(1)

SECONDA UNIVERSITA' DEGLI STUDI DI NAPOLI

DIPARTIMENTO DI INGEGNERIA INDUSTRIALE E DELL'INFORMAZIONE

Scuola Politecnica e delle Scienze di Base

Teoria dei Segnali/Telecomunicazioni 2

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SOLUZIONI

Schizzare i seguenti segnali e valutarne l'energia e la potenza. Inoltre se ne valuti e se ne schizzi al meglio la trasformata di Fourier.

(1) $s(t) = e^{-t}u(t+3)$;

(2) $s(t) = 1 + \cos^2(\pi t + \frac{\pi}{4})$;

(3) Per il segnale periodico

$$s(t) = \sum_{k=-\infty}^{\infty} \left[\Pi\left(\frac{t}{4} - k\right) - \Lambda\left(\frac{t}{2} - 2k\right)\right], \tag{1}$$

eseguire uno schizzo, valutare la potenza, la trasformata di Fourier e confrontarla con la relativa serie di Fourier esponenziale.

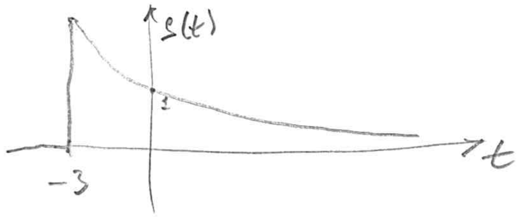
(4) Un sistema lineare ha risposta impulsiva $h(t) = e^{-t}u(t+1)$. Valutare la risposta armonica in modulo e fase. Valutare inoltre l'uscita del sistema quando all'ingresso é posto il segnale $s(t) = u(t+3) - u(t-7)$ (si utilizzi il metodo grafico).

(5) Si consideri il segnale

$$X(t) = 1 + S(t)\cos^2(4000\pi t), \tag{2}$$

dove $S(t)$ è un processo aleatorio SSL avente spettro di potenza piatto nella banda [200 500] Hz e potenza P_0. Ricavare la autocorrelazione di $S(t)$, la autocorrelazione e lo spettro di potenza medio di $X(t)$ commentando sulla stazionariet.

(1) $s(t) = e^{-t} u(t+3)$



$P_s = 0$;

$$\mathcal{E}_s \triangleq \int_{-\infty}^{+\infty} s^2(t)\,dt = \int_{-3}^{\infty} e^{-t2}\,dt = \left.\frac{e^{-2t}}{-2}\right]_{-3}^{\infty}$$

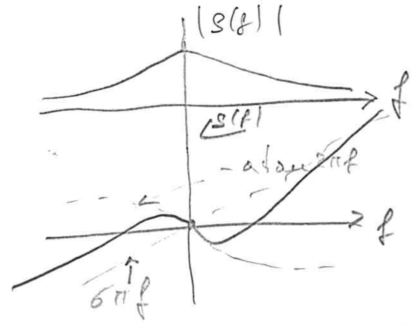
$$= \frac{0 - e^{6}}{-2} = \frac{e^{6}}{2}$$

$$S(f) = \int_{-3}^{\infty} e^{-t} e^{-j2\pi f t}\,dt = \int_{-3}^{+\infty} e^{-(1+j2\pi f)t}\,dt = \left.\frac{e^{-(1+j2\pi f)t}}{-(1+j2\pi f)}\right]_{-3}^{\infty}$$

$$= \frac{0 - e^{+(1+j2\pi f)3}}{-(1+j2\pi f)} = e^{3}\,\frac{e^{j6\pi f}}{1+j2\pi f}$$

