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Ia Prova Intracorso AA 2022-23
**Teoria dei Segnali**
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lunedí 24 ottobre 2022

SOLUZIONI

**1.** Schizzare i seguenti segnali e valutarne l'energia, la potenza e la trasformata di Fourier:

(a)[10 pt] $s(t) = \Pi\left(\frac{t}{2}\right) + 3\Pi\left(\frac{2t-3}{2}\right) + 2\Lambda\left(t-3\right)$;

(b)[10 pt] $s(t) = e^{2t}u(-t+1)$;

(c)[10 pt] $s(t) = 2 - \cos(\frac{\pi}{2}t + \frac{1}{10}) + \cos^2 3\pi t$.

(d)[10 pt] $s(t) = \sum_{k=-\infty}^{\infty} \Lambda\left(\frac{t-1-5k}{2}\right)$

**2.**[30 pt] Usando il metodo grafico valutare la risposta nel dominio del tempo di un sistema lineare (anticausale) avente risposta impulsiva $h(t) = e^{\alpha t}u(-t)$ al cui ingresso é posto il segnale $s(t) = u(t-3) - u(t-8)$.

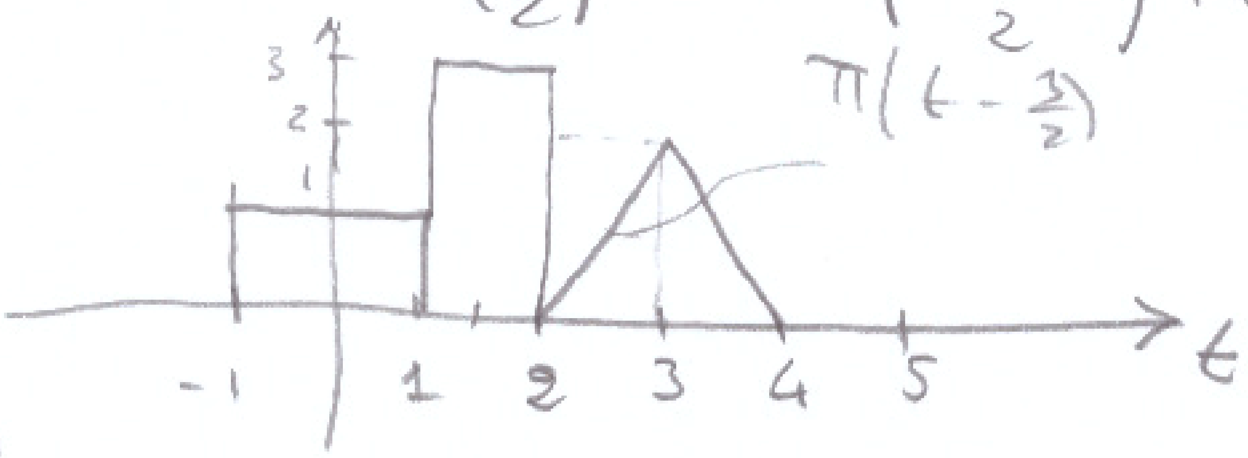
**3.**[30 pt] Un sistema lineare ha risposta impulsiva

$$h(t) = 2e^{-\alpha(t-1)}u(t-1). \tag{1}$$

Valutare la risposta armonica del sistema e l'uscita corrispondente all'ingresso

$$x(t) = \frac{1}{2} - \cos \pi t + 5\cos^2(5t-4). \tag{2}$$

1.(a) $s(t) = \Pi\left(\frac{t}{2}\right) + 3\,\Pi\left(\frac{2t-3}{2}\right) + 2\Lambda(t-3)$




Segnale di energia, $P_s = 0$

$$\mathcal{E}_s = \int_{-1}^{1} 1\,dt + \int_{1}^{2} 9\,dt + 2\int_{2}^{3} (2t-4)^2\,dt = 2 + 9 + 2\left[\int_{2}^{3} 4t^2\,dt + \int_{2}^{3} 16\,dt - 16\int_{2}^{3} t\,dt\right]$$

$$= 11 + 2\left[\left[4\frac{t^3}{3}\right]_2^3 + 16 - \left[\cancel{16}^{\,8}\,\frac{t^2}{\cancel{2}}\right]_2^3\right] =$$

$$= 11 + 2\left(4\,\frac{27-8}{3} + 16 - 8(9-4)\right) =$$

$$= 11 + 2\left(4\cdot\frac{19}{3} + 16 - 40\right) = 11 + 2\,\frac{4\cdot 19 + 48 - 120}{3}$$

