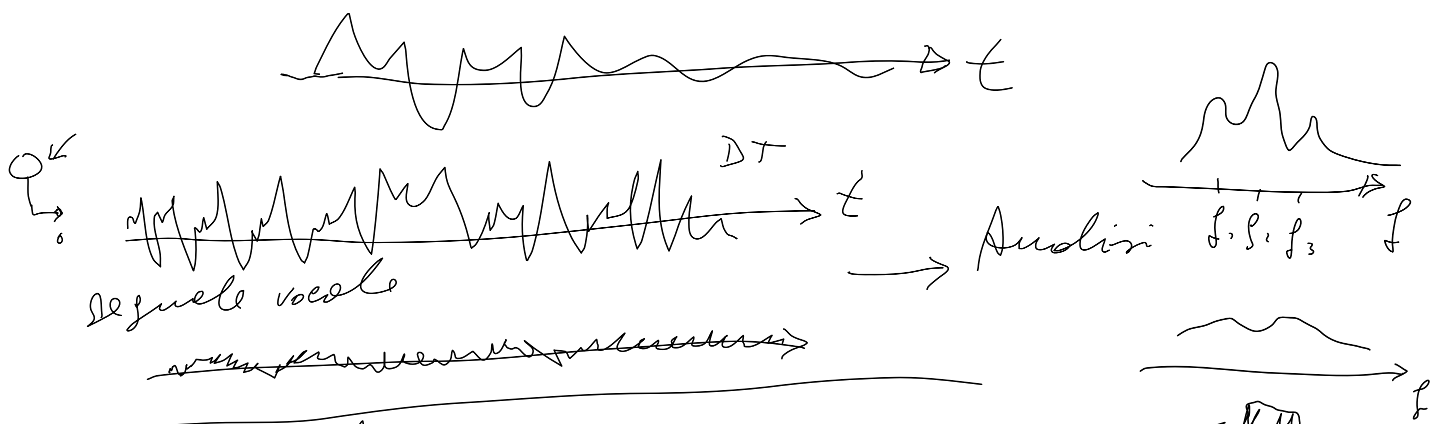
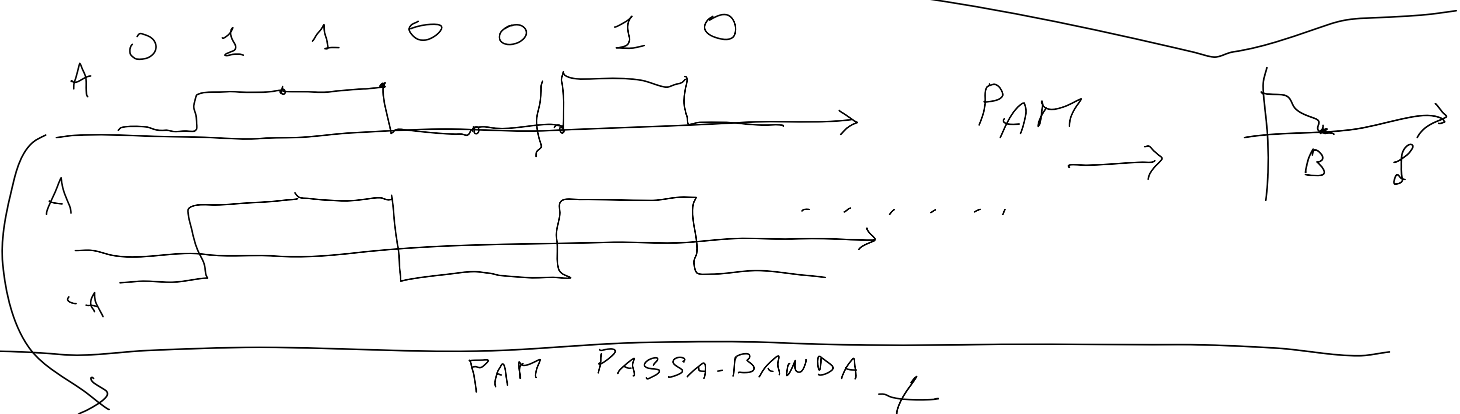
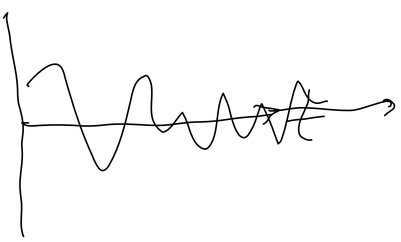
$s(t)$