$$|S(f)| = \frac{e^{3}}{|1+j2\pi f|} = \frac{e^{3}}{\sqrt{1+4\pi^2 f^2}}$$

$$\angle S(f) = 6\pi f - \arctan 2\pi f$$



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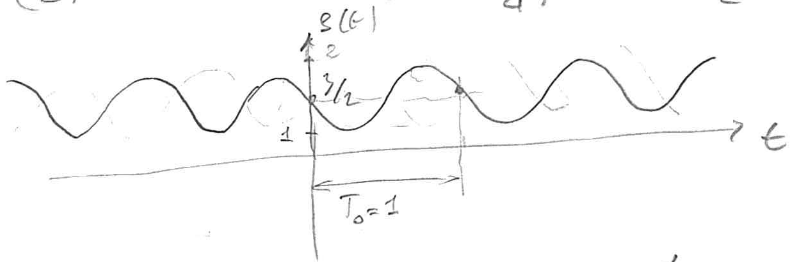
(2) $s(t) = 1 + \cos^2\left(\pi t + \frac{\pi}{4}\right) = 1 + \frac{1}{2} + \frac{1}{2}\cos\left(2\pi t + \frac{\pi}{2}\right) = \frac{3}{2} + \frac{1}{2}\cos\left(2\pi t + \frac{\pi}{2}\right)$



$\mathcal{E}_s = \infty$; segnale di potenza

$$P_s = \frac{\text{Energia nel periodo}}{\text{periodo}} = \frac{1}{1}\int_0^1 \left(\frac{3}{2} + \frac{1}{2}\cos\left(2\pi t + \frac{\pi}{2}\right)\right)^2 dt$$

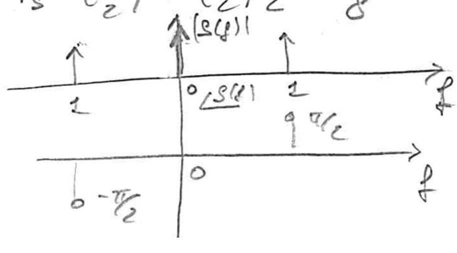
$$= \int_0^1 \frac{9}{4}\,dt + \int_0^1 \frac{1}{4}\cos^2\left(2\pi t + \frac{\pi}{2}\right)dt + \underbrace{\cancel{\int_0^1 \frac{3}{2}\cos\left(2\pi t + \frac{\pi}{2}\right)dt}}_{0}$$

$$= \frac{9}{4} + \int_0^1 \left(\frac{1}{8} + \frac{1}{8}\cos(4\pi t + \pi)\right)dt = \frac{9}{4} + \frac{1}{8} + \underbrace{\cancel{\int_0^1 \frac{1}{8}\cos(4\pi t + \pi)\,dt}}_{0}$$

$$= \frac{18+1}{8} = \frac{19}{8}$$; Più semplicemente $P_s = \left(\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2 \frac{1}{2} = \frac{19}{8}$

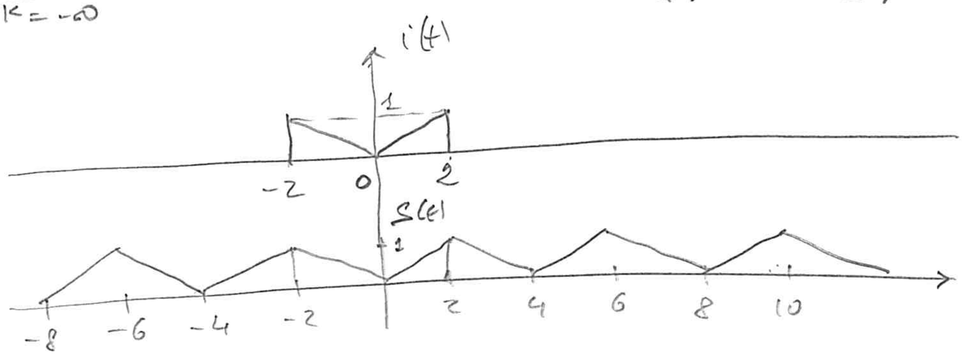
$$S(f) = \mathcal{F}\left[\frac{3}{2} + \frac{1}{2}\cos\left(2\pi f + \pi/2\right)\right]$$

$$= \frac{3}{2}\delta(f) + \frac{1}{4}e^{j\pi/2}\delta(f-1) + \frac{1}{4}e^{-j\frac{\pi}{2}}\delta(f+1)$$

