UNIVERSITA' DEGLI STUDI DELLA CAMPANIA Luigi Vanvitelli
SCUOLA POLITECNICA E DELLE SCIENZE DI BASE
Dipartimento di Ingegneria
Corso di Laurea in Ingegneria Elettronica e Informatica

SOLUZIONI

Ia Prova Intracorso
Teoria dei Segnali
Prof. Francesco A. N. Palmieri
giovedì 18 ottobre 2018

1. Schizzare i seguenti segnali e valutarne l'energia, la potenza e la trasformata di Fourier:

(a)[4 pt] $s(t) = 2\Pi\left(\frac{1}{4}t - \frac{1}{2}\right) - \Lambda(t-2)$;

(b)[3 pt] $s(t) = e^{-|t|}\sin(2\pi f_0 t)$;

(c)[3 pt] $s(t) = 1 + \cos 2\pi t + \sin^2(4\pi t + 2)$.

(d)[3 pt] $s(t) = 2\ \mathrm{sinc}(2t+1)$

2.[10 pt] Usando il metodo grafico valutare la risposta nel dominio del tempo di un sistema lineare (non causale) avente risposta impulsiva $h(t) = e^{-\alpha|t|}$ al cui ingresso é posto il segnale $s(t) = u(t-1) - u(t-2)$.

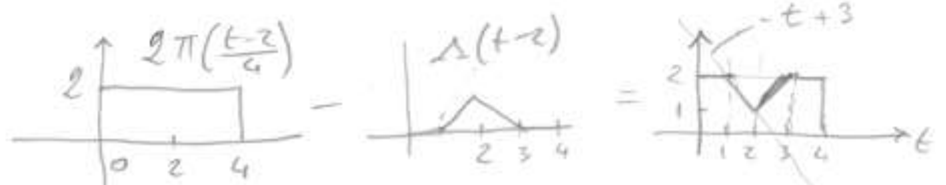
3.[10 pt] Si consideri il sistema lineare (non causale) avente risposta impulsiva

$$h(t) = a(t)\ \cos 2\pi f_0 t, \qquad a(t) = 2b\ \mathrm{sinc}^2 bt\ \cos \pi bt \tag{1}$$

e se ne valuti e disegni la risposta armonica (sia $f_0 >> b$). Siano $f_0 = 100$ KHz e $b = 4$ kHz, valutare se il sistema é distorcente (in fase e/o in ampiezza) per il segnale

$$s(t) = \sin(200000\pi t + 0.5) + 7\cos(201000\pi t). \tag{2}$$

(a) $s(t) = 2\Pi\left(\frac{1}{4}t - \frac{1}{2}\right) - \Lambda(t-2) = 2\Pi\left(\frac{t-2}{4}\right) - \Lambda(t-2)$



Segnale di energia, $P_s = 0$

$$\mathcal{E}_s = \int_{-\infty}^{+\infty} s^2(t)\,dt = 2\left[\int_0^1 (2)^2 dt + \int_1^2 (-t+3)^2 dt\right] = 2\left[4 + \int_1^2 (t^2+9-6t)\,dt\right]$$

$$= 2\left[4 + \frac{t^3}{3}\Big]_1^2 + 9t\Big]_1^2 - 6\,\frac{t^2}{2}\Big]_1^2\right] =$$

$$= 2\left[4 + \frac{8-1}{3} + 9 - 6\,\frac{4-1}{2}\right] = 2\left[4 + \frac{7}{3} + \cancel{9} - \cancel{9}\right] = 2\,\frac{12+7}{3} = \frac{38}{3}$$

$$S(f) = 8\,e^{-j2\pi f2}\,\mathrm{sinc}\,4f - e^{-j2\pi f2}\,\mathrm{sinc}^2 f$$

$$= e^{-j4\pi f}\left(8\,\mathrm{sinc}\,4f - \mathrm{sinc}^2 f\right)$$

(b) $s(t) = e^{-|t|}\sin 2\pi f_0 t$

$$S(f) = \mathcal{F}[e^{-|t|}] * \mathcal{F}[\sin 2\pi f_0 t]$$

$$\mathcal{F}[e^{-|t|}] = \int_{-\infty}^{0} e^{t} e^{-j2\pi f t}\,dt + \int_0^{\infty} e^{-t} e^{-j2\pi f t}\,dt =$$

