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SOLUZIONI

UNIVERSITA' DEGLI ~~STUDI DELLA CAMPANIA Luigi Vanvitelli~~
SCUOLA POLITECNICA E DELLE SCIENZE DI BASE
Dipartimento di Ingegneria
Corso di Laurea in Ingegneria Elettronia e Informatica

Ia Prova Intracorso
Teoria dei Segnali
Prof. Francesco A. N. Palmieri
giovedi 24 ottobre 2019

1. Schizzare i seguenti segnali e valutarne l'energia, la potenza e la trasformata di Fourier:

(a)[3 pt] $s(t) = \Lambda\left(\frac{t}{2}\right) + \Lambda\left(\frac{t}{2}+1\right)$;

(b)[3 pt] $s(t) = 3e^{-3t}\cos(2\pi f_0 t)u(t)$;

(c)[3 pt] $s(t) = 1 - \cos 2\pi t + \cos^2 5\pi t$.

(d)[4 pt] $s(t) = 1 - 2\sum_{k=-\infty}^{\infty} \Lambda\left(\frac{t-10k}{5}\right)$

2.[10 pt] Usando il metodo grafico valutare la risposta nel dominio del tempo di un sistema lineare (anticausale) avente risposta impulsiva $h(t) = e^{\alpha t}u(-t)$ al cui ingresso é posto il segnale $s(t) = u(t+1) - u(t)$.

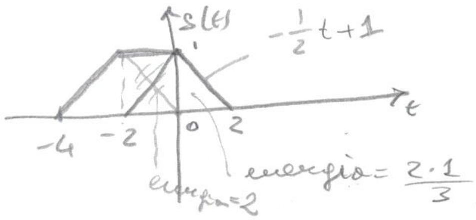
3.[10 pt] Si consideri il sistema lineare (non causale) avente risposta impulsiva

$$h(t) = 2B\ \mathrm{sinc}^2 Bt\ \cos \pi Bt \tag{1}$$

con $B = 15$ KHze se ne valuti e disegni la risposta armonica. Valutare la risposta del sistema all'ingresso

$$s(t) = \sin(10000\pi t + 0.7) + 3\cos(22500\pi t) + \sin(40000\pi t). \tag{2}$$

1 (a) $s(t) = \Lambda\left(\frac{t}{2}\right) + \Lambda\left(\frac{t}{2}+1\right) = \Lambda\left(\frac{t}{2}\right) + \Lambda\left(\frac{t+2}{2}\right)$



$P_s = 0$

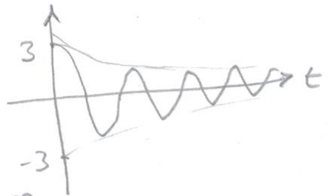
$$\mathcal{E}_s = \frac{2\cdot 1}{3}\cdot 2 + 2 = \frac{4}{3} + 2 = \frac{4+6}{3} = \frac{10}{3}$$

$$S(f) = 2\,\mathrm{sinc}^2 2f + e^{j2\pi f 2}\, 2\,\mathrm{sinc}^2 2f$$

$$= 2\,\mathrm{sinc}^2 2f\left(1 + e^{j4\pi f}\right)$$

(b) $s(t) = 3e^{-3t}\cos 2\pi f_0 t \, u(t)$

$P_s = 0$



$$\mathcal{E}_s = \int_0^\infty 9 e^{-6t}\cos^2 2\pi f_0 t \, dt = 9\int_0^\infty e^{-6t}\left(\frac{1}{2}+\frac{1}{2}\cos 4\pi f_0 t\right) dt$$

$$= \frac{9}{2}\int_0^\infty e^{-6t} dt + \frac{9}{2}\int_0^\infty e^{-6t}\cos 4\pi f_0 t \, dt$$

$$= \frac{9}{2} \left.\frac{e^{-6t}}{-6}\right]_0^\infty + \frac{9}{2}\int_0^\infty e^{-6t}\left(\frac{e^{j4\pi f_0 t} + e^{-j4\pi f_0 t}}{2}\right) dt$$

$$= \frac{\cancel{9}^{3}}{2}\,\frac{1}{\cancel{6}2} + \frac{9}{4}\int_0^\infty e^{(-6+j4\pi f_0)t} dt + \frac{9}{4}\int_0^\infty e^{(-6-j4\pi f_0)t} dt$$