$$= 11 + 2\,\frac{76+48-120}{3} = 11 + 2\cdot\frac{4}{3} =$$

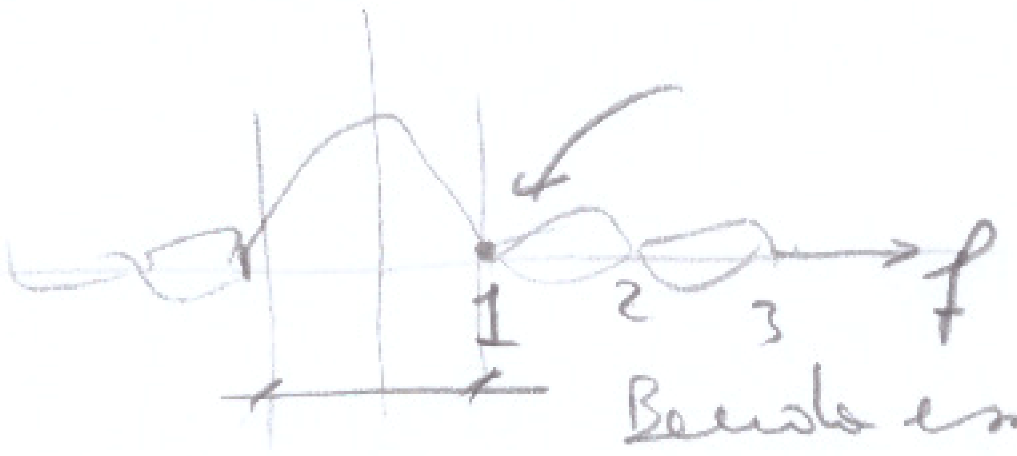
$$= \frac{33+8}{3} = \frac{41}{3}$$

$16 \times 3 = 48$

$19 \times 4 = 76$

$76 + 48 = 124$

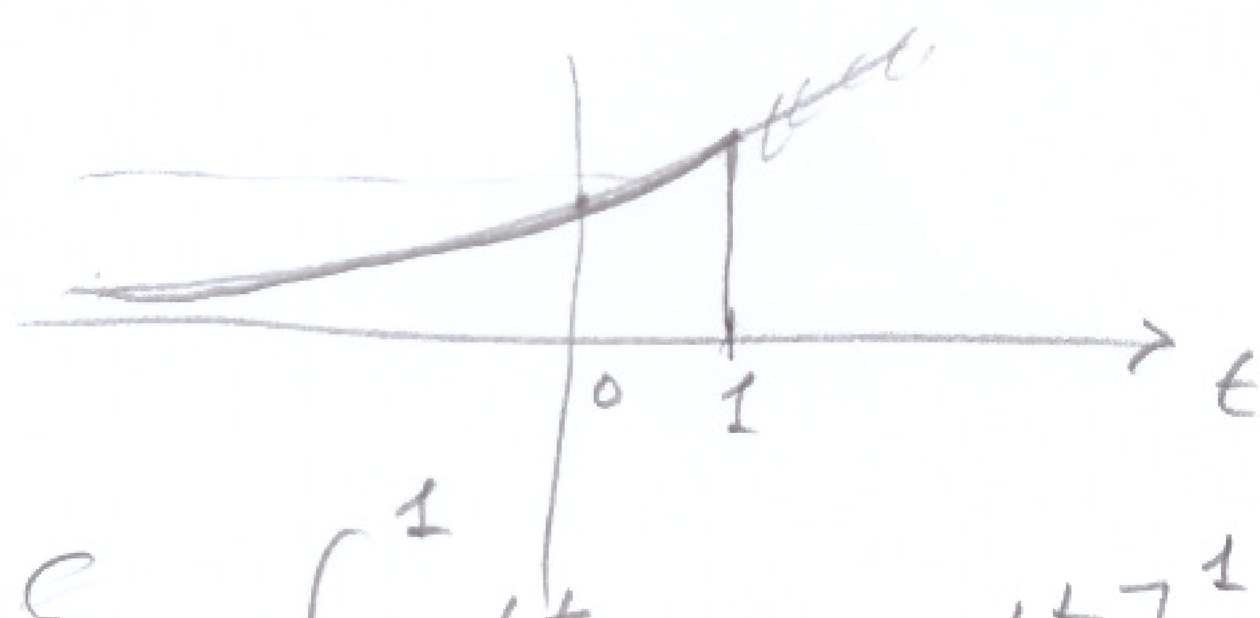
$$S(f) = 2\,\mathrm{sinc}\,2f + 3\,e^{-j2\pi f\frac{3}{2}}\,\mathrm{sinc}\,f + 2\,e^{-j2\pi f 3}\,\mathrm{sinc}^2 f$$




Banda energetica = 1 Hz

1.(b) $s(t) = e^{2t}\, u(-t+1)$

$u(-t+1) \Rightarrow \begin{cases} 1 & -t+1>0 \Rightarrow t<1 \\ 0 & \text{altrove} \end{cases}$




segnale di energia

$\mathcal{E}_s = \int_{-\infty}^{1} e^{4t}\, dt = \left. \frac{e^{4t}}{4} \right]_{-\infty}^{1} = \frac{e^4}{4}$ $\qquad P_s = 0$