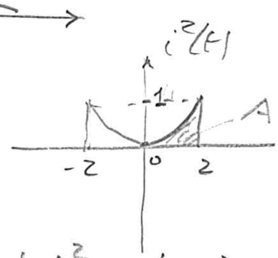


(3) $s(t)=\sum_{k=-\infty}^{+\infty}\left[\Pi\left(\frac{t}{4}-k\right)-\Lambda\left(\frac{t}{2}-2k\right)\right]=\sum_{k=-\infty}^{+\infty}\left[\Pi\left(\frac{t-4k}{4}\right)-\Lambda\left(\frac{t-4k}{2}\right)\right]$ 3

$=\sum_{k=-\infty}^{+\infty} i(t-4k)$; $i(t)=\Pi\left(\frac{t}{4}\right)-\Lambda\left(\frac{t}{2}\right)$ periodo $T_0=4$



$P_s=\frac{\mathcal{E}_i}{T_0}=\frac{\int_{-2}^{2} i^2(t)\,dt}{4}=\frac{2A}{4}=\frac{2\,\frac{2\cdot 1}{3}}{4}=\frac{1}{3}$



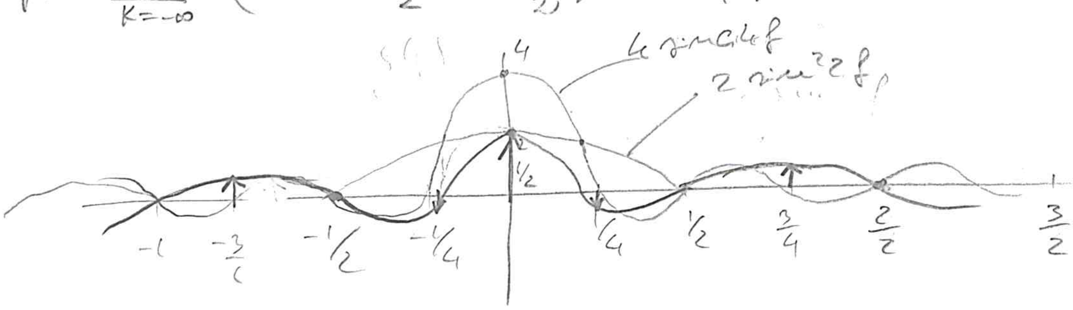
Più formalmente $i(t)=\begin{cases}\frac{1}{2}t & 0<t<2\\ -\frac{1}{2}t & -2<t<0\\ 0 & \text{altrove}\end{cases}$ $\quad i^2(t)=\begin{cases}\frac{1}{4}t^2 & 0<t<2\\ \frac{1}{4}t^2 & -2<t<2\\ 0 & \text{altrove}\end{cases}$

$\int_{-2}^{2} i^2(t)\,dt=2\int_0^2 i^2(t)\,dt=2\int_0^2\frac{1}{4}t^2\,dt=\frac{1}{2}\left.\frac{t^3}{3}\right]_0^2=\frac{8}{6}=\frac{4}{3}$; $P_s=\frac{1}{4}\frac{4}{3}=\frac{1}{3}$

$S(f)=I(f)*\frac{1}{T_0}\sum_{k=-\infty}^{+\infty}\delta\left(f-\frac{k}{T_0}\right)=\sum_{k=-\infty}^{+\infty}\frac{1}{T_0}I\left(\frac{k}{T_0}\right)\delta\left(f-\frac{k}{T_0}\right)$

