Sol_30102017-1

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Dipartimento di Ingegneria Industriale e dell'Informazione
Corso di Laurea in Ingegneria Elettronia e Informatica

Ia Prova Intracorso (SOLUZIONI)
Teoria dei Segnali
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lunedì 30 ottobre 2017

1. Schizzare i seguenti segnali e valutarne l'energia, la potenza e la trasformata di Fourier:

(a)[4 pt] $s(t) = 2\Pi\left(\frac{t}{2} - 2\right) - \Lambda\left(t - 4\right)$;

(b)[3 pt] $s(t) = e^{-|t+5|}$;

(c)[3 pt] $s(t) = 2 + \cos^2 \pi t$.

(d)[3 pt] $s(t) = 1 - \sum_{k=-\infty}^{\infty} \Lambda\left(t - 2k\right)$

2.[10 pt] Usando il metodo grafico valutare la risposta nel dominio del tempo di un sistema lineare (non causale) avente risposta impulsiva $h(t) = e^{\alpha t}u(-t)$ al cui ingresso é posto il segnale $s(t) = u(t) - u(t-1)$.

3.[10 pt] Si consideri il sistema lineare (non causale) avente risposta impulsiva

$$h(t) = 2B \,\mathrm{sinc} Bt \, \cos 2\pi f_0 t, \tag{1}$$

e se ne valuti la risposta armonica. Se $f_0 = 30$ KHz e $B = 3$ KHz, dire se il sistema é distorcente (in fase e/o in ampiezza) per il segnale

$$s(t) = 5\sin(56000\pi t + 0.5) + \cos(59000\pi t). \tag{2}$$

1. (a) $s(t) = 2\pi\left(\frac{t}{2}-2\right) - \Lambda(t-4) = 2\pi\left(\frac{t-4}{2}\right) - \Lambda(t-4)$

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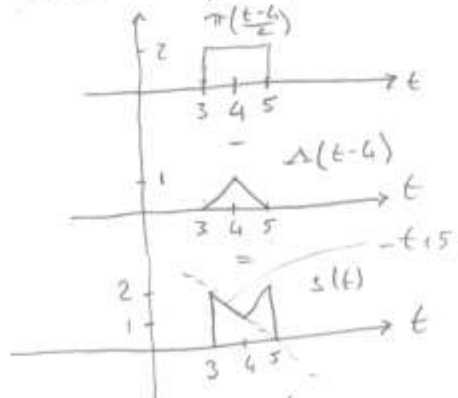

Segnale di energia, $P_s = 0$

$$\mathcal{E}_s = 2\int_3^4 (-t+5)^2 dt = 2\int_3^4 (t^2+25-10t)dt = 2\left(\frac{t^3}{3}\Big]_3^4 + 25t\Big]_3^4 - 10\frac{t^2}{2}\Big]_3^4\right)$$

$$= 2\,\frac{64-27}{3} + 2\cdot 25 - \cancel{20}^{10}\,\frac{16-9}{\cancel{2}}$$

$$= 2\,\frac{37}{3} + 50 - 70 = \frac{7\cancel{4}+150-210}{3} = \frac{14}{3}$$

oppure con la somma delle aree (vedi appunti on line)

$$\mathcal{E}_s = 2\left(\frac{H^2\cdot \text{Base}}{3} + (H+\text{shift})\cdot\text{base}\cdot\text{shift}\right)$$

$$= 2\left(\frac{1\cdot 1}{3} + (1+1)\,1\cdot 1\right) = 2\left(\frac{1}{3}+2\right) = \frac{14}{3}$$

$$S(f) = 2\,e^{-j2\pi f4}\,2\,\text{sinc}\,2f - e^{-j2\pi f4}\,\text{sinc}^2 f$$

$$= e^{-j8\pi f}\left(4\,\text{sinc}\,2f - \text{sinc}^2 f\right)$$

(b) $s(t) = e^{-|t+5|}$

Segnale di energia, $P_s = 0$

$$\mathcal{E}_s = \int_{-\infty}^{+\infty} s^2(t)\,dt = \int_{-\infty}^{-5} e^{(t+5)2}\,dt + \int_{-5}^{\infty} e^{-(t+5)2}\,dt = 2\int_{-5}^{\infty} e^{-2(t+5)}\,dt$$

$$= 2\,e^{-10}\int_{-5}^{\infty} e^{-2t}\,dt = 2e^{-10}\,\frac{e^{-2t}}{-2}\Big]_{-5}^{\infty} = 2\,e^{-10}\,\frac{e^{+10}}{2} = 1$$

$$S(f) = \int_{-\infty}^{+\infty} s(t)\, e^{-j2\pi f t}\, dt = \int_{-\infty}^{-5} e^{(t+5)} e^{-j2\pi f t}\, dt + \int_{-5}^{\infty} e^{-(t+5)} e^{-j2\pi f t}\, dt$$