$S(f) = \int_{-\infty}^{1} e^{2t} e^{-j2\pi f t}\, dt = \int_{-\infty}^{1} e^{t(2-j2\pi f)}\, dt = \left. \frac{e^{t(2-j2\pi f)}}{2-j2\pi f} \right]_{-\infty}^{1}$

$= \frac{e^{2-j2\pi f}}{2-j2\pi f}$

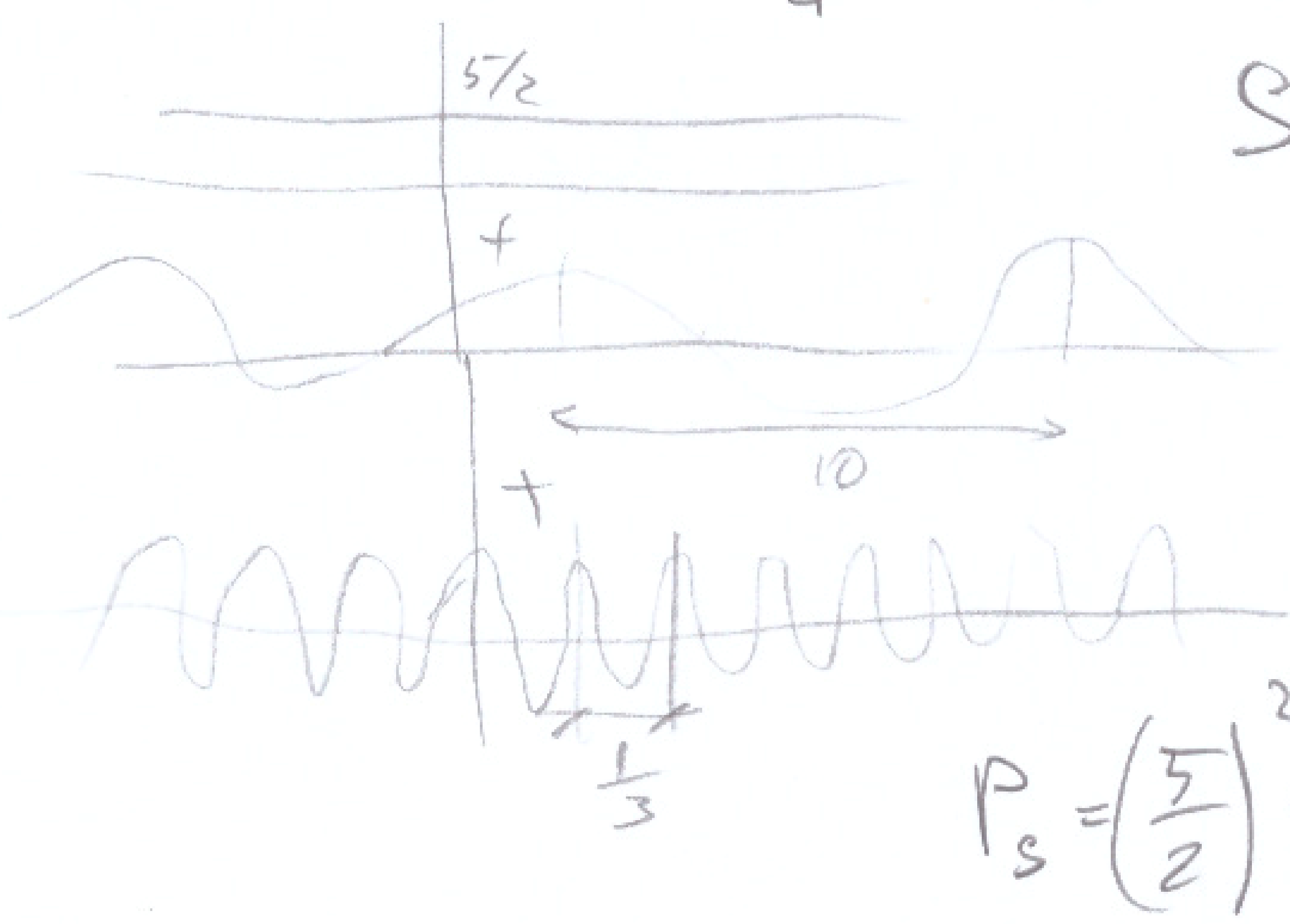
1.(e) $s(t) = 2 - \cos\left(\frac{\pi}{2}t + \frac{1}{10}\right) + \cos^2 3\pi t$

$= 2 - \cos\left(\frac{\pi}{2}t + \frac{1}{10}\right) + \frac{1}{2} + \frac{1}{2}\cos 6\pi t$

$= \frac{5}{2} - \cos\left(\frac{\pi}{2}t + \frac{1}{10}\right) + \frac{1}{2}\cos 6\pi t$