$$= \frac{3}{4} + \frac{9}{4} \left.\frac{e^{(-6+j4\pi f_0)t}}{-6+j4\pi f_0}\right]_0^\infty + \frac{9}{4} \left.\frac{e^{(-6-j4\pi f_0)t}}{-6-j4\pi f_0}\right]_0^\infty$$

$$= \frac{3}{4} + \frac{9}{4}\left(\frac{1}{6-j4\pi f_0} + \frac{1}{6+j4\pi f_0}\right) = \frac{3}{4} + \frac{9}{4}\,\frac{6+\cancel{j4\pi f_0}+6-\cancel{j4\pi f_0}}{36+16\pi^2 f_0^2}$$

$$= \frac{3}{4} + \frac{9}{\cancel{4}}\,\frac{\cancel{12}3}{36+16\pi^2 f_0^2} = \frac{3}{4} + \frac{27}{36+16\pi^2 f_0^2}$$

$$S(f) = 3\,\mathcal{F}\left[e^{-3t}u(t)\right] * \mathcal{F}\left[\cos 2\pi f_0 t\right] \rightarrow \frac{1}{2}\delta(f-f_0) + \frac{1}{2}\delta(f+f_0)$$

$$\int_0^{\infty} e^{-3t} e^{-j2\pi f t}\,dt = \left.\frac{e^{-(3+j2\pi f)t}}{-(3+j2\pi f)}\right]_0^{\infty} = \frac{1}{3+j2\pi f}$$

$$S(f) = \frac{3}{2}\left(\frac{1}{3+j2\pi(f-f_0)} + \frac{1}{3+j2\pi(f+f_0)}\right)$$

(c)

$$s(t) = 1 - \cos 2\pi t + \cos^2 5\pi t$$

$$= 1 - \cos 2\pi t + \frac{1}{2} + \frac{1}{2}\cos 10\pi t$$

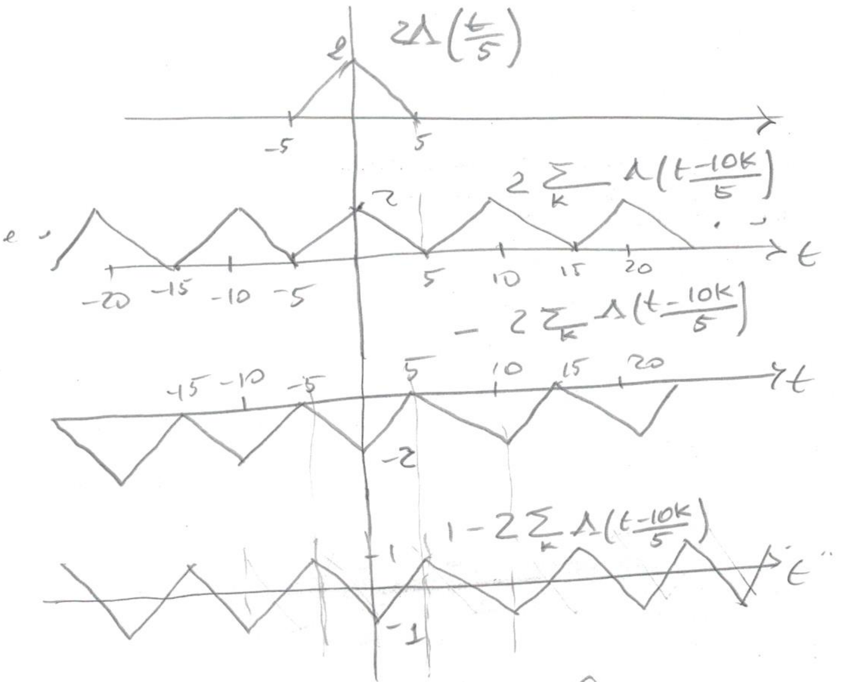
$$= \frac{3}{2} - \cos 2\pi t + \frac{1}{2}\cos 10\pi t$$

$\mathcal{E}_s = \infty$
segnale di potenza

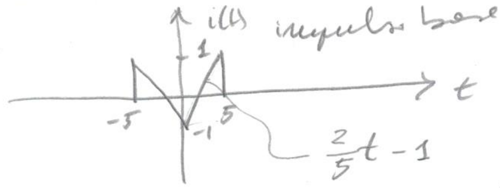
$$P_s = \frac{9}{4} + \frac{1}{2} + \frac{1}{4}\frac{1}{2} = \frac{18+4+1}{8} = \frac{23}{8}$$

$$S(f) = \frac{3}{2}\delta(f) - \frac{1}{2}\delta(f-1) - \frac{1}{2}\delta(f+1) + \frac{1}{4}\delta(f-5) + \frac{1}{4}\delta(f+5)$$

