dt\right]$$

$$= \lim_{T\to\infty} \frac{1}{2T}\left[2T + \int_{-T}^T \frac12\, dt + \int_{-T}^T \frac12 \cos 4\pi t\, dt + \cancel{2}\left.\frac{\sin 2\pi t}{\cancel{2}\pi}\right|_{-T}^T\right]$$

$$= \lim_{T\to\infty} \frac{1}{2T}\left[2T+T+\frac12 \left.\frac{\sin 4\pi t}{4\pi}\right|_{-T}^T + \frac{\sin 2\pi T}{\pi} - \frac{\sin 2\pi(-T)}{\pi}\right]$$

$$= \lim_{T\to\infty}\frac{1}{2T}\left[3T + \frac{\sin 4\pi T}{8\pi} - \frac{\sin 4\pi(-T)}{8\pi} + \frac{\sin 2\pi T}{\pi}+\frac{\sin 2\pi T}{\pi}\right]$$

$$= \lim_{T\to\infty}\frac{1}{2T}\left[3T+\frac{\sin4\pi T}{8\pi}+\frac{\sin 4\pi T}{8\pi} + 2\frac{\sin 2\pi T}{\pi}\right]$$

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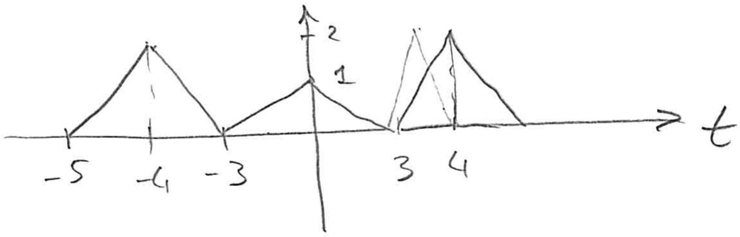
$$= \lim_{T\to\infty}\left[\frac{3}{2} + \overbrace{\cancel{\frac{\sin 4\pi T}{4\pi T}}}^{0} + 2\overbrace{\cancel{\frac{\sin 2\pi T}{\pi T}}}^{0}\right] = \frac{3}{2}$$

Che poteva ottenersi semplicemente facendo

$$P_s = (1)^2 + \frac{1^2}{2} = \frac{3}{2}$$

(3) $s(t) = 2\Lambda(t+4) + \Lambda\left(\frac{t}{3}\right) + 2\Lambda(t-4)$



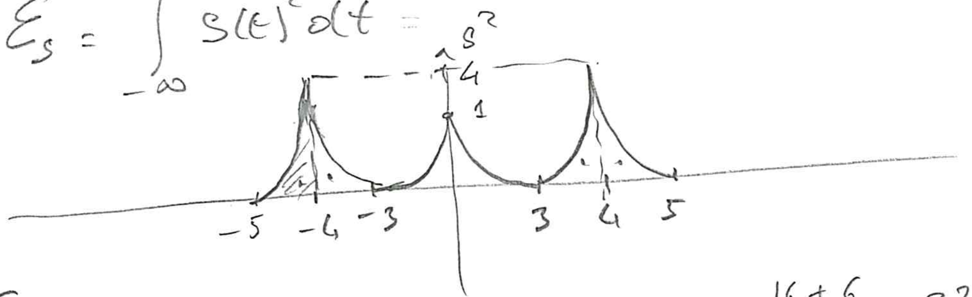
$$S(f) = 2e^{j2\pi f4}\operatorname{sinc}^2 f + 3\operatorname{sinc}^2 3f + 2e^{-j2\pi f4}\operatorname{sinc}^2 f$$

$$= 2\operatorname{sinc}^2 f\left(\frac{e^{j8\pi f} + e^{-j8\pi f}}{2}\right)2 + 3\operatorname{sinc}^2 3f$$

$$= 4\operatorname{sinc}^2 f\cos 8\pi f + 3\operatorname{sinc}^2 3f$$

Segnale di energia $P_s = 0$

$$\mathcal{E}_s = \int_{-\infty}^{+\infty} s(t)^2\,dt =$$



$$\mathcal{E}_s = \frac{1\cdot 4}{3}\cdot 4 + \frac{3\cdot 1}{3}\cdot 2 = \frac{16}{3} + 2 = \frac{16+6}{3} = \frac{22}{3}$$

