UNIVERSITA' DEGLI STUDI DELLA CAMPANIA Luigi Vanvitelli
Dipartimento di Ingegneria
Corso di Laurea in Ingegneria Elettronia e Informatica

Ia Prova Intracorso AA 2023-24
Teoria dei Segnali
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lunedí 26 ottobre 2023

SOLUZIONI

1. Schizzare i seguenti segnali e valutarne l'energia, la potenza e la trasformata di Fourier:

(a)[10 pt] $s(t) = \Pi\left(\frac{t-3}{4}\right) + 2\Lambda\left(\frac{t-3}{2}\right)$;

(b)[10 pt] $s(t) = e^{-3t}u(t+5)$;

(c)[10 pt] $s(t) = 3 - \cos(\frac{\pi}{5}t + \frac{1}{5}) + \cos^2 2\pi t$.

(d)[10 pt] $s(t) = -\frac{1}{2} + \sum_{k=-\infty}^{\infty} \Lambda\left(\frac{t-1-8k}{3}\right)$

2.[30 pt] Usando il metodo grafico valutare la risposta nel dominio del tempo di un sistema lineare avente risposta impulsiva $h(t) = e^{-\alpha t}u(t-5)$ al cui ingresso é posto il segnale $s(t) = u(t+3) - u(t-4)$.

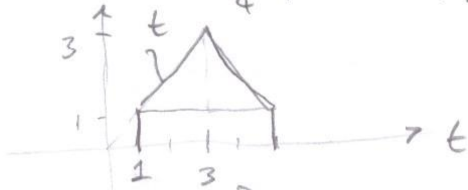
3.[30 pt] Un sistema lineare ha risposta impulsiva

$$h(t) = 10e^{-\alpha(t-4)}u(t-1). \tag{1}$$

Valutare la risposta armonica del sistema e l'uscita corrispondente all'ingresso

$$x(t) = \frac{1}{4} - \cos \pi t + 5\cos^2(10t-1). \tag{2}$$

(1) (a) $s(t) = \Pi\left(\frac{t-3}{4}\right) + 2\,\Lambda\left(\frac{t-3}{2}\right)$

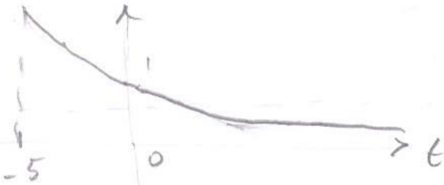


Segnale di energia, $P_s = 0$

$$\mathcal{E}_s = 2\int_1^3 s^2(t)\,dt = 2\int_1^3 t^2\,dt = 2\cdot\frac{t^3}{3}\Big]_1^3 = \frac{2}{3}(27-1) = \frac{52}{3}$$

$$S(f) = \left(4\,\mathrm{sinc}\,4f + 2\cdot 2\,\mathrm{sinc}^2 2f\right)e^{-j2\pi f 3}$$

(b) $s(t) = e^{-3t}u(t+5)$



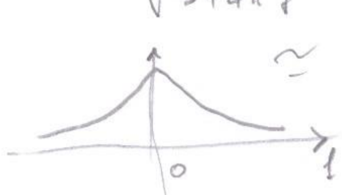
Segnale di energia, Potenza = 0

$$\mathcal{E}_s = \int_{-5}^{\infty} s(t)^2\,dt = \int_{-5}^{\infty} e^{-6t}\,dt = \frac{e^{-6t}}{-6}\Big]_{-5}^{\infty} = \frac{e^{30}}{6}$$

$$S(f) = \int_{-5}^{\infty} e^{-3t}e^{-j2\pi f t}\,dt = \int_{-5}^{\infty} e^{-(3+j2\pi f)t}\,dt$$

$$= \frac{e^{-(3+j2\pi f)t}}{-(3+j2\pi f)}\Big]_{-5}^{\infty} = \frac{e^{(3+j2\pi f)5}}{3+j2\pi f} = \frac{e^{15}\,e^{j10\pi f}}{3+j2\pi f}$$