$$= \int_{-\infty}^{0} e^{(1-j2\pi f)t}\,dt + \int_0^{\infty} e^{-(1+j2\pi f)t}\,dt = \frac{e^{(1-j2\pi f)t}}{1-j2\pi f}\Bigg]_{-\infty}^{0} + \frac{e^{-(1+j2\pi f)t}}{-(1+j2\pi f)}\Bigg]_0^{\infty}$$

$$= \frac{1}{1-j2\pi f} + \frac{1}{1+j2\pi f} = \frac{1+\cancel{j2\pi f}+1-\cancel{j2\pi f}}{1+4\pi^2 f^2} = \frac{2}{1+4\pi^2 f^2}$$

Poiché $\mathcal{F}[\sin 2\pi f_0 t] = \frac{1}{2j}\delta(f-f_0) - \frac{1}{2j}\delta(f+f_0)$

$$S(f) = \frac{1}{2j}\,\frac{2}{1+4\pi^2(f-f_0)^2} + \frac{1}{\cancel{2}j}\,\frac{\cancel{2}}{1+4\pi^2(f+f_0)^2}$$

$$\mathcal{E}_s = \int_{-\infty}^{+\infty} e^{-2|t|} \sin^2 2\pi f_0 t \, dt = 2\int_0^{\infty} e^{-2t} \sin^2 2\pi f_0 t \, dt = \qquad (2)$$

$$= \underbrace{\cancel{2}\int_0^{\infty} e^{-2t} \frac{1}{\cancel{2}} \, dt}_{\mathrm{I}} - \cancel{2}\underbrace{\int_0^{\infty} e^{-2t} \frac{1}{\cancel{2}} \cos 4\pi f_0 t \, dt}_{\mathrm{II}}$$

$$\mathrm{I} \quad \int_0^{\infty} e^{-2t} \, dt = \left.\frac{e^{-2t}}{-2}\right]_0^{\infty} = \frac{1}{2}$$

$$\mathrm{II} \quad \underbrace{\int_0^{\infty} e^{-2t} \cos 4\pi f_0 t \, dt}_{\text{int}} = \left.\frac{e^{-2t}}{-2} \cos 4\pi f_0 t\right]_0^{\infty} - \int_0^{\infty} \frac{e^{-2t}}{-2}(-4\pi f_0) \sin 4\pi f_0 t \, dt \quad (\text{per parti})$$

$$= +\frac{1}{2} - 2\pi f_0 \int_0^{\infty} e^{-2t} \sin 4\pi f_0 t \, dt$$

$$= \frac{1}{2} - 2\pi f_0 \left[\left.\frac{e^{-2t}}{-2} \sin 4\pi f_0 t\right]_0^{\infty} - \int_0^{\infty} \frac{e^{-2t}}{-2} (4\pi f_0) \cos 4\pi f_0 t \, dt \right]$$

$$= \frac{1}{2} - 2\pi f_0 \frac{1}{2} 4\pi f_0 \underbrace{\int_0^{\infty} e^{-2t} \cos 4\pi f_0 t \, dt}_{\text{int}}$$

$$\text{int} = \frac{1}{2} - 4\pi^2 f_0^2 \, \text{int}$$

$$\text{int} = \frac{1/2}{1 + 4\pi^2 f_0^2}$$

$$\mathcal{E}_s = \frac{1}{2} - \frac{1}{2} \frac{1}{1 + 4\pi^2 f_0^2} = \frac{1}{2} \frac{\cancel{1} + 4\pi^2 f_0^2 - \cancel{1}}{1 + 4\pi^2 f_0^2} = \frac{2\pi^2 f_0^2}{1 + 4\pi^2 f_0^2}$$