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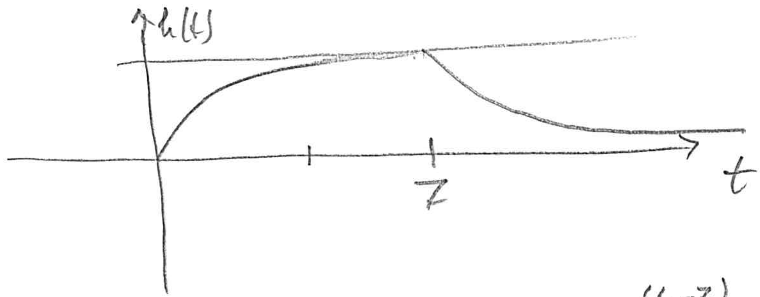
(4)

$$h(t) = (1-e^{-t})u(t) - \left((1-e^{-(t-7)})u(t-7)\right)$$



$$h(t) = u(t) - e^{-t}u(t) - u(t-7) + e^{-(t-7)}u(t-7)$$

$$= \left(u(t) - u(t-7)\right) - e^{-t}u(t) + e^{-(t-7)}u(t-7)$$

$$= \Pi\left(\frac{t-\frac{7}{2}}{7}\right) - e^{-t}u(t) + e^{-(t-7)}u(t-7)$$

$$\mathcal{F}\left[e^{-t}u(t)\right] = \int_0^\infty e^{-t}e^{-j2\pi f t}\,dt = \left[\frac{e^{-(1+j2\pi f)t}}{-(1+j2\pi f)}\right]_0^\infty = \frac{1}{1+j2\pi f}$$

$$\boxed{H(f) = 7e^{-j2\pi f\frac{7}{2}}\,\mathrm{sinc}\,7f - \frac{1}{1+j2\pi f} + e^{-j2\pi f 7}\frac{1}{1+j2\pi f}}$$

$$= 7e^{-j7\pi f}\frac{\sin \pi 7 f}{\pi 7 f} - \frac{1}{1+j2\pi f} + \frac{e^{-j14\pi f}}{1+j2\pi f}$$

$$= \frac{e^{j7\pi f}\sin 7\pi f\,(1+j2\pi f) - \pi f + \pi f\, e^{-j14\pi f}}{\pi f\,(1+j2\pi f)}$$

$$= \frac{(\cos 7\pi f + j\sin 7\pi f)\sin 7\pi f\,(1+j2\pi f) - \pi f + \pi f(\cos 14\pi f - j\sin 14\pi f)}{\pi f + j2\pi^2 f^2}$$

$$= \frac{(\cos 7\pi f + j\sin 7\pi f + j2\pi f\cos 7\pi f - 2\pi f\sin 7\pi f)\sin 7\pi f - \pi f \ldots}{\pi f + j2\pi^2 f^2}$$

$$\frac{+\pi f\cos 14\pi f - j\pi f\sin 14\pi f}{} = \frac{N_R(f) + j N_I(f)}{D_R(f) + j D_I(f)}$$

$N_R(f) = \cos 7\pi f \sin 7\pi f - 2\pi f \sin^2 7\pi f - \pi f + \pi f \cos 14\pi f$

$N_I(f) = \sin^2 7\pi f + 2\pi f \cos 7\pi f \cdot \sin 7\pi f - \pi f \sin 14\pi f$

$D_R(f) = \pi f$

$D_I(f) = 2\pi^2 f^2$

$$|H(f)|^2 = \frac{N_R^2(f) + N_I^2(f)}{D_R^2(f) + D_I^2(f)} \qquad \angle H(f) = \mathrm{atan2}(N_I(f)/N_R(f)) - \mathrm{atan2}(D_I(f)/D_R(f))$$