$\frac{\pi}{2} = 2\pi f_1 \qquad \cancel{3}\, 6\pi = 2\pi f_2$

$f_1 = \frac{1}{4} \qquad f_2 = 3$



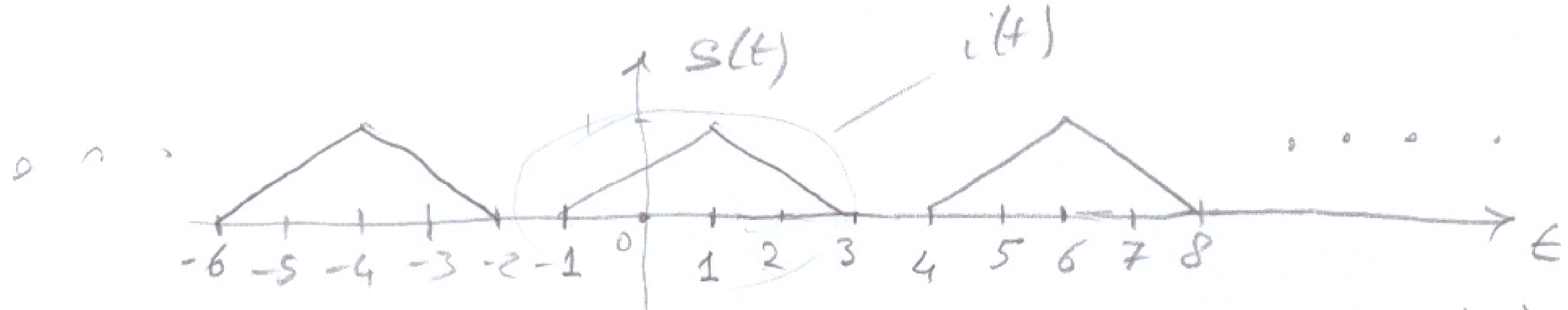

$S(f) = \frac{5}{2}\delta(f) - \frac{1}{2}e^{-j\frac{1}{10}}\delta\left(f-\frac{1}{4}\right) - \frac{1}{2}e^{-j\frac{1}{10}}\delta\left(f+\frac{1}{4}\right)$

$+ \frac{1}{4}\delta(f-3) + \frac{1}{4}\delta(f+3)$

$P_s = \left(\frac{5}{2}\right)^2 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 \frac{1}{2} = \frac{25}{4} + \frac{1}{2} + \frac{1}{8}$