DALLE TAVOLE:

$$\boxed{\int e^{ax}\cos x\,dx = e^{ax}\frac{(a\cos x+\sin x)}{a^2+1}+c}$$

③

oppure

(II) $$\int_0^{\infty} e^{-2t}\cos 4\pi f_0 t\,dt = \int_0^{\infty} e^{-\frac{2x}{4\pi f_0}}\cos x\,\frac{dx}{4\pi f_0}$$

$$x = 4\pi f_0 t$$

$$t = \frac{x}{4\pi f_0}$$

$$dt = \frac{dx}{4\pi f_0} \qquad a = -\frac{1}{2\pi f_0}$$

$$= \frac{1}{4\pi f_0}\int_0^{\infty} e^{ax}\cos x\,dx =$$

$$= \frac{1}{4\pi f_0}\left. e^{-\frac{1}{2\pi f_0}x}\frac{\left(-\frac{1}{2\pi f_0}\cos x+\sin x\right)}{\frac{1}{4\pi^2 f_0^2}+1}\right]_0^{\infty}$$

$$= -\frac{1}{4\pi f_0}\left(\frac{+\frac{1}{2\pi f_0}}{4\pi^2 f_0^2+1}\right) = \frac{1/2}{4\pi^2 f_0^2+1}$$

oppure

(II) $$\int_0^{\infty} e^{-2t}\cos 4\pi f_0 t\,dt = \int_0^{\infty} e^{-2t}\frac{e^{j4\pi f_0 t}+e^{-j4\pi f_0 t}}{2}\,dt$$

$$= \frac{1}{2}\int_0^{\infty} e^{-(2-j4\pi f_0)t}\,dt + \frac{1}{2}\int_0^{\infty} e^{-(2+j4\pi f_0)t}\,dt$$

$$= \frac{1}{2}\left.\frac{e^{-(2-j4\pi f_0)t}}{-(2-j4\pi f_0)}\right]_0^{\infty} + \frac{1}{2}\left.\frac{e^{-(2+j4\pi f_0)t}}{-(2+j4\pi f_0)}\right]_0^{\infty}$$

$$= \frac{1}{2}\left(\frac{1}{2-j4\pi f_0}+\frac{1}{2+j4\pi f_0}\right) = \frac{1}{2}\,\frac{2+\cancel{j4\pi f_0}+2-\cancel{j4\pi f_0}}{4+16\pi^2 f_0^2}$$

$$= \frac{4}{2(4+16\pi^2 f_0^2)} = \frac{1/2}{1+4\pi^2 f_0^2}$$

c) $s(t) = 1 + \cos 2\pi t + \sin^2(4\pi t + 2) = 1 + \cos 2\pi t + \frac{1}{2} - \frac{1}{2}\sin(8\pi t + 4)$

$= \frac{3}{2} + \cos 2\pi t - \frac{1}{2}\sin(8\pi t + 4)$ 4

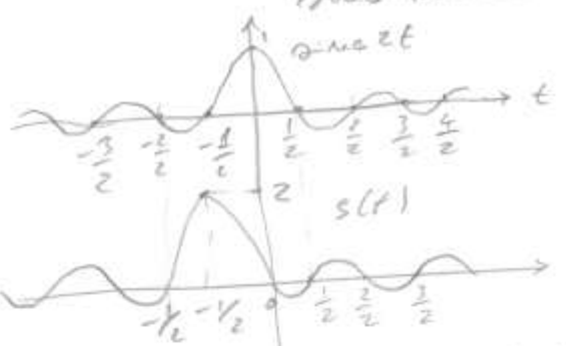
Segnale di potenza, $\mathcal{E}_s = \infty$

$P_s = \frac{9}{4} + \frac{1}{2} + \frac{1}{8} = \frac{18+4+1}{8} = \frac{23}{8}$

$S(f) = \frac{3}{2}\delta(f) + \frac{1}{2}\delta(f-1) + \frac{1}{2}\delta(f+1) - \frac{1}{4j}e^{j4}\delta(f-4) + \frac{1}{4j}e^{-j4}\delta(f+4)$

(d) $s(t) = 2\,\mathrm{sinc}(2t+1) = 2\,\mathrm{sinc}(2(t+\frac{1}{2}))$

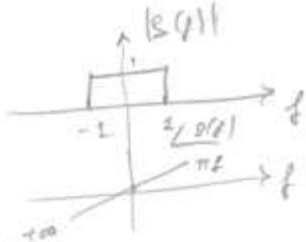
sinc traslato a sinistra di 1/2



poiché $\Pi\left(\frac{t}{a}\right) \overset{\mathcal{F}}{\leftrightarrow} a\,\mathrm{sinc}\,af$