$$|S(f)| = e^{15}\frac{1}{\sqrt{9+4\pi^2 f^2}} \qquad \angle S(f) = 10\pi f - \mathrm{tg}^{-1}\frac{2\pi f}{3}$$



$$(c)\quad s(t) = 3 - \cos\left(\frac{\pi}{5}t + \frac{1}{5}\right) + \cos^2 2\pi t = 3 - \cos\left(\frac{\pi}{5}t + \frac{1}{5}\right) + \frac{1}{2} + \frac{1}{2}\cos 4\pi t \qquad 2.$$

$$= \frac{7}{2} - \cos\left(\frac{\pi}{5}t + \frac{1}{5}\right) + \frac{1}{2}\cos 4\pi t$$

$2\pi f_1 = \frac{\pi}{5}$ $\qquad$ $f_2 = 2\ \text{Hz}.$

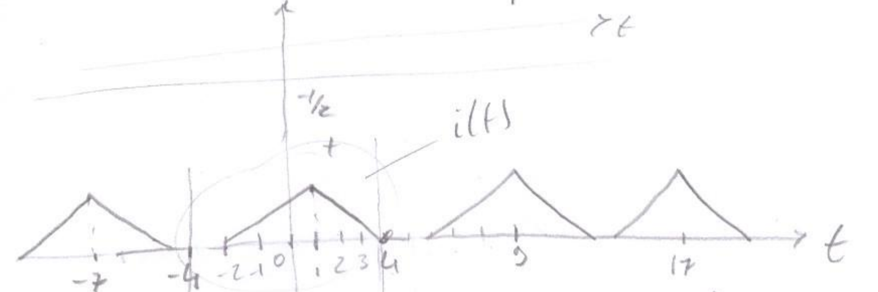
$f_1 = \frac{1}{10}\ \text{Hz}$

Segnale di potenza, $\mathcal{E}_s = \infty$

$$P_s = \left(\frac{7}{2}\right)^2 + (1)^2\frac{1}{2} + \left(\frac{1}{2}\right)^2\frac{1}{2} = \frac{49}{4} + \frac{1}{2} + \frac{1}{8} = \frac{98+4+1}{8} = \frac{103}{8}$$

$$S(f) = 3\,\delta(f) - \frac{e^{j\frac{1}{5}}}{2}\delta\left(f - \frac{1}{10}\right) - \frac{e^{-j\frac{1}{5}}}{2}\delta\left(f + \frac{1}{10}\right) + \frac{1}{4}\delta(f-2) + \frac{1}{4}\delta(f+2)$$

$$(d)\quad s(t) = -\frac{1}{2} + \underbrace{\sum_{k=-\infty}^{+\infty} \Lambda\left(\frac{t-1-8k}{3}\right)}_{s_p(t)}$$



$$i(t) = \Lambda\left(\frac{t-1}{3}\right) \longleftrightarrow I(f) = e^{-j2\pi f}\, 3\,\mathrm{sinc}^2 3f$$

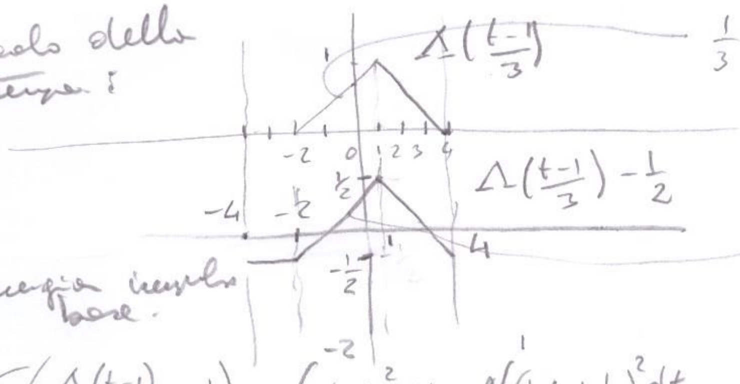
$$\mathcal{F}[s_p(t)] = \mathcal{F}\left[i(t) * \sum_{k=-\infty}^{+\infty}\delta(t - k8)\right] = I(f)\cdot\frac{1}{8}\sum_{k=-\infty}^{+\infty}\delta\left(f - \frac{k}{8}\right)$$