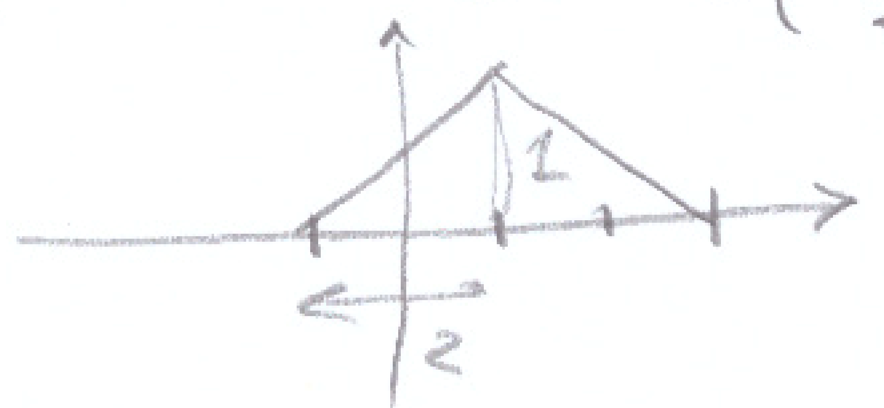
$= \frac{50+4+1}{8} = \frac{55}{8}$

1.(d) $s(t) = \sum_{k=-\infty}^{+\infty} \Lambda\left(\frac{t-1-5k}{2}\right)$




Segnale di potenza. $\mathcal{E}_s = \infty$

$i(t) = \Lambda\left(\frac{t-1}{2}\right)$



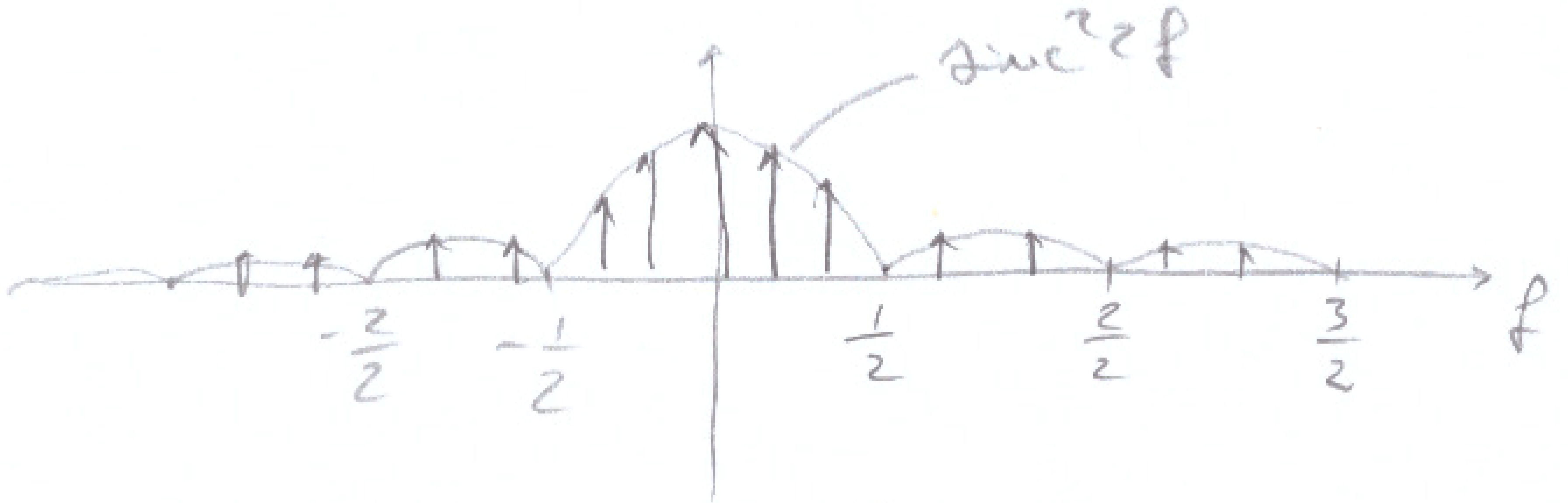

$P_s = \frac{\mathcal{E}_i}{T} = \frac{\mathcal{E}_i}{5} = \frac{4}{15}$

$\mathcal{E}_i = 2\frac{2\cdot 1}{3} = \frac{4}{3}$

$s(t) = i(t) * \sum_{k=-\infty}^{+\infty} \delta(t-5k)$

$S(f) = I(f) \cdot \frac{1}{T} \sum_{k=-\infty}^{+\infty} \delta\left(f - \frac{k}{5}\right)$ $\quad I(f) = e^{-j2\pi f}\, 2\, \mathrm{sinc}^2 2f$

$2f = n$




$S(f) = \frac{2}{5} \sum_{k=-\infty}^{+\infty} e^{-j2\pi\frac{k}{5}}\, \mathrm{sinc}^2 \frac{2k}{5}\, \delta\left(f - \frac{k}{5}\right)$