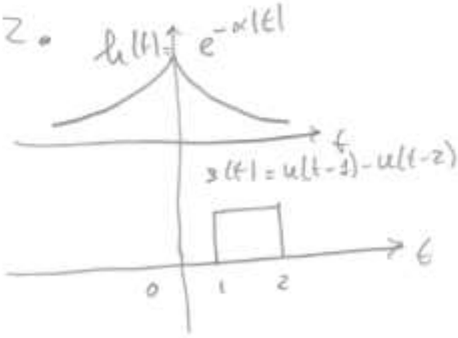
per dualità $\Pi\left(\frac{f}{a}\right) \overset{\mathcal{F}}{\leftrightarrow} a\,\mathrm{sinc}\,at$

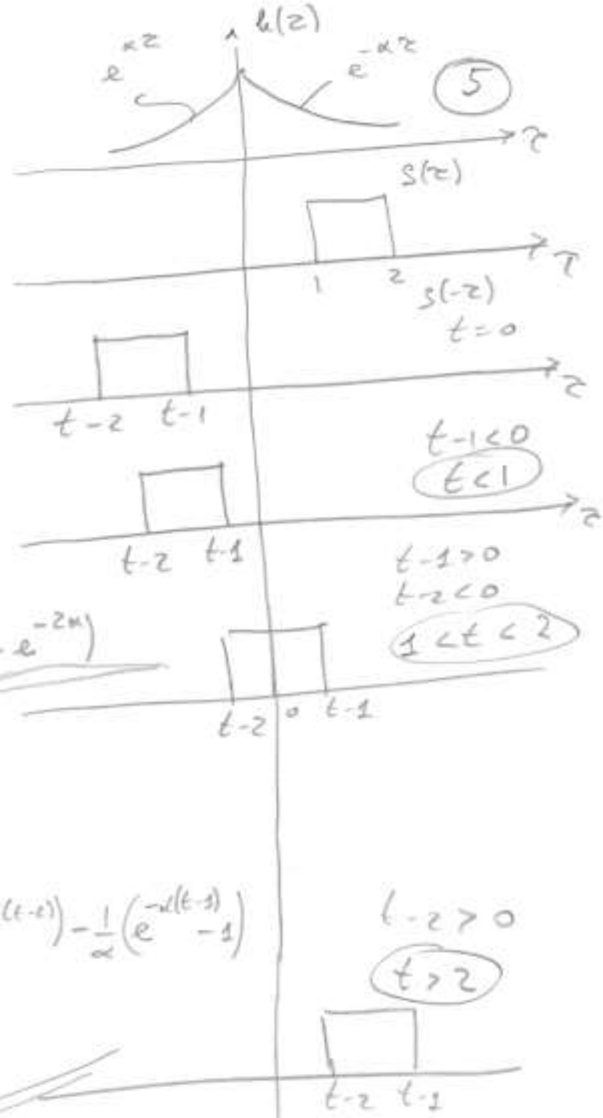
$S(f) = e^{j2\pi f\frac{1}{2}}\,\Pi\left(\frac{f}{2}\right)$



$\mathcal{E}_s = \int_{-\infty}^{+\infty} s^2(t)\,dt = \int_{-\infty}^{+\infty} |S(f)|^2\,df = \int_{-\infty}^{+\infty} \Pi\left(\frac{f}{2}\right) df = 2$

2. $h(t) = e^{-\alpha|t|}$





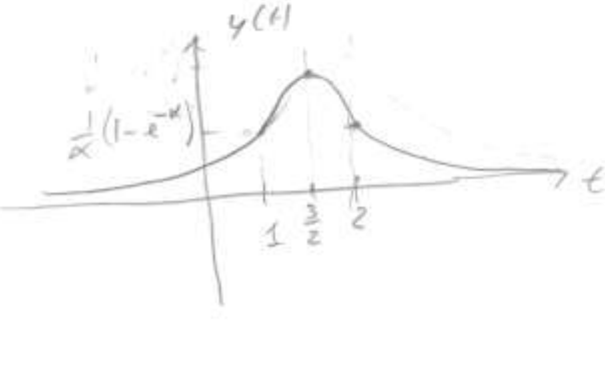
$$y(t) = \int_{t-2}^{t-1} e^{\alpha\tau} d\tau = \left.\frac{e^{\alpha\tau}}{\alpha}\right|_{t-2}^{t-1} \Leftarrow$$