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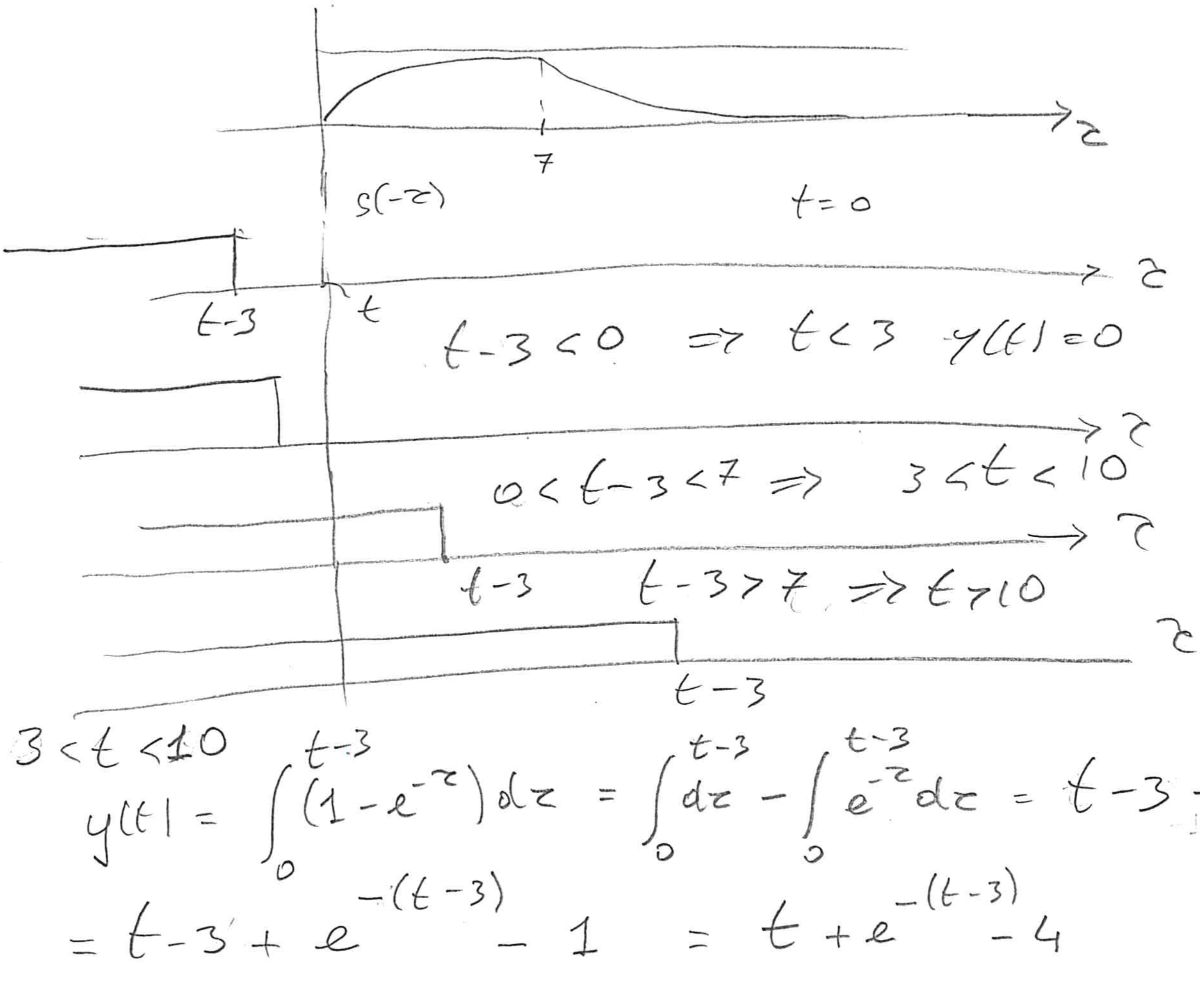
(5) $s(t) \rightarrow \boxed{h(t)} \rightarrow y(t) = \int_{-\infty}^{+\infty} h(\tau)\, s(t-\tau)\, d\tau \qquad s(t) = u(t-3)$



$3 < t < 10$

$$y(t) = \int_0^{t-3} (1-e^{-\tau})\, d\tau = \int_0^{t-3} d\tau - \int_0^{t-3} e^{-\tau} d\tau = t-3 + \left.\frac{e^{-\tau}}{+1}\right]_0^{t-3}$$

$$= t-3 + e^{-(t-3)} - 1 = t + e^{-(t-3)} - 4$$

$t > 10$

$$y(t) = \int_0^{t-3} (1-e^{-\tau})\, d\tau + \int_0^{t-3} (1-e^{-(\tau-7)})\, u(\tau-7)\, d\tau$$

