SOLUZIONI

Ia pI. 1

SECONDA UNIVERSITA' DEGLI STUDI DI NAPOLI
DIPARTIMENTO DI INGEGNERIA INDUSTRIALE E DELL'INFORMAZIONE
Scuola Politecnica e delle Scienze di Base

Teoria dei Segnali/Telecomunicazioni 2
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lunedi 20 ottobre 2014

Schizzare i seguenti segnali e valutarne l'energia e la potenza. Inoltre se ne valuti e si schizzi al meglio la trasformata di Fourier.

(1) $s(t) = e^{-2t}u(t-3)$;

(2) $s(t) = \cos^2(\pi t + \frac{\pi}{4})$;

(3) $s(t) = 2\Lambda\left(\frac{t+5}{3}\right) + \Lambda\left(\frac{t}{2}\right) + 2\Lambda\left(\frac{t-5}{3}\right)$.

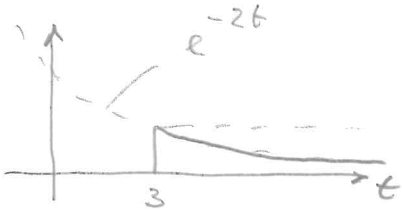
(4) Per il segnale periodico

$$s(t) = -1 + \sum_{k=-\infty}^{\infty}\left[\Pi\left(\frac{t}{2} - 2k\right) + \Lambda\left(t - 4k\right)\right], \qquad (1)$$

eseguire uno schizzo, valutare la potenza, la trasformata di Fourier e confrontarla con la relativa serie di Fourier esponenziale.

(5) Un sistema lineare ha risposta impulsiva $h(t) = e^{-2t}u(t-1)$. Valutare la risposta armonica in modulo e fase. Valutare inoltre l'uscita del sistema quando all'ingresso é posto il segnale $s(t) = u(t-6) - u(t-7)$ (si utilizzi il metodo grafico).

(1) $s(t) = e^{-2t} u(t-3)$



TapI. 2

Segnale di energia

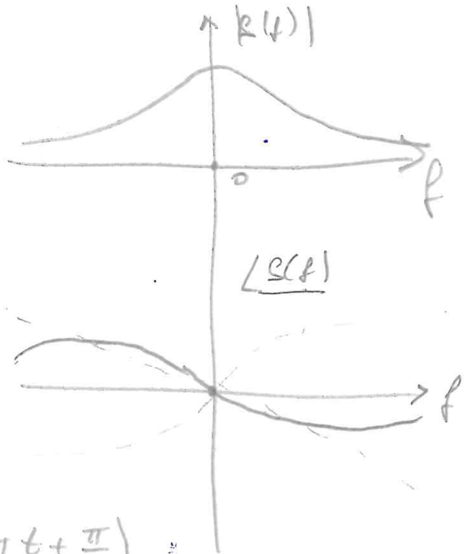
$P_s = 0$

$$\mathcal{E}_s = \int_{-\infty}^{+\infty} |s(t)|^2 dt = \int_3^{\infty} e^{-2t \cdot 2} dt = \left.\frac{e^{-4t}}{-4}\right]_3^{\infty} = \frac{e^{-12}}{4}$$

$$S(f) = \int_3^{\infty} e^{-2t} e^{-j2\pi f t} dt = \int_3^{\infty} e^{-(2+j2\pi f)t} dt = \left.\frac{e^{-(2+j2\pi f)t}}{-(2+j2\pi f)}\right]_3^{\infty}$$

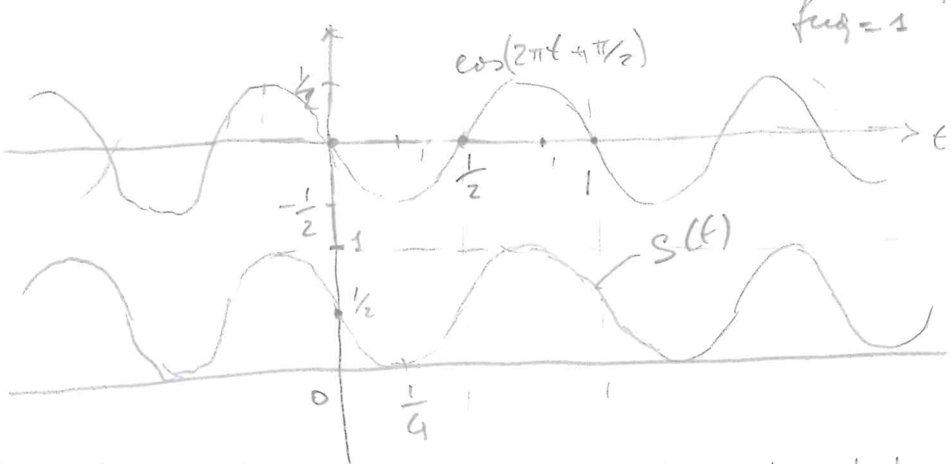
$$= \frac{e^{-(2+j2\pi f)3}}{(2+j2\pi f)} = \frac{e^{-6}}{2} e^{-j6\pi f} \frac{1}{1+j\pi f}$$