$$= \frac{1}{\alpha}\left(e^{\alpha(t-1)} - e^{\alpha(t-2)}\right) = \frac{1}{\alpha} e^{\alpha t}\left(e^{-\alpha} - e^{-2\alpha}\right)$$

$$y(t) = \int_{t-2}^{0} e^{\alpha\tau} d\tau + \int_{0}^{t-1} e^{-\alpha\tau} d\tau =$$

$$= \left.\frac{e^{\alpha\tau}}{\alpha}\right|_{t-2}^{0} + \left.\frac{e^{-\alpha\tau}}{-\alpha}\right|_{0}^{t-1} = \frac{1}{\alpha}\left(1 - e^{\alpha(t-2)}\right) - \frac{1}{\alpha}\left(e^{-\alpha(t-1)} - 1\right)$$

$$y(t) = \int_{t-2}^{t-1} e^{-\alpha\tau} d\tau = \left.\frac{e^{-\alpha\tau}}{-\alpha}\right|_{t-2}^{t-1} = \frac{e^{-\alpha(t-2)} - e^{-\alpha(t-1)}}{\alpha} = \frac{1}{\alpha} e^{-\alpha t}\left(e^{2\alpha} - e^{\alpha}\right)$$



3.

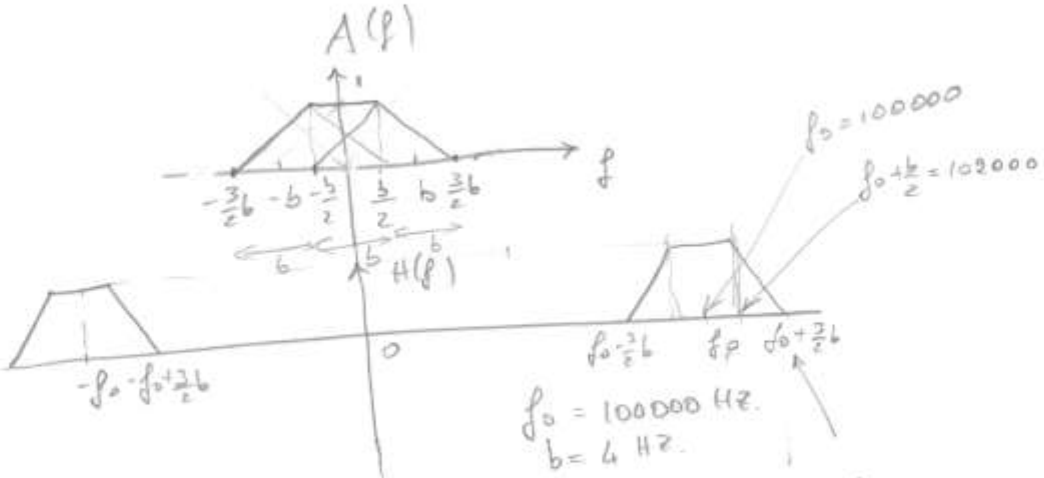
$$h(t) = a(t)\cos 2\pi f_0 t \qquad a(t) = 2b\,\mathrm{sinc}^2 bt \cos \pi b t \qquad 6$$

$$H(f) = A(f) * \mathcal{F}[\cos 2\pi f_0 t] = \frac{1}{2}A(f-f_0) + \frac{1}{2}A(f+f_0)$$

$$A(f) = \mathcal{F}[2b\,\mathrm{sinc}^2 bt] * \mathcal{F}[\cos \pi b t]$$

$$= 2\,\Lambda\left(\frac{f}{b}\right) * \left(\frac{1}{2}\delta\left(f-\frac{b}{2}\right) + \frac{1}{2}\delta\left(f+\frac{b}{2}\right)\right)$$

$$= \Lambda\left(\frac{f-b/2}{b}\right) + \Lambda\left(\frac{f+b/2}{b}\right)$$



$$s(t) = \sin(200000\pi t + 0.5) + 7\cos(201000\pi t)$$

$f_1 = 100000$ $\quad f_2 = 100500$

Entrambi le frequenze cadono all'interno della parte piatta della risposta armonica. Pertanto rimangono non distorte in ampiezza. La fase è nulla quindi non c'è neanche distorsione di fase.