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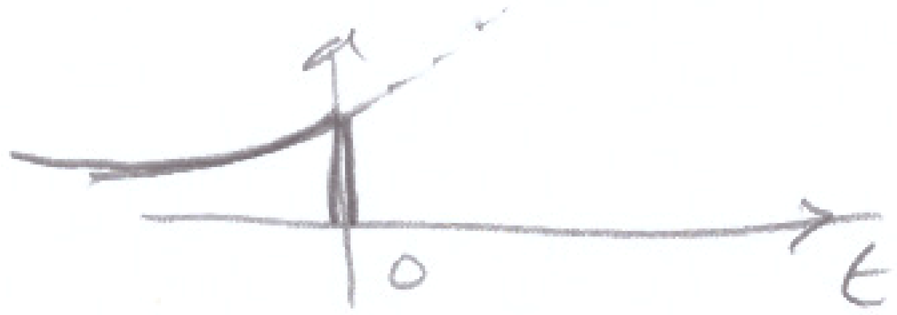
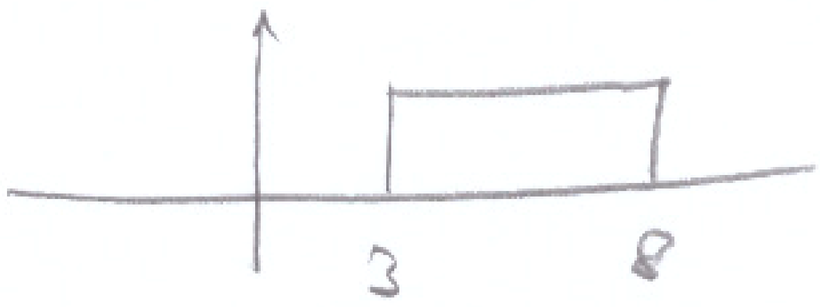
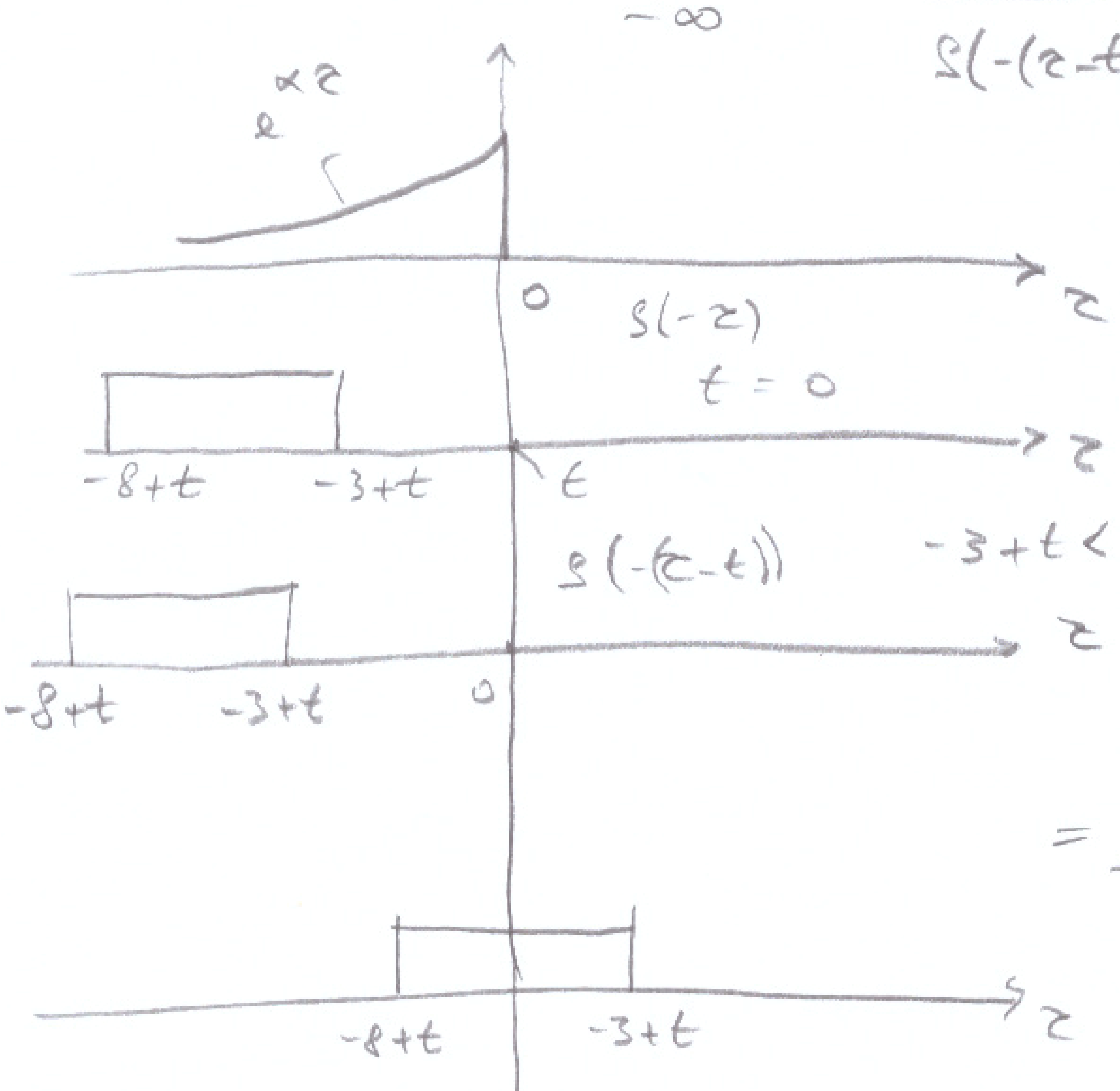
2. $h(t) = e^{\alpha t} u(-t)$ $\qquad$ $s(t) = u(t-3) - u(t-8)$





$$y(t) = (h * s)(t) = \int_{-\infty}^{+\infty} h(\tau)\, \underbrace{s(t-\tau)}_{s(-(\tau-t))}\, d\tau$$