$$= e^{-j2\pi f}\, 3\,\mathrm{sinc}^2 3f\, \frac{1}{8}\sum_{k=-\infty}^{+\infty}\delta\left(f - \frac{k}{8}\right)$$

$$= \frac{3}{8}\sum_{k=-\infty}^{+\infty} e^{-j2\pi\frac{k}{8}}\,\mathrm{sinc}^2\frac{3k}{8}\,\delta\left(f - \frac{k}{8}\right)$$

$$\mathcal{F}[s(t)] = \frac{3}{8}\sum_{k=-\infty}^{+\infty} e^{-j2\pi\frac{k}{8}}\,\mathrm{sinc}^2\frac{3k}{8}\,\delta\left(f - \frac{k}{8}\right) - \frac{1}{2}\delta(f)$$

Calcolo della potenza:



$\frac{1}{3}t+\frac{2}{3}$

$\frac{1}{3}t+\frac{2}{3}-\frac{1}{2} = \frac{1}{3}t+\frac{4-3}{6} = \frac{1}{3}t+\frac{1}{6}$

Energia impulso base.

$$\mathcal{E}\left(\frac{\Lambda(t-1)}{3}-\frac{1}{2}\right) = \int_{-4}^{-2}\left(-\frac{1}{2}\right)^2 dt + 2\int_{-2}^{1}\left(\frac{1}{3}t+\frac{1}{6}\right)^2 dt$$

$$= \frac{1}{4}\cdot 2 + 2\int_{-2}^{1}\frac{1}{9}t^2\,dt + \frac{4}{18}\int_{-2}^{1} t\,dt + \frac{2}{36}\int_{-2}^{1} dt$$

$$= \frac{1}{2} + \frac{2}{9}\,\frac{t^3}{3}\Big]_{-2}^{1} + \frac{4}{18}\,\frac{t^2}{2}\Big]_{-2}^{1} + \frac{1}{18}\,t\Big]_{-2}^{1}$$

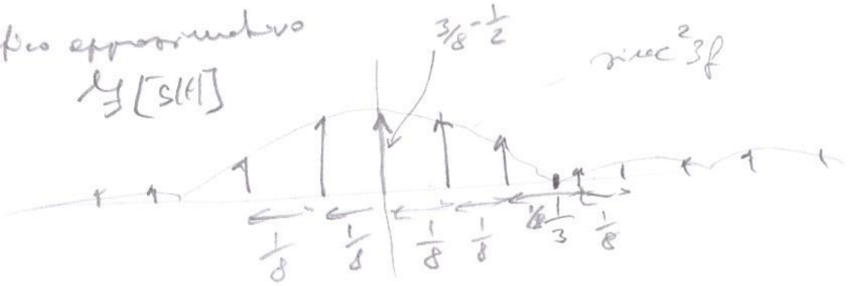
$$= \frac{1}{2} + \frac{2}{\cancel{9}}\,\frac{\cancel{1+8}}{3} + \frac{\cancel{4}^{\,2}}{\cancel{18}\,6}\,\frac{\cancel{1-4}}{\cancel{2}} + \frac{1}{18}(1+2)$$

$$= \frac{1}{2} + \frac{2}{3} - \frac{1}{3} + \frac{\cancel{3}}{\cancel{18}\,6}$$

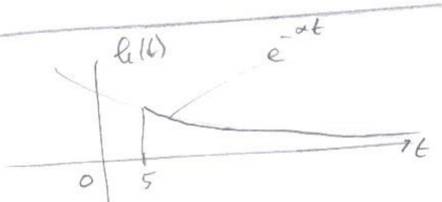
$$= \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{3+2+1}{6} = \frac{6}{6} = 1$$