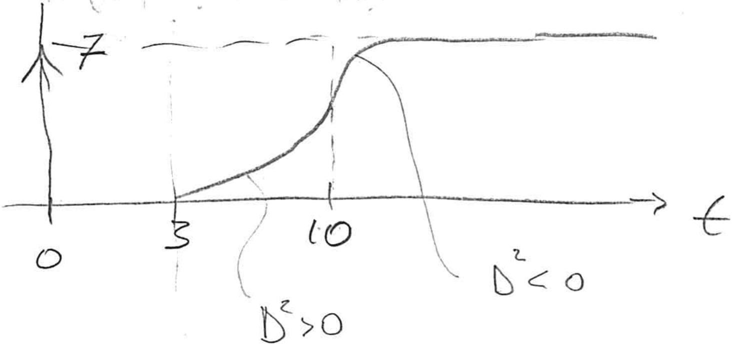
$$= \underbrace{\int_0^{t-3}(1-e^{-\tau})\,d\tau}_{\text{già calcolato.}} - \int_7^{t-3}\left(1-e^{-(\tau-7)}\right)d\tau$$

$$\int_7^{t-3} 1 - e^{7}\int_7^{t-3} e^{-\tau}\,d\tau = t-3-7+e^{7}\left[\frac{e^{-\tau}}{+1}\right]_7^{t-3}$$

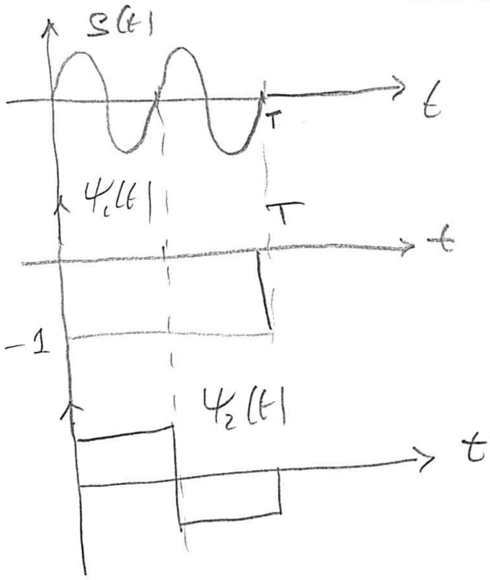
$$= t-10+e^{7}\left(e^{-(t-3)}-e^{-7}\right) = t-10+e^{7}e^{-(t-3)}-1$$

$$y(t) = \cancel{t}+e^{-(t-3)}-4-\cancel{t}+10-e^{7}e^{-(t-3)}+1$$

$$= 7+(1-e^{7})\,e^{-(t-3)}$$



(6) $s(t) = \sin\frac{4\pi}{T}t$



$$\hat{s}(t) = c_1\varphi_1(t) + c_2\varphi_2(t)$$

$$c_1 = \frac{\int_0^T s(t)\varphi_1(t)}{\int_0^T \varphi_1^2(t)\,dt} = \frac{-\int_0^T \sin\frac{4\pi}{T}t\,dt}{T} = 0$$

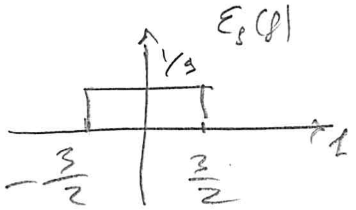
$$c_2 = \frac{\overset{0}{\int_0^{T/2}\sin\frac{4\pi}{T}t\,dt} - \overset{0}{\int_{T/2}^{T}\sin\frac{4\pi}{T}t\,dt}}{T} = 0$$

$$\hat{s}(t) = 0$$

(7) $s(t) = \mathrm{sinc}\, 3t = \frac{1}{3}\, 3\, \mathrm{sinc}\, 3t$

$$S(f) = \frac{1}{3}\, \Pi\left(\frac{f}{3}\right)$$

$$\mathcal{E}_s(f) = |S(f)|^2 = \frac{1}{9}\, \Pi\left(\frac{f}{3}\right)$$



$$r_s(\tau) = \mathcal{F}^{-1}\left[\mathcal{E}_s(f)\right] = \frac{1}{9}\, 3\, \mathrm{sinc}\, 3\tau = \frac{1}{3}\, \mathrm{sinc}\, 3\tau$$