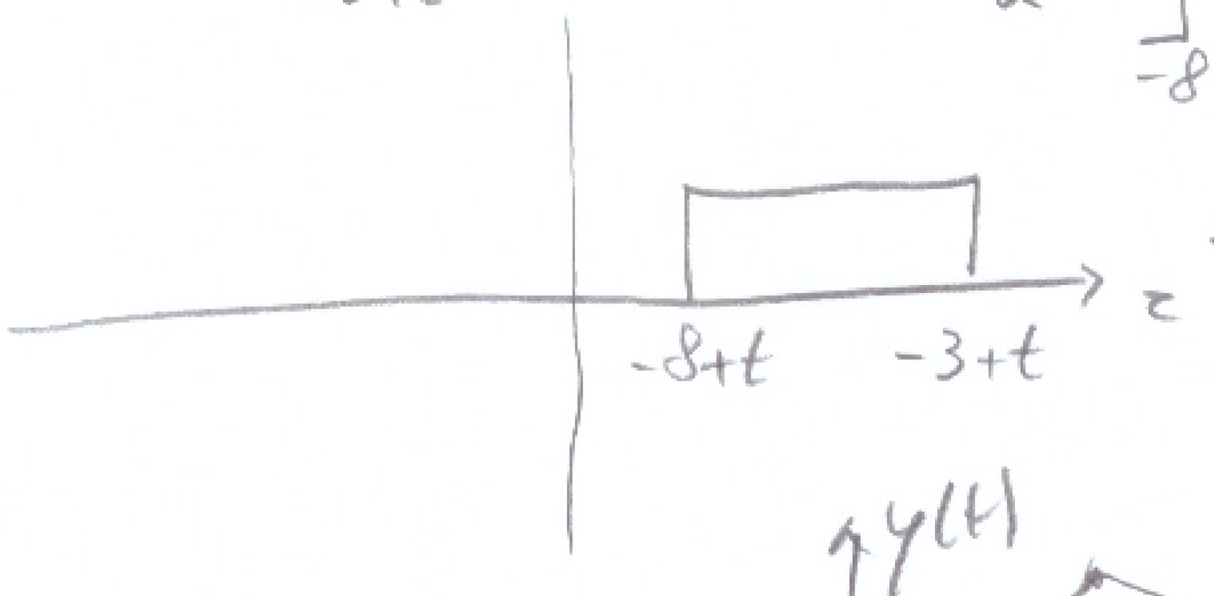

$\boxed{t<3}$

$-3+t<0 \Rightarrow$

$$y(t) = \int_{-8+t}^{-3+t} e^{\alpha\tau}\, d\tau = \frac{e^{\alpha\tau}}{\alpha}\Bigg]_{-8+t}^{-3+t} = \frac{e^{\alpha(-3+t)} - e^{\alpha(-8+t)}}{\alpha} = e^{\alpha t}\, \frac{e^{-3\alpha} - e^{-8\alpha}}{\alpha}$$

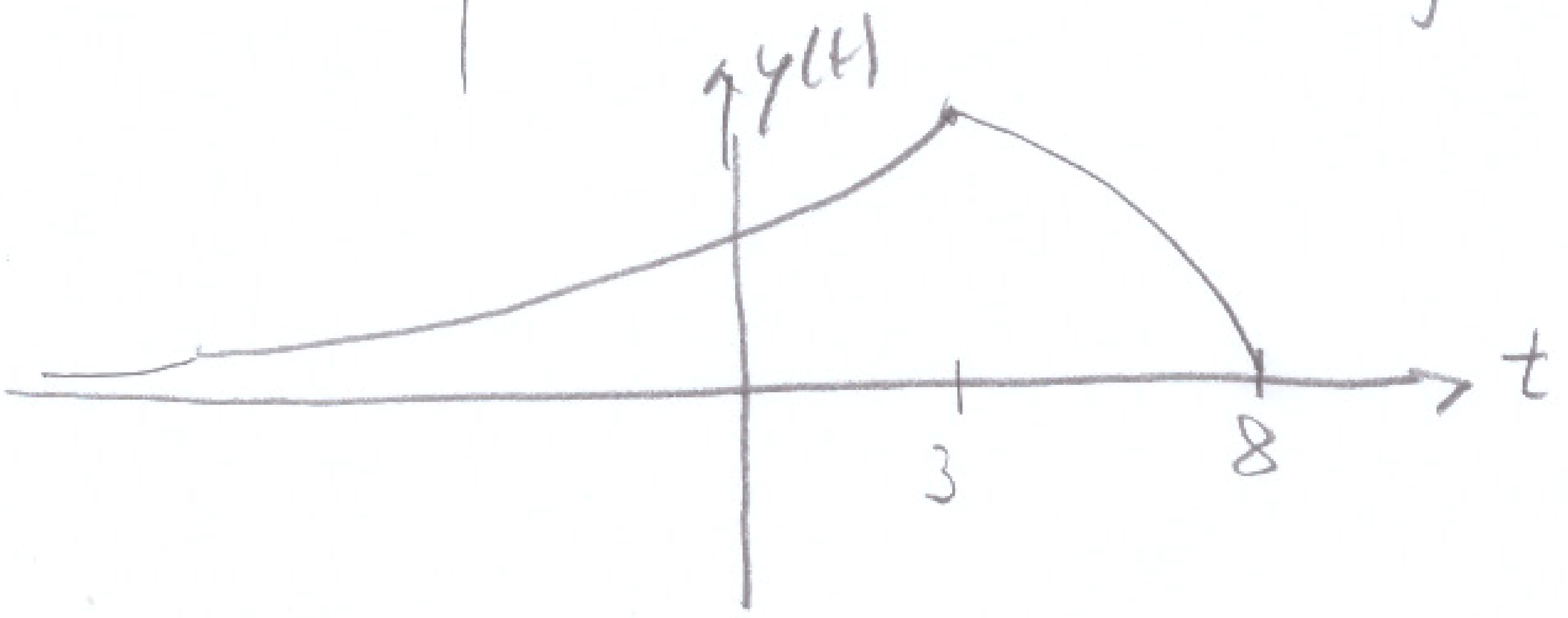
$\left.\begin{array}{l} -8+t<0 \\ -3+t>0 \end{array}\right\}$ $\boxed{3<t<8}$

$$y(t) = \int_{-8+t}^{0} e^{\alpha\tau}\, d\tau = \frac{e^{\alpha\tau}}{\alpha}\Bigg]_{-8+t}^{0} = \frac{1 - e^{\alpha(-8+t)}}{\alpha}$$