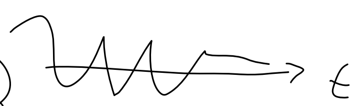


PAM PASSA-BANDA

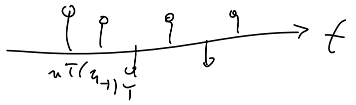


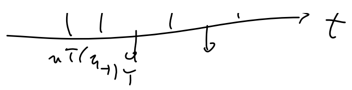
$S(t): t \in \mathcal{T} \rightarrow \mathbb{R}$

$\mathcal{T}$ continuo
tempo-continuo

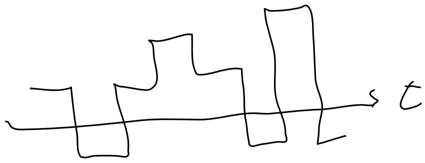


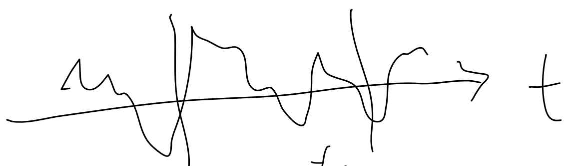
$\mathcal{T}$ discreto
tempo-discreto

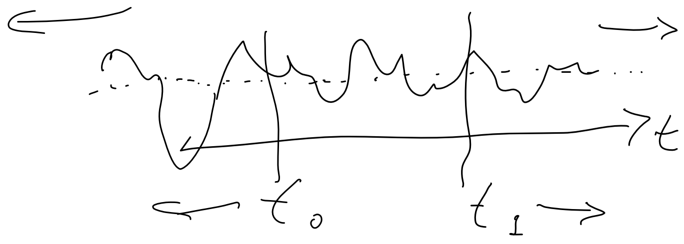


Tempo-discreto 

A continuo

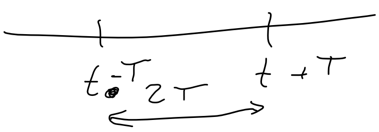
A discreto 





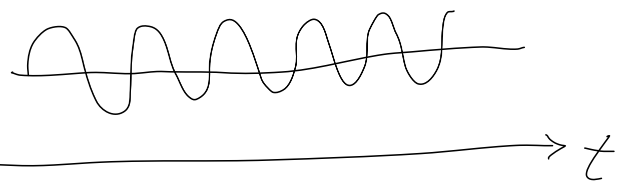
$$\mu_s(t_0,t_1) = \frac{1}{t_1-t_0}\int_{t_0}^{t_1} s(t)\,dt$$

$$\mu_s = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} s(t)\,dt$$



Media asintotica

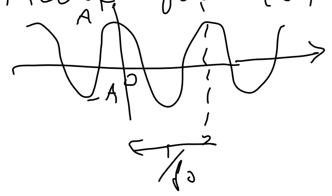
Esempio



$[f_0]$ Hz

$$s(t) = A\cos 2\pi f_0 t + b$$

$A\cos(2\pi f_0 t + \varphi_0)$



DC

$$\mu_s = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}(A\cos 2\pi f_0 t + b)\,dt$$

$$= \lim_{T\to\infty}\frac{1}{2T}\left[\int_{-T}^{T} A\cos 2\pi f_0 t\,dt + \int_{-T}^{T} b\,dt\right]$$