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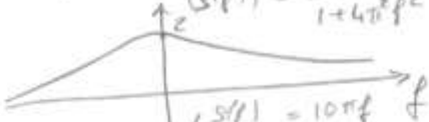
$$= e^{5} \int_{-\infty}^{-5} e^{(1-j2\pi f)t}\, dt + e^{-5} \int_{-5}^{\infty} e^{-(1+j2\pi f)t}\, dt$$

$$= e^{5} \left. \frac{e^{(1-j2\pi f)t}}{1-j2\pi f} \right]_{-\infty}^{-5} + e^{-5} \left. \frac{e^{-(1+j2\pi f)t}}{-(1+j2\pi f)} \right]_{-5}^{+\infty} =$$

$$= e^{5} \frac{e^{-(1-j2\pi f)5}}{1-j2\pi f} + e^{-5} \frac{e^{(1+j2\pi f)5}}{1+j2\pi f}$$

$$= \frac{e^{j10\pi f}}{1-j2\pi f} + \frac{e^{j10\pi f}}{1+j2\pi f} = e^{j10\pi f} \frac{1+j2\pi f + 1 - j2\pi f}{1+4\pi^2 f^2} = e^{j10\pi f} \frac{2}{1+4\pi^2 f^2}$$

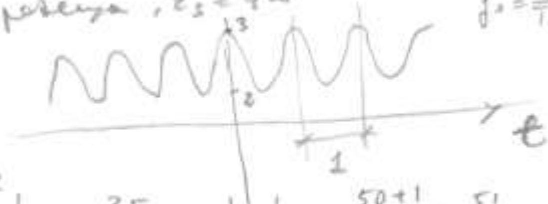
$|S(f)| = \frac{2}{1+4\pi^2 f^2}$



$\angle S(f) = 10\pi f$

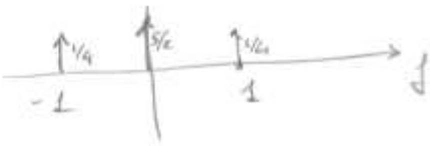
(c) $s(t) = 2 + \cos^2 \pi t = 2 + \frac{1}{2} + \frac{1}{2}\cos 2\pi t = \frac{5}{2} + \frac{1}{2}\cos 2\pi t$

Segnale di potenza, $E_s = +\infty$ $\qquad f_0 = \frac{1}{T_0} = 1$



$$P_s = \left(\frac{5}{2}\right)^2 + \left(\frac{1}{2}\right)^2 \frac{1}{2} = \frac{25}{4} + \frac{1}{4}\frac{1}{2} = \frac{50+1}{8} = \frac{51}{8}$$

$$S(f) = \frac{5}{2}\delta(f) + \frac{1}{4}\delta(f-1) + \frac{1}{4}\delta(f+1)$$



d) $s(t) = 1 - \sum_{k=-\infty}^{+\infty} \Delta(t-2k)$

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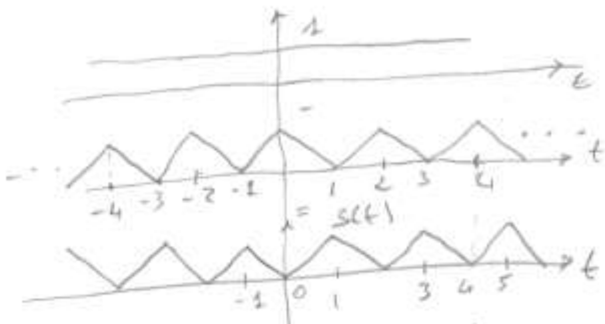


Segnale di Potenza, $\mathcal{E}_s = \infty$

Poiché periodico

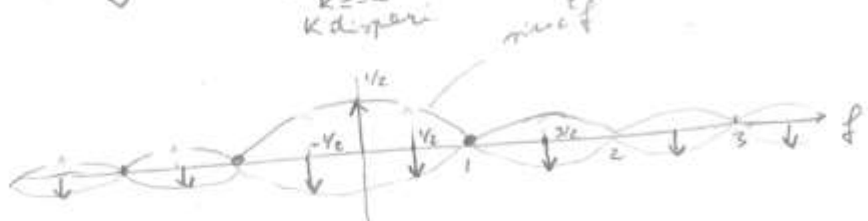
$$P_s = \frac{\mathcal{E}_{periodo}}{periodo} = \frac{\mathcal{E}(\Delta(t))}{2} = \frac{2 \cdot \frac{1}{3}}{2} = \frac{1}{3}$$

$$\sum_{k=-\infty}^{+\infty} \Delta(t-2k) = \Delta(t) * \sum_{k=-\infty}^{+\infty} \delta(t-2k)$$