$I(f)=\mathcal{F}\left[\Pi\left(\frac{t}{4}\right)-\Lambda\left(\frac{t}{2}\right)\right]=4\,\mathrm{sinc}\,4f-2\,\mathrm{sinc}^2\,2f$

$S(f)=\sum_{k=-\infty}^{+\infty}\left(\mathrm{sinc}\,k-\frac{1}{2}\mathrm{sinc}^2\frac{k}{2}\right)\delta\left(f-\frac{k}{4}\right)$



sinc k contribuisce solo per $k=0$; $\mathrm{sinc}^2\frac{k}{2}$ è diverso da zero per $k=0$ e per k dispari: $\pm\frac{1}{4}$, $\pm\frac{3}{4}$; $\frac{5}{4}$; ecc.

Quindi c'è una componente continua $\frac{1}{2}\delta(f)$ e armoniche dispari

$-\frac{1}{2}\mathrm{sinc}^2\frac{k}{2}\,\delta\left(f-\frac{k}{4}\right)$ k dispari.

(3) cont.

Il confronto con la serie di Fourier esponenziale è noto in generale

$$s(t) = \mathcal{F}^{-1}[S(f)] = \mathcal{F}^{-1}\left[\sum_{k=-\infty}^{+\infty} \frac{1}{T_0} I\left(\frac{k}{T_0}\right)\delta\left(f-\frac{k}{T_0}\right)\right]$$

$$= \sum_{k=-\infty}^{+\infty} \underbrace{\frac{1}{T_0} I\left(\frac{k}{T_0}\right)}_{c_k} e^{-j2\pi\frac{k}{T_0}t} \qquad c_k = \mathrm{sinc}\, k - \frac{1}{2}\,\mathrm{sinc}^2\frac{k}{2}$$

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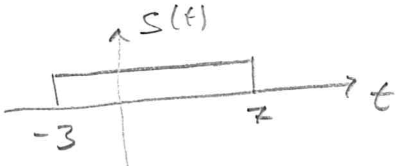
(4) $h(t) = e^{-t}u(t+1)$

$$H(f) = \mathcal{F}[h(t)] = \int_{-1}^{\infty} e^{-t}e^{-j2\pi f t}\,dt = \int_{-1}^{\infty} e^{-(1+j2\pi f)t}\,dt = \left.\frac{e^{-(1+j2\pi f)t}}{-(1+j2\pi f)}\right]_{-1}^{\infty}$$

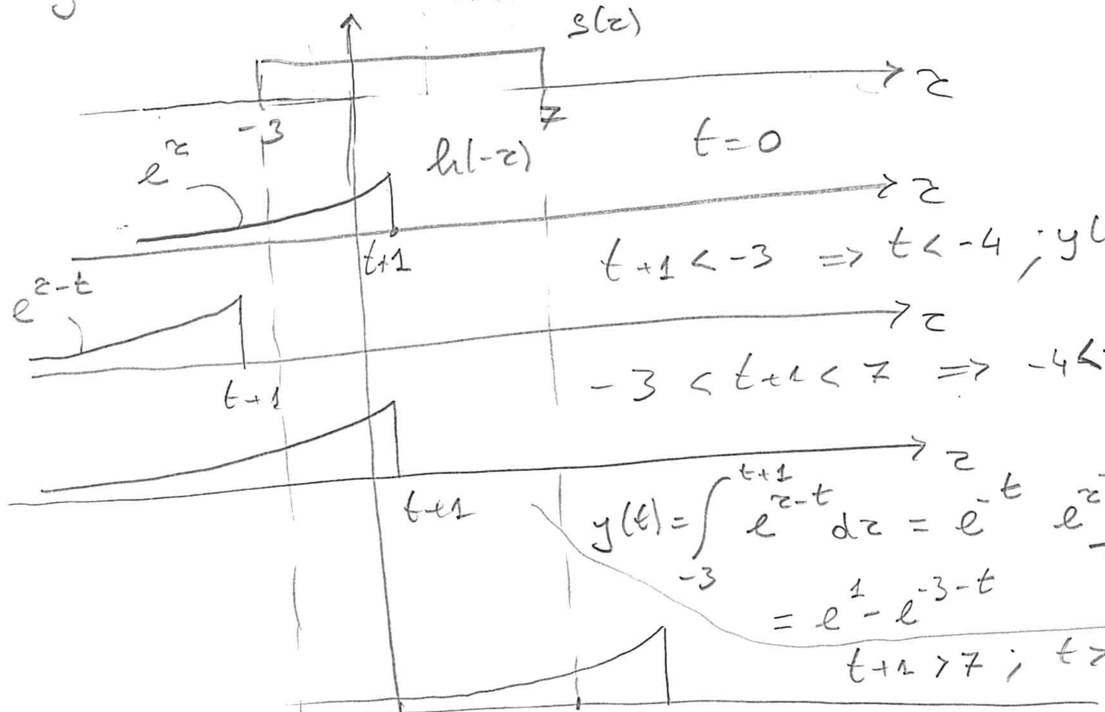
$$= \frac{0 - e^{+(1+j2\pi f)}}{-(1+j2\pi f)} = e^{1}\,\frac{e^{j2\pi f}}{1+j2\pi f}$$