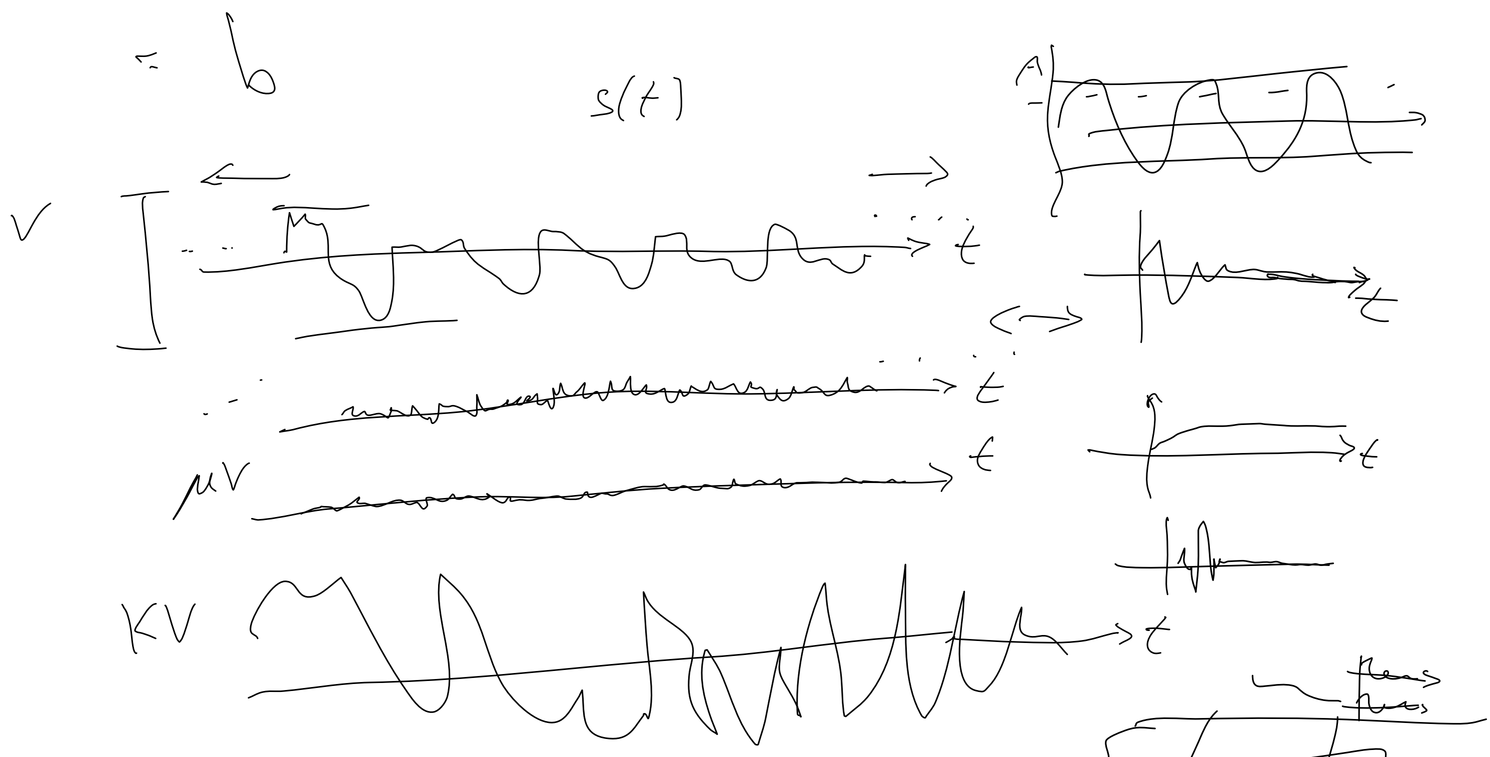
$$= \lim \frac{1}{2T}\left[A\,\frac{\sin 2\pi f_0 t}{2\pi f_0}\right]_{-T}^{T} + b2T =$$

$$= \lim_{T\to\infty}\frac{1}{2T}\left[A\,\frac{\sin 2\pi f_0 T + \sin 2\pi f_0 T}{2\pi f_0} + b\,2T\right]$$

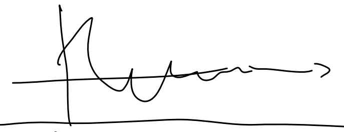
$[-1 \quad 1]$

$$= \lim_{T\to\infty}\left[\underbrace{\frac{A}{2T}\,\frac{2\sin 2\pi f_0 T}{2\pi f_0}}_{\to 0} + b\right]$$