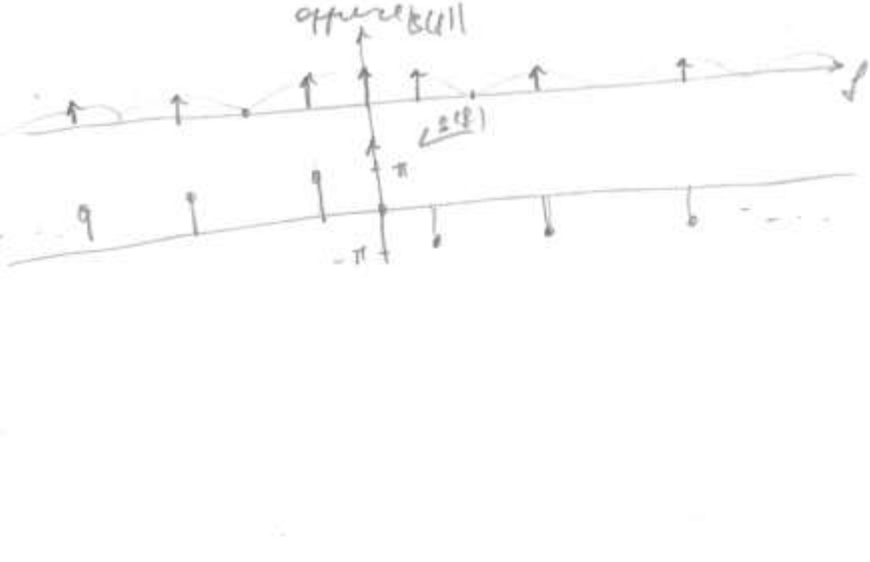
$$\mathcal{F}[\downarrow] = \mathrm{sinc}^2 f \, \frac{1}{2} \sum_{k=-\infty}^{+\infty} \delta\left(f - \frac{k}{2}\right) = \frac{1}{2} \sum_{k=-\infty}^{+\infty} \mathrm{sinc}^2 \frac{k}{2} \, \delta\left(f - \frac{k}{2}\right)$$

$$S(f) = \delta(f) - \frac{1}{2} \sum_{k=-\infty}^{+\infty} \mathrm{sinc}^2 \frac{k}{2} \, \delta\left(f - \frac{k}{2}\right)$$

$$= \frac{1}{2}\delta(f) - \frac{1}{2} \sum_{\substack{k \neq 0 \\ k=-\infty \\ k \text{ dispari}}} \mathrm{sinc}^2 \frac{k}{2} \, \delta\left(f - \frac{k}{2}\right)$$



oppure



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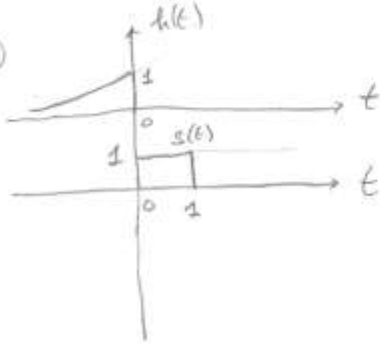

$$y(t)=\int_{-\infty}^{+\infty} h(\tau)\,s(t-\tau)\,d\tau$$

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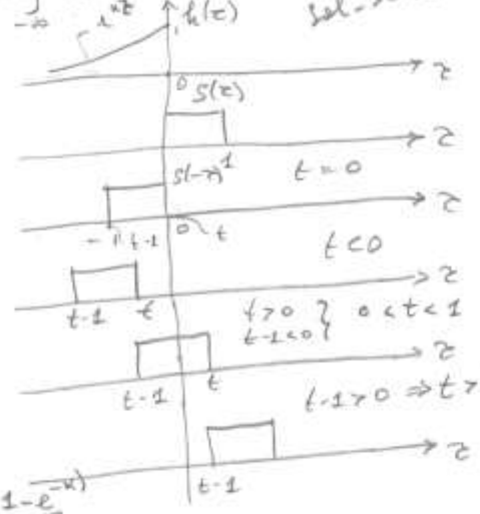
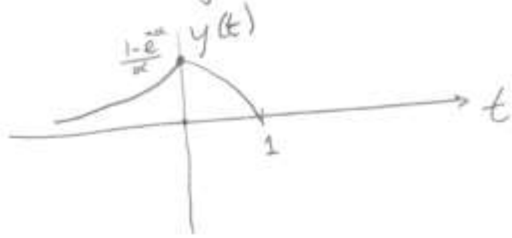