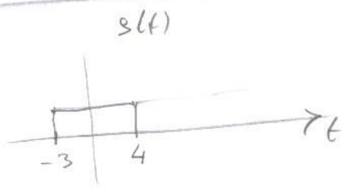
$$P_s = \frac{\mathcal{E}}{T} = \frac{1}{8}$$

Grafico approssimativo $\mathcal{F}[s(t)]$

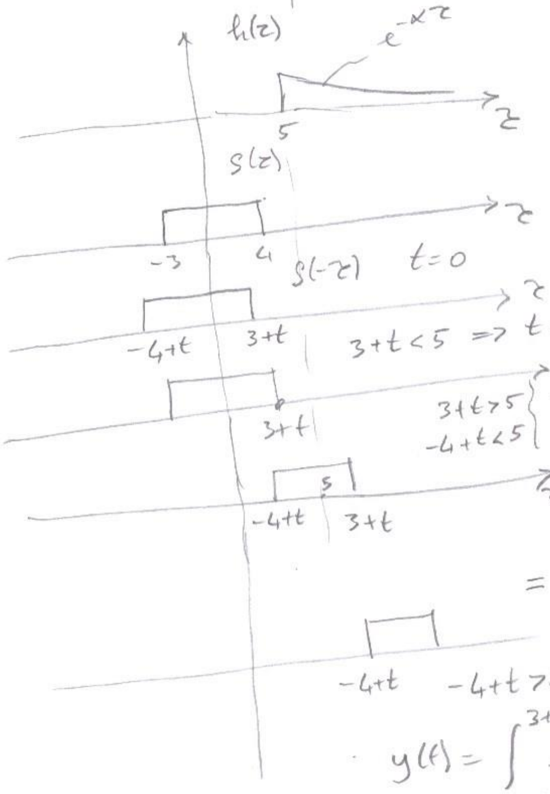


(2)





$y(t) = (h * s)(t)$



$t=0$

$3+t<5 \Rightarrow t<2 \qquad y(t)=0$

$\left.\begin{array}{l}3+t>5\\ -4+t<5\end{array}\right\} \Rightarrow 2<t<9$

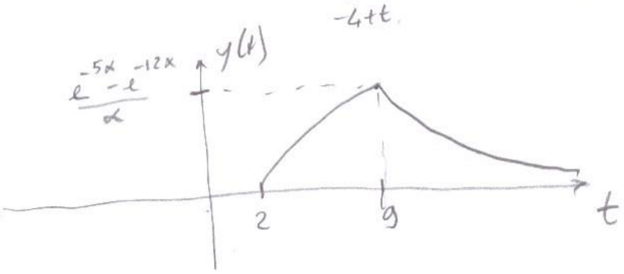
$$y(t)=\int_5^{3+t} e^{-\alpha\tau}\,d\tau = \left.\frac{e^{-\alpha\tau}}{-\alpha}\right]_5^{3+t}$$

$$= \frac{e^{-\alpha(3+t)}-e^{-\alpha 5}}{-\alpha} = \frac{e^{-\alpha 5}-e^{-\alpha 3}e^{-\alpha t}}{\alpha}$$

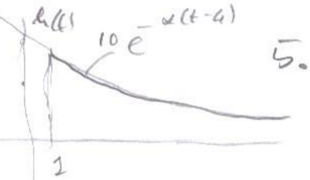
$-4+t>5 \Rightarrow t>9$

$$y(t)=\int_{-4+t}^{3+t} e^{-\alpha\tau}\,d\tau = \left.\frac{e^{-\alpha\tau}}{-\alpha}\right]_{-4+t}^{3+t} = \frac{e^{-\alpha(3+t)}-e^{-\alpha(-4+t)}}{-\alpha}$$

$$= \frac{e^{-\alpha t}\left(e^{\alpha 4}-e^{-\alpha 3}\right)}{\alpha}$$



③ $$h(t) = 10\, e^{-\alpha(t-4)}\, u(t-1)$$ 5.