$$\begin{cases} |S(f)| = \dfrac{e^{-6}}{2} \dfrac{1}{|1+j\pi f|} = \dfrac{e^{-6}}{2} \dfrac{1}{\sqrt{1+\pi^2 f^2}} \\ \angle S(f) = -6\pi f - \mathrm{tg}^{-1} \pi f \end{cases}$$



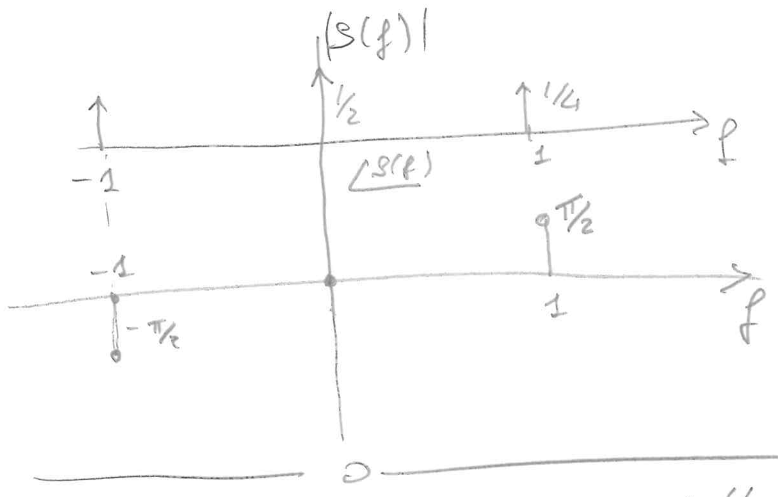
(2) $s(t) = \cos^2(\pi t + \frac{\pi}{4}) = \frac{1}{2} + \frac{1}{2} \cos(2\pi t + \frac{\pi}{2})$

freq = 1



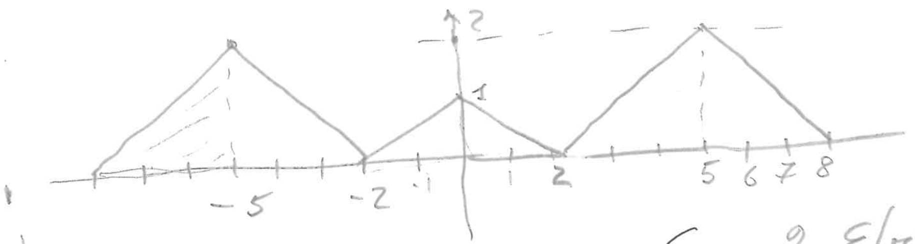
Segnale di Potenza ; $\mathcal{E}_s = \infty$; $P_s = \frac{1}{4} + \frac{1}{4}\frac{1}{2} = \frac{3}{8}$

$$S(f) = \frac{1}{2}\delta(f) + \frac{1}{4}\delta(f-1)e^{j\frac{\pi}{2}} + \frac{1}{4}\delta(f+1)e^{-j\frac{\pi}{2}}$$



16K
3
48

(3) $s(t) = 2\Lambda\left(\frac{t+5}{3}\right) + \Lambda\left(\frac{t}{2}\right) + 2\Lambda\left(\frac{t-5}{3}\right)$



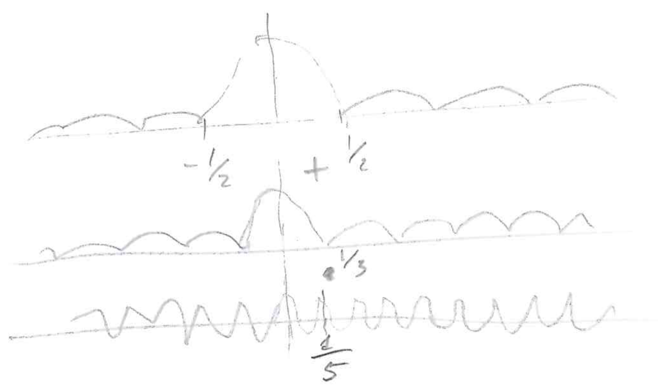
Segnale di energia; $P_s = 0$; $\mathcal{E}_s = 2\,\mathcal{E}(\text{primo triang.}) + \mathcal{E}(\text{triang. centrale})$