$= b$



ENERGIA



$$\boxed{\mathcal{E}_s = \int_{-\infty}^{+\infty} |s(t)|^2 dt < \infty}$$

L_2 $\quad P_s = 0$

$$\boxed{s(t) = s_R(t) + j\, s_I(t)}$$

$s = s_R + j s_I$

$|s| = \sqrt{s_R^2 + s_I^2}$

$\angle s = \mathrm{tg}^{-1} \frac{s_I}{s_R}$

$s_R = |s| \cos \angle s$

$s_I = |s| \sin \angle s$

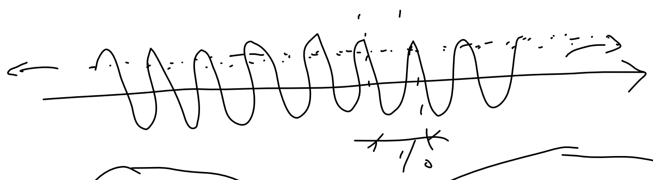
POTENZA

$$\boxed{P_s = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |s(t)|^2 dt}$$

$\mathcal{E}_s = \infty$

$\mathcal{E}_s$

$s(t) = A \cos(2\pi f_0 t + \varphi_0)$



$P_s^{RMS} = \sqrt{P_s}$

$P_s = ?$

$$P_s = \frac{A^2}{2} \qquad P_s^{RMS} = \frac{A}{\sqrt{2}}$$

$\mathcal{E}_s = \infty$

$$\sqrt{\lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |s(t)|^2 dt}$$

$[s(t)] = V$

$[P_s] = V^2$

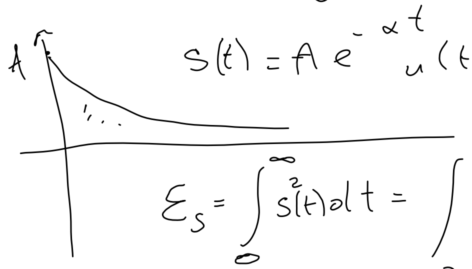
$[P_s^{RMS}] = V$

$= \frac{A^2}{2}$

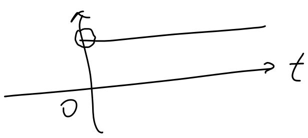
$u(t) = \{ 1 \quad t \geq 0$

$$= \frac{A^2}{2}$$

$$s(t) = A e^{-\alpha t} u(t)$$

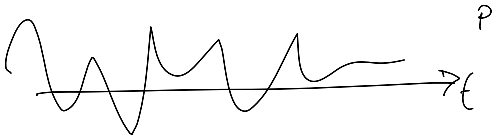


$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$



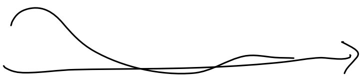
$$\mathcal{E}_s = \int_0^{\infty} s^2(t)\,dt = \int_0^{\infty} A^2 e^{-2\alpha t}\,dt$$

$$= A^2 \left. \frac{e^{-2\alpha t}}{-2\alpha} \right]_0^{\infty} = A^2 \frac{1}{2\alpha} = \frac{A^2}{2\alpha}$$



POTENZA

$$P_s = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |s(t)|^2\,dt$$

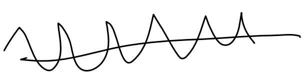


ENERGIA

$$\mathcal{E}_s = \int_{-\infty}^{+\infty} |s(t)|^2\,dt$$

SEGNALI CANONICI

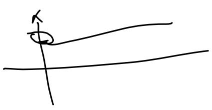
$$s(t) = A\cos(2\pi f_0 t + \varphi_0)$$



$$= A\cos(\omega_0 t + \varphi_0)$$

$$[\omega_0] = rad/sec$$

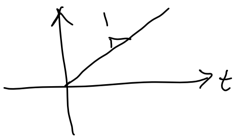
$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$