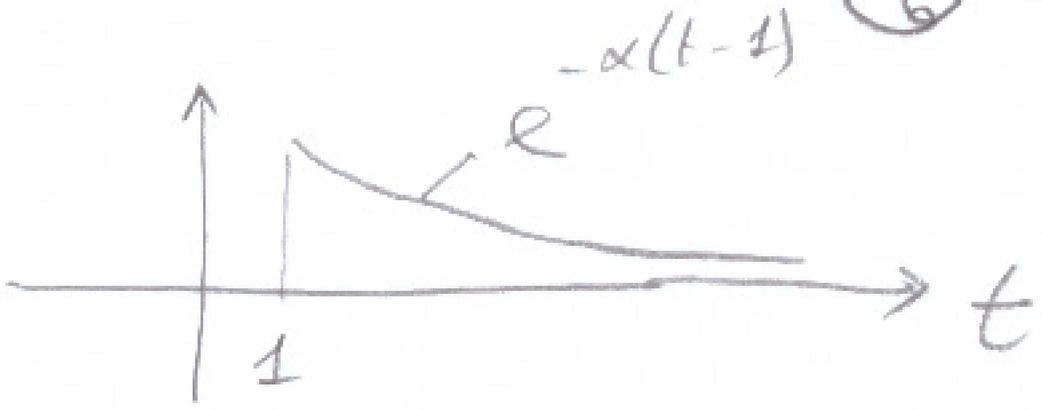



$-8+t>0$ $\quad$ $\boxed{t>8}$

$y(t) = 0$



3. $h(t) = 2\, e^{-\alpha(t-1)} u(t-1)$




$$H(f) = 2 \int_1^{\infty} e^{-\alpha(t-1)} e^{-j2\pi f t}\, dt$$

$$= 2e^{\alpha} \int_1^{\infty} e^{-t(\alpha + j2\pi f)}\, dt = 2e^{\alpha} \left. \frac{e^{-t(\alpha+j2\pi f)}}{-(\alpha + j2\pi f)} \right]_1^{\infty}$$

$$= 2e^{\alpha} \frac{e^{-(\alpha+j2\pi f)}}{\alpha + j2\pi f} = 2\,\frac{e^{-j2\pi f}}{\alpha + j2\pi f}$$

$$|H(f)| = \frac{2}{\sqrt{\alpha^2 + 4\pi^2 f^2}} \qquad \angle H(f) = -2\pi f - tg^{-1}\frac{2\pi f}{\alpha}$$

$$s(t) = \frac{1}{2} - \cos \pi t + 5\cos^2(5t-4) = \frac{1}{2} - \cos \pi t + \frac{5}{2} + \frac{5}{2}\cos(10t-8)$$

$$= 3 - \underbrace{\cos \pi t}_{\substack{2\pi f_1 = \pi \\ f_1 = \frac{1}{2}}} + \frac{5}{2}\cos(10t-8)$$

$$\cancel{2}\pi f_2 = \cancel{10}\,5 \qquad f_2 = \frac{5}{\pi}$$

$$y(t) = (h * s)(t)$$

$$S(f) = \frac{1}{2}\delta(f) - \frac{1}{2}\delta\left(f - \frac{1}{2}\right) - \frac{1}{2}\delta\left(f + \frac{1}{2}\right) + \frac{5}{4}e^{j8}\delta\left(f - \frac{5}{\pi}\right) + \frac{5}{4}e^{-j8}\delta\left(f + \frac{5}{\pi}\right)$$

$$y(t) = \frac{1}{2}\cdot|H(0)| - |H(f_1)|\cos(\pi t + \angle H(f_1)) + \frac{5}{2}|H(f_2)|\cos(10t - 8 + \angle H(f_2))$$

$$|H(0)| = \frac{2}{\alpha}$$

$$|H(f_1)| = \frac{2}{\sqrt{\alpha^2 + 4\pi^2 \frac{1}{4}}} = \frac{2}{\sqrt{\alpha^2+\pi^2}}; \quad \angle H(f_1) = -\cancel{2}\pi\frac{1}{\cancel{2}} - tg^{-1}\frac{\cancel{2}\pi}{\alpha}\frac{1}{\cancel{2}}$$

$$= -\pi - tg^{-1}\frac{\pi}{\alpha}$$

$$|H(f_2)| = \frac{2}{\sqrt{\alpha^2 + 4\cancel{\pi^2}\frac{25}{\cancel{\pi^2}}}} = \frac{2}{\sqrt{\alpha^2+100}}; \quad \angle H(f_2) = -2\cancel{\pi}\frac{5}{\cancel{\pi}} - tg^{-1}\frac{2\cancel{\pi}}{\alpha}\cdot\frac{5}{\cancel{\pi}}$$

$$= -10 - tg^{-1}\frac{10}{\alpha}$$