$= 2\left(\frac{3\cdot 2}{3}\cdot 2^2\right) + \frac{2\cdot 1^2}{3}\cdot 2$ (dall'area sottesa dalle parabole)

$= 16 + \frac{4}{3} = \frac{52}{3}$

$S(f) = 2e^{j2\pi f5}\, 3\,\mathrm{sinc}^2 3f + 2\,\mathrm{sinc}^2 2f + 2e^{-j2\pi f5}\, 3\,\mathrm{sinc}^2 3f$

$= 2\,\mathrm{sinc}^2 2f + 12\cos 10\pi f \cdot \mathrm{sinc}^2 3f$

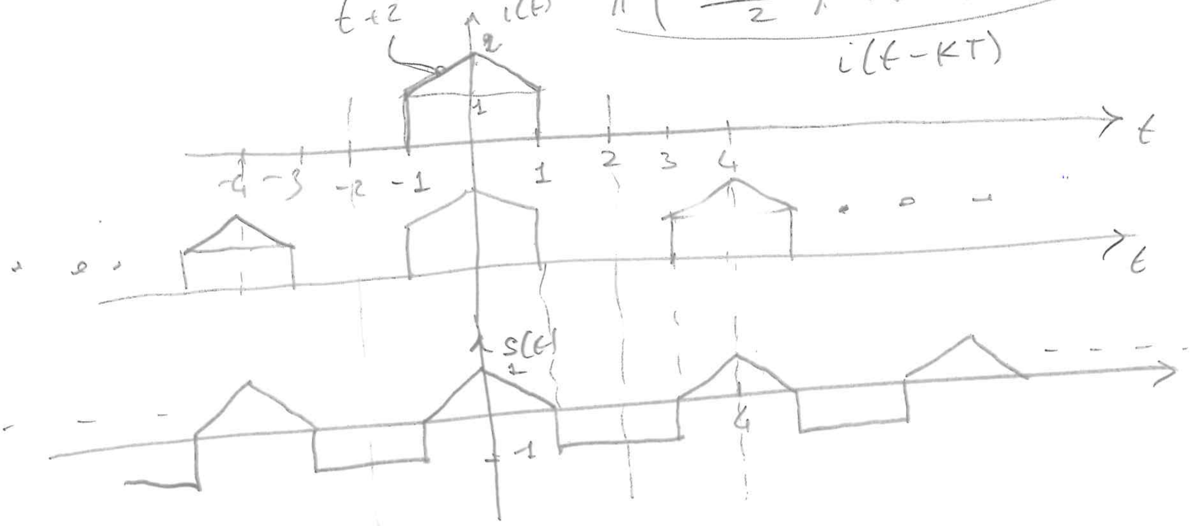


J_aPI.4

(4)

$$s(t) = -1 + \sum_{k=-\infty}^{+\infty}\left[\Pi\left(\frac{t}{2}-2k\right)+\Lambda(t-4k)\right]$$

$$\underbrace{\Pi\left(\frac{t-4k}{2}\right)+\Lambda(t-4k)}_{i(t-kT)}$$



$$s(t) = -1 + i(t) * \sum_{k=-\infty}^{+\infty}\delta(t-4k)$$

$$\begin{cases} S(f) = -\delta(f) + I(f)\cdot\frac{1}{4}\sum_{k=-\infty}^{+\infty}\delta\left(f-\frac{k}{4}\right) \\ I(f) = 2\,\mathrm{sinc}\,2f + \mathrm{sinc}^2 f \end{cases}$$

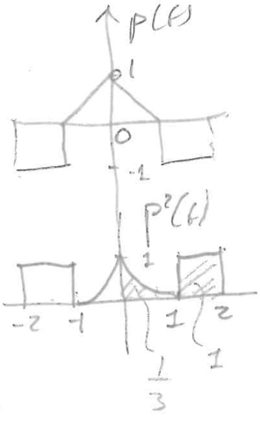
$$S(f) = -\delta(f) + \frac{1}{4}\sum_{k=-\infty}^{+\infty}\left[2\,\mathrm{sinc}\,2\frac{k}{4} + \mathrm{sinc}^2\frac{k}{4}\right]\delta\left(f-\frac{k}{4}\right)$$

La serie di Fourier esponenziale è

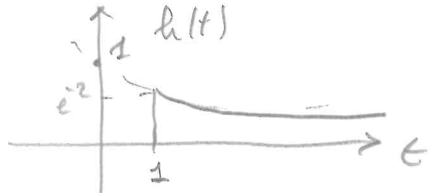
$$s(t) = \sum_{k=-\infty}^{+\infty} c_k\, e^{j2\pi \frac{k}{T_0}t} = \sum_{k=-\infty}^{+\infty}\frac{1}{T}I\left(\frac{k}{T}\right)\delta\left(f-\frac{k}{T_0}\right), \quad T_0 = 4$$