(d) $s(t) = 1 - 2\sum_{k=-\infty}^{+\infty} \Lambda\left(\frac{t-10k}{5}\right)$



Segnale di Potenza, $\mathcal{E}_s = \infty$



$i(t) = -2\Lambda\left(\frac{t}{5}\right) + \Pi\left(\frac{t}{10}\right)$

$$P_s = \frac{\mathcal{E}_i}{T} \ ; \ \mathcal{E}_i = 2\int_0^5 \left(\frac{2}{5}t - 1\right)^2 dt = 2\left[\int_0^5 \frac{4}{25}t^2\,dt + \int_0^5 1\,dt - \int_0^5 \frac{4}{5}t\,dt\right]$$

$$= 2\left[\frac{4}{25}\frac{t^3}{3}\Big]_0^5 + 5 + \frac{4}{5}\frac{t^2}{2}\Big]_0^5\right]$$

$$= 2\left[\frac{4}{\cancel{25}}\,\frac{\overset{5}{\cancel{125}}}{3} + 5 - \frac{\overset{2}{\cancel{4}}}{\cancel{5}}\,\frac{\overset{5}{\cancel{25}}}{\cancel{2}}\right]$$

$$= 2\left[\frac{20}{3} + 5 - 10\right] = \cancel{2}\,\frac{20+15-30}{\cancel{6}\,3} = \frac{10}{3}$$

$$P_s = \frac{\cancel{10}}{3}\,\frac{1}{\cancel{10}} = \frac{1}{3}$$

$$s(t) = i(t) * \sum_{k=-\infty}^{+\infty} \delta(t-10k)$$

$$S(f) = I(f) \cdot \frac{1}{5} \sum_{k=-\infty}^{+\infty} \delta\left(f - \frac{k}{10}\right)$$

$$I(f) = -2.5\,\mathrm{sinc}^2 5f + 10\,\mathrm{sinc}\,10f$$

$$S(f) = \sum_{k=-\infty}^{+\infty} \underbrace{\left(-2\,\mathrm{sinc}^2 \frac{\cancel{5}k}{\cancel{10}\,2} + 2\,\mathrm{sinc}\,\cancel{10}\frac{k}{\cancel{10}}\right)} \delta\left(f - \frac{k}{10}\right)$$

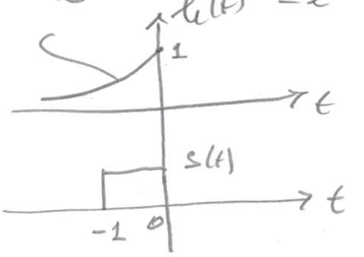
Nota che: per $k=0$ $\;= 0$ (Nessuna componente continua)

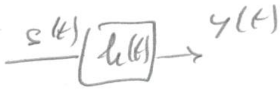
$\mathrm{sinc}\,k = 0 \;\; \forall k \neq 0$

$\mathrm{sinc}^2 \frac{k}{2} = 0 \;\; \forall k$ pari.

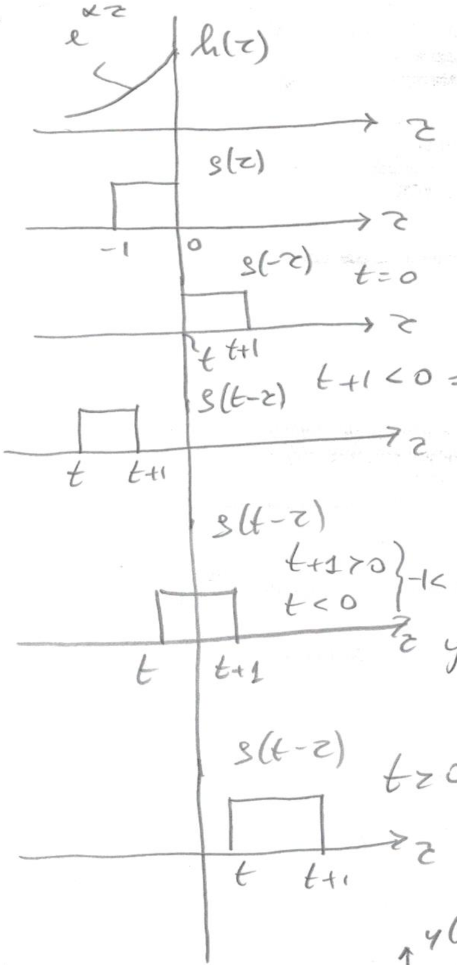
$$= 2 \sum_{\substack{k=-\infty \\ k \text{ dispari}}}^{+\infty} \mathrm{sinc}^2 \frac{k}{2}\, \delta\left(f - \frac{k}{10}\right)$$