$\boxed{t<0}$ $$y(t)=\int_{t-1}^{t} e^{\alpha\tau}\,d\tau = \left.\frac{e^{\alpha\tau}}{\alpha}\right]_{t-1}^{t} = \frac{e^{\alpha t}-e^{\alpha(t-1)}}{\alpha} = \frac{e^{\alpha t}(1-e^{-\alpha})}{\alpha}$$

$\boxed{0<t<1}$ $$y(t)=\int_{t-1}^{0} e^{\alpha\tau}\,d\tau = \left.\frac{e^{\alpha\tau}}{\alpha}\right]_{t-1}^{0} = \frac{1-e^{\alpha(t-1)}}{\alpha}$$

$\boxed{t>1}$ $$y(t)=0$$



(3) $\xrightarrow{s(t)} \boxed{h(t)} \longrightarrow y(t)$

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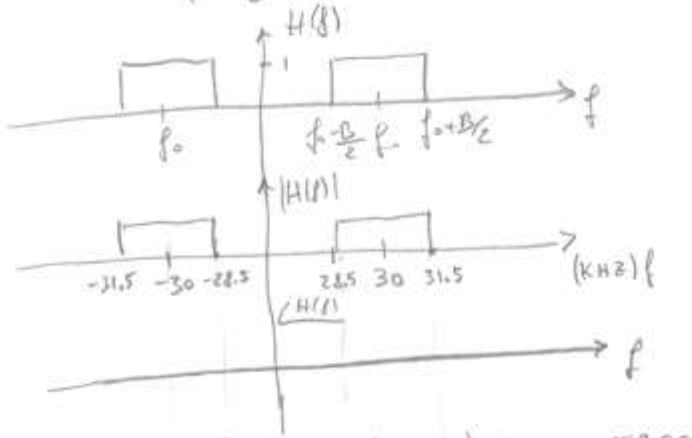
$h(t) = 2B \,\mathrm{sinc}\, Bt \cos 2\pi f_0 t$

$H(f) = 2 \quad \mathcal{F}[B \,\mathrm{sinc}\, Bt] \times \mathcal{F}[\cos 2\pi f_0 t]$

$= 2 \quad \Pi\left(\frac{f}{B}\right) * \left(\frac{1}{2}\delta(f-f_0) + \frac{1}{2}\delta(f+f_0)\right)$

$= \Pi\left(\frac{f-f_0}{B}\right) + \Pi\left(\frac{f+f_0}{B}\right)$



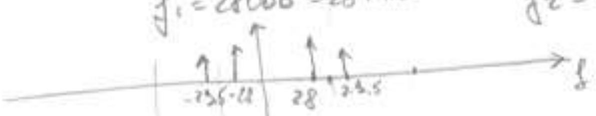
$B = 3\,\mathrm{KHz}$

$f_0 = 30\,\mathrm{KHz}$.

$s(t) = 5 \sin(56000\pi t + 0.5) + \cos 59000\pi t$

$= 5 \sin(2\pi f_1 t + 0.5) + \cos 2\pi f_2 t$

$f_1 = 28000 = 28\,\mathrm{KHz}$ $\qquad f_2 = 29500 = 29.5\,\mathrm{KHz}$



Sistema distorcente in ampiezza perché f_1 viene eliminato (fuori banda). Non distorcente in fase perché $\angle H(f) = 0 \ \forall f$.

L'uscita è:

$Y(f) = H(f) S(f) = \left(\Pi\left(\frac{f-f_0}{B}\right) + \Pi\left(\frac{f+f_0}{B}\right)\right)\left(\frac{5}{2}\delta(f-f_1)e^{j0.5} + \frac{5}{2}\delta(f+f_1)e^{-j0.5}\right.$

$\left. + \frac{1}{2}\delta(f-f_2) + \frac{1}{2}\delta(f+f_2)\right)$

$= \frac{1}{2}\delta(f-f_2) + \frac{1}{2}\delta(f+f_2)$

$y(t) = \cos 2\pi f_2 t$