Quindi
$$\begin{cases} c_0 = -1 + \frac{3}{4} = -\frac{1}{4} \\ c_k = 2\,\mathrm{sinc}\,\frac{k}{2} + \mathrm{sinc}^2\frac{k}{4} \quad \forall k \neq 0 \end{cases}$$

$$P_s = \frac{\mathcal{E}_{p(t)}}{4} = \frac{1}{4}\,2\left(\frac{1}{3}+1\right) = \frac{2}{3}$$



(5) $h(t) = e^{-2t}\,u(t-1)$

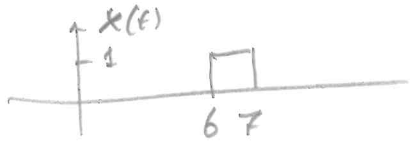


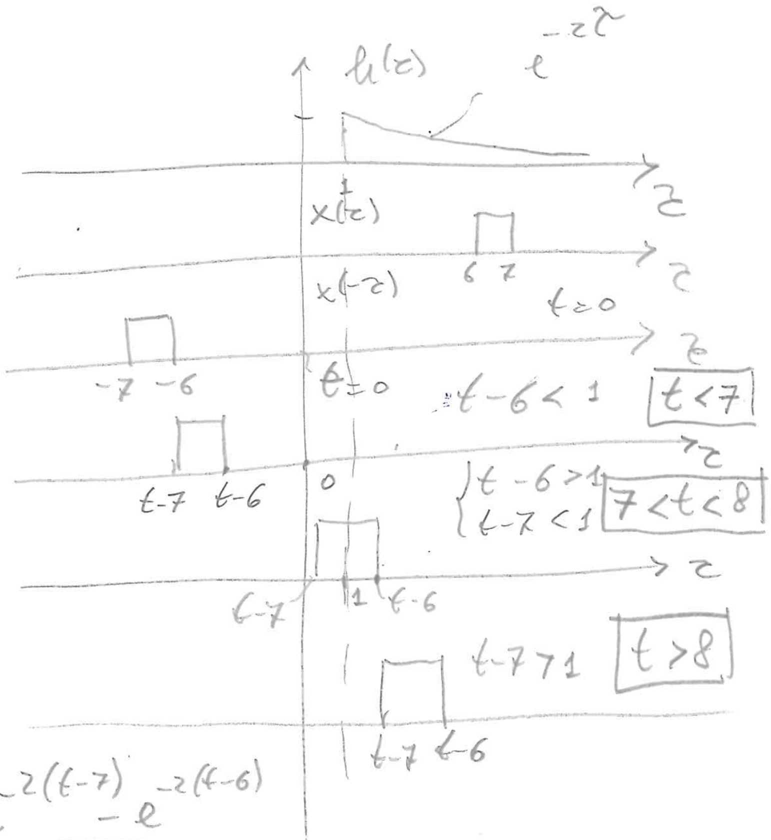
$$H(f) = \int_1^\infty e^{-2t} e^{-j2\pi f t}\,dt = \int_1^\infty e^{-(2+j2\pi f)t}\,dt = \left.\frac{e^{-(2+j2\pi f)t}}{-(2+j2\pi f)}\right]_1^\infty$$

$$= \frac{e^{-(2+j2\pi f)}}{2(1+j\pi f)} = \frac{e^{-2}}{2}\, e^{-j2\pi f}\, \frac{1}{1+j\pi f}$$

$$\begin{cases} |H(f)| = \dfrac{e^{-2}}{2}\dfrac{1}{|1+j\pi f|} = \dfrac{e^{-2}}{2}\dfrac{1}{\sqrt{1+\pi^2 f^2}} \\[2ex] \angle H(f) = -2\pi f - tg^{-1}\pi f \end{cases}$$

$x(t) = u(t-6) - u(t-7)$





$\boxed{t<7}$ $y(t) = 0$

$\boxed{7<t<8}$

$$y(t) = \int_1^{t-6} e^{-2\tau}\,d\tau = \left.\frac{e^{-2\tau}}{-2}\right]_1^{t-6} =$$

$$= \frac{e^{-2} - e^{-2(t-6)}}{2}$$

$\boxed{t>8}$

$$y(t) = \int_{t-7}^{t-6} e^{-2\tau}\,d\tau = \left.\frac{e^{-2\tau}}{-2}\right]_{t-7}^{t-6} = \frac{e^{-2(t-7)} - e^{-2(t-6)}}{2}$$