$$|H(f)| = \frac{e^{1}}{\sqrt{1+4\pi f^2}} \;;\quad \angle H(f) = 2\pi f - \mathrm{arctg}\, 2\pi f$$

$$s(t) = u(t+3) - u(t-7)$$



$$y(t) = (h * s)(t) = \int_{-\infty}^{+\infty} s(\tau)\,h(t-\tau)\,d\tau$$



$t=0$

$t+1 < -3 \Rightarrow t < -4 \;;\; y(t) = 0$

$-3 < t+1 < 7 \Rightarrow -4 < t < 6$

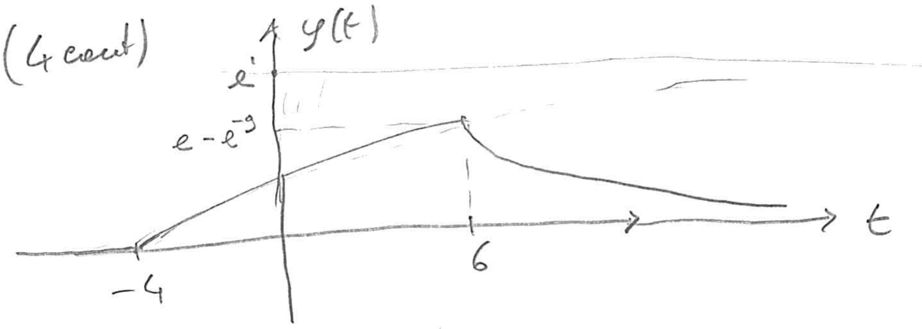
$$y(t) = \int_{-3}^{t+1} e^{\tau - t}\,d\tau = e^{-t}\left. e^{\tau}\right]_{-3}^{t+1} = e^{-t}\left(e^{t+1} - e^{-3}\right)$$

$$= e^{1} - e^{-3-t}$$

$t+1 > 7 \;;\; t > 6$

$$y(t) = \int_{-3}^{7} e^{\tau - t}\,d\tau = e^{-t}\left. e^{\tau}\right]_{-3}^{7} = e^{-t}\left(e^{7} - e^{-3}\right)$$

(4 cont)



(5)

$$X(t) = 1 + S(t)\cos^2 4000\pi t = 1 + \frac{S(t)}{2} + \frac{S(t)}{2}\cos \underbrace{8000\pi t}_{2\pi f_0 t}$$