② $e^{\alpha t}$ $h(t) = e^{\alpha t} u(-t)$ $s(t) = u(t+1) - u(t)$





$$y(t) = \int_{-\infty}^{+\infty} h(\tau)\, s(t-\tau)\, d\tau$$



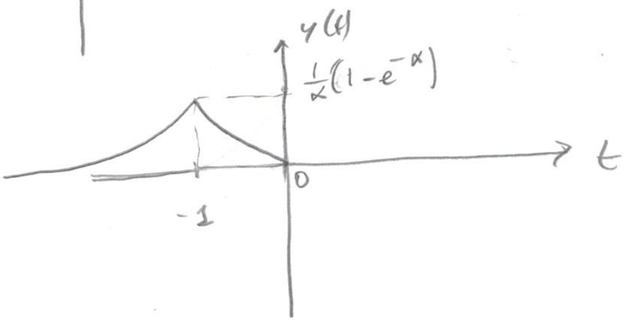
$t+1<0 \Rightarrow t<-1$ $y(t) = \int_t^{t+1} e^{\alpha\tau}\, d\tau = \left.\frac{e^{\alpha\tau}}{\alpha}\right]_t^{t+1}$

$= \frac{1}{\alpha}\left(e^{\alpha(t+1)} - e^{\alpha t}\right) = \frac{1}{\alpha} e^{\alpha t}\left(e^{\alpha} - 1\right)$

$\left.\begin{array}{l} t+1>0 \\ t<0 \end{array}\right\} -1<t<0$

$y(t) = \int_t^0 e^{\alpha\tau}\, d\tau = \left.\frac{e^{\alpha\tau}}{\alpha}\right]_t^0 = \frac{1-e^{\alpha t}}{\alpha}$

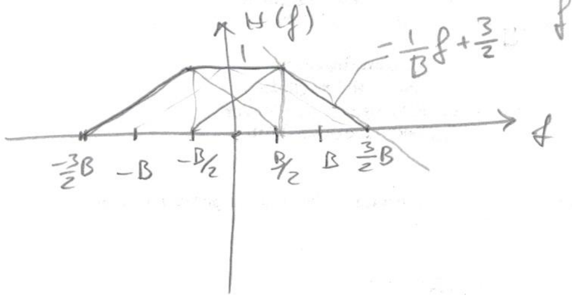
$t>0$ $y(t) = 0$



(3) $h(t) = 2B\,\text{sinc}^2 Bt\; \cos \pi B t$

$$H(f) = \cancel{2}\, \Lambda\left(\frac{f}{B}\right) * \left(\frac{1}{\cancel{2}}\delta\left(f - \frac{B}{2}\right) + \frac{1}{\cancel{2}}\delta\left(f + \frac{B}{2}\right)\right)$$

$$= \Lambda\left(\frac{f - B/2}{B}\right) + \Lambda\left(\frac{f + B/2}{B}\right)$$



$f \in \left[\frac{B}{2}, \frac{3}{2}B\right] = [7500, 22500]$

45000

$B/2 = 7500$

$B = 15000$

$\frac{3}{2}B = 22500$

$s(t) = \sin(\underbrace{10000}_{f_1 = 5000}\pi t + 0{,}7) + 3\cos \underbrace{22500}_{f_2 = 11250\,Hz}\pi t + \sin \underbrace{40000}_{f_3 = 20000}\pi t$

Solo distorsione di ampiezza poiché la fase è nulla

$y(t) = H(f_1)\sin(2\pi f_1 t + 0{,}7) + 3H(f_2)\cos 2\pi f_2 t + H(f_3)\sin 2\pi f_3 t$

$H(f_1) = 1$

$H(f_2) = -\frac{1}{B}f_2 + \frac{3}{2}$

$H(f_3) = -\frac{1}{B}f_3 + \frac{3}{2}$