$$H(f) = \int_1^{\infty} 10\, e^{-\alpha t} e^{4\alpha} e^{-j2\pi f t}\, dt$$

$$= 10 e^{4\alpha} \int_1^{\infty} e^{-(\alpha + j2\pi f)t}\, dt = 10 e^{4\alpha} \left. \frac{e^{-(\alpha+j2\pi f)t}}{-(\alpha+j2\pi f)} \right]_1^{\infty}$$

$$= \frac{10 e^{4\alpha}\, e^{-(\alpha+j2\pi f)}}{\alpha + j2\pi f} = 10 e^{3\alpha} \frac{e^{-j2\pi f}}{\alpha + j2\pi f}$$

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$|H(f)| = \frac{10\, e^{3\alpha}}{\sqrt{\alpha^2 + 4\pi^2 f^2}} \qquad \angle H(f) = -2\pi f - \operatorname{tg}^{-1} \frac{2\pi f}{\alpha}$$

$$x(t) = \frac{1}{4} - \cos \pi t + 5\left(\frac{1}{2}\cos(20t-2) + \frac{1}{2}\right)$$

$$= \frac{1}{4} - \cos\pi t + \frac{5}{2}\cos(20t-2) + \frac{5}{2}$$

$$= \frac{11}{4} - \underbrace{\cos \pi t} + \frac{5}{2}\cos(\underline{20}t - 2)$$

$$2\pi f_1 = \pi \qquad 2\pi f_2 = 20$$

$$f_1 = \frac{1}{2} \qquad f_2 = \frac{10}{\pi}$$

La componente a frequenza zero: $\frac{11}{4} \Rightarrow$ ($|H(0)|$ guadagno statico)

$$y_0(t) = \frac{11}{4} \cdot |H(0)| = \frac{11}{\cancel{4}_2} \frac{\cancel{10}^{5}\, e^{3\alpha}}{\alpha} = \frac{55}{2\alpha} e^{3\alpha}$$

La componente a frequenza $f_1 = \frac{1}{2}$: $-\cos \pi t \Rightarrow$

$$y_1(t) = -|H(f_1)| \cos(\pi t + \angle H(f_1)) = -\frac{10 e^{3\alpha}}{\sqrt{\alpha^2 + 4\pi^2 \left(\frac{1}{2}\right)^2}} \cos\left(\pi t + \left(-2\pi \frac{1}{2} - \operatorname{tg}^{-1} \frac{2\pi}{\alpha}\frac{1}{2}\right)\right)$$

$$= -\frac{10 e^{3\alpha}}{\sqrt{\alpha^2 + \pi^2}} \cos\left(\pi t - \pi - \operatorname{tg}^{-1}\frac{\pi}{\alpha}\right)$$

la componente a frequenza $f_2 = \frac{10}{\pi}$: $\frac{5}{2}\cos(20t-2) \Rightarrow$ 6.

$$y_2(t) = \frac{5}{2}|H(f_2)| \cos(20t - 2 + \angle H(f_2))$$

$$= \frac{5}{2} \frac{10e^{3\alpha}}{\sqrt{\alpha^2 + 4\pi^2\frac{100}{\pi^2}}} \cos\left(20t - 2 + \left(-2\pi\frac{5}{\pi} - tg^{-1}\frac{2\pi}{\alpha}\frac{10}{\pi}\right)\right)$$

$$= \frac{25}{\sqrt{\alpha^2+400}} \cdot \cos\left(20t - 2 - 10 - tg^{-1}\frac{20}{\alpha}\right)$$

$$= \frac{25}{\sqrt{\alpha^2+400}} \cdot \cos\left(20t - 12 - tg^{-1}\frac{20}{\alpha}\right)$$

$$y(t) = y_0(t) + y_1(t) + y_2(t)$$