$f_0 = 4000$

$$R_X(t;\tau) = E[X(t)X(t-\tau)] = E\Big[\Big(1 + \tfrac{1}{2}S(t) + \tfrac{S(t)}{2}\cos 2\pi f_0 t\Big)\cdot \Big(1 + \tfrac{1}{2}S(t-\tau) + \tfrac{S(t-\tau)}{2}\cos 2\pi f_0 (t-\tau)\Big)\Big] =$$

$$= 1 + \tfrac{1}{2}\cancel{E[S(t-\tau)]}^{0} + \frac{\cancel{E[S(t-\tau)]}^{0}}{2}\cos 2\pi f_0 (t-\tau)$$

$$+ \tfrac{1}{2}\cancel{E[S(t)]}^{0} + \tfrac{1}{4}E[S(t)S(t-\tau)] + \tfrac{1}{4}E[S(t)S(t-\tau)]\cos 2\pi f_0 (t-\tau)$$

$$+ \tfrac{1}{2}\cancel{E[S(t)]}^{0}\cos 2\pi f_0 t + \tfrac{1}{4}E[S(t)S(t-\tau)]\cos 2\pi f_0 t +$$

$$+ \tfrac{1}{4}E[S(t)S(t-\tau)]\cos 2\pi f_0 t\cos 2\pi f_0 (t-\tau)$$

$$= 1 + \tfrac{1}{4}R_S(\tau) + \tfrac{1}{4}R_S(\tau)\cos 2\pi f_0 (t-\tau) + \tfrac{1}{4}R_S(\tau)\cos 2\pi f_0 t$$

$$+ \tfrac{1}{4}R_S(\tau)\cos 2\pi f_0 t \cos 2\pi f_0 (t-\tau)$$

processo non stazionario ma ciclostazionario con periodo $\frac{1}{f_0}$. Valutiamo la autocorrelazione media

$$\overline{R_X(\tau)} = f_0 \int_{-\frac{1}{2f_0}}^{\frac{1}{2f_0}} R_X(t;\tau)\,dt = 1 + \tfrac{1}{4}R_S(\tau)$$

$$+ \tfrac{1}{4}R_S(\tau)\, f_0 \cancel{\int \cos 2\pi f_0 (t-\tau)\,dt}^{0} + \tfrac{1}{4}R_S(\tau) f_0 \cancel{\int \cos 2\pi f_0 t\,dt}^{0}$$

$$+ \tfrac{1}{4}R_S(\tau) f_0 \int\Big(\tfrac{1}{2}\cos 2\pi f_0 \tau + \tfrac{1}{2}\cancel{\cos 2\pi f_0 (2t-\tau)}^{0}\Big)dt$$

$$= 1 + \tfrac{1}{4}R_S(\tau) + \tfrac{1}{8}R_S(\tau)\cos 2\pi f_0 \tau$$

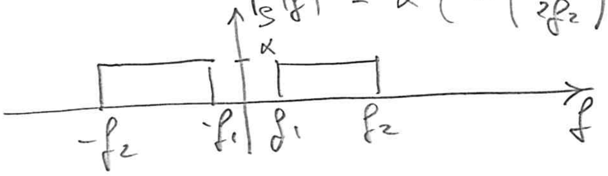
(5 cont)

$$\overline{P_X(f)} = \mathcal{F}\left[\overline{R_X(\tau)}\right] = \delta(f) + \frac{1}{4} P_S(f) + \frac{1}{16} P_S(f-f_0) + \frac{1}{16} P_S(f+f_0)$$

$$P_S(f) = \alpha\left(\Pi\left(\frac{f}{2f_2}\right) - \Pi\left(\frac{f}{2f_1}\right)\right)$$

$$P_0 = (f_2-f_1)\alpha 2$$

$$\boxed{\alpha = \frac{P_0}{2(f_2-f_1)}}$$



$$R_S(\tau) = \mathcal{F}^{-1}\left[P_S(f)\right] = \alpha 2f_2 \,\mathrm{sinc}\, 2f_2\tau - \alpha 2f_1 \,\mathrm{sinc}\, 2f_1\tau$$