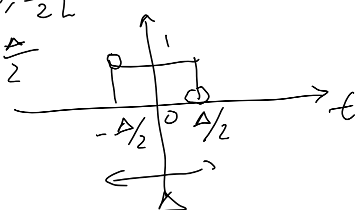
$$\omega_0 = 2\pi f_0$$

$$f_0 = \frac{\omega_0}{2\pi}$$

$$r(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

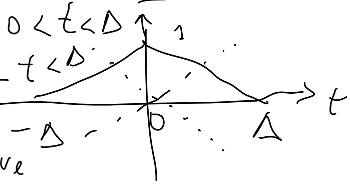


$$\Pi\left(\frac{t}{\Delta}\right) = \begin{cases} 0 & t \in \left]-\infty, -\frac{\Delta}{2}\right[\\ 1 & -\frac{\Delta}{2} < t < \frac{\Delta}{2} \\ 0 & t \geq \frac{\Delta}{2} \end{cases}$$



Impulso rettangolare

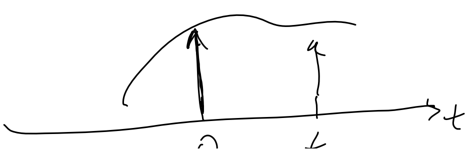
$$\Lambda\left(\frac{t}{\Delta}\right) = \begin{cases} -\frac{1}{\Delta}t + 1 & 0 < t < \Delta \\ \frac{1}{\Delta}t + 1 & -\Delta < t < 0 \\ 0 & \text{altrove} \end{cases}$$



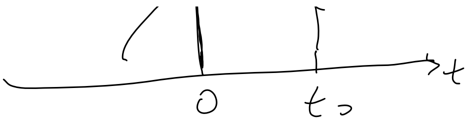
Impulso triangolare

impulso di Dirac

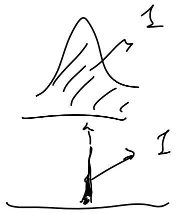
$$\delta(t) = \begin{cases} +\infty & t = 0 \\ 0 & \text{altrove} \end{cases}$$!!



$$\int^{+\infty} \delta(t)\,dt = 1$$

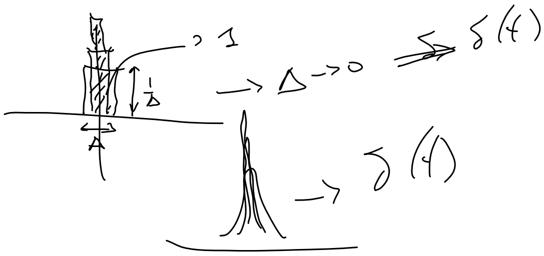


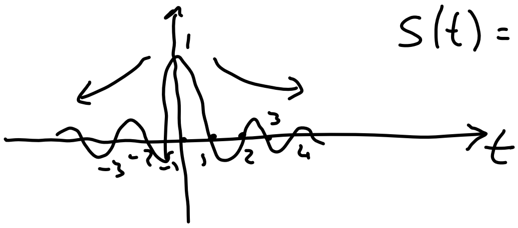
$$\int_{-\infty}^{+\infty} \delta(t)\,dt = 1$$



$$\int_{-\infty}^{+\infty} \delta(t)\, f(t)\,dt = f(0) \quad !!$$

$$\int_{-\infty}^{+\infty} \delta(t-t_0)\, f(t)\,dt = f(t_0)$$





$$s(t) = \text{sinc}\, t \triangleq \frac{\sin \pi t}{\pi t}$$

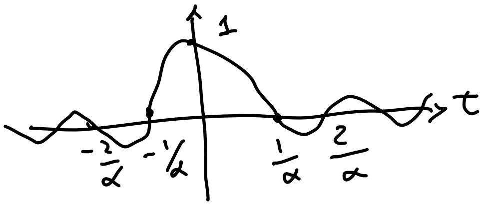
$$\sin \pi t = 0$$
$$\cancel{\pi} t = k \cancel{\pi}$$

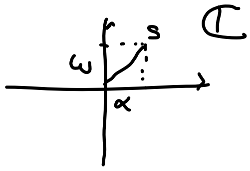
$$s(0) = \frac{0}{0}\ ? \quad \left.\frac{\cancel{\pi}\cos \pi t}{\cancel{\pi}}\right|_{t=0} = 1$$

$$s(t) = \text{sinc}\, \alpha t$$

$$\alpha t = K$$

$$\frac{\sin \pi \alpha t}{\pi \alpha t} \Rightarrow \frac{\cancel{\pi\alpha}\cos \pi \alpha t}{\cancel{\pi\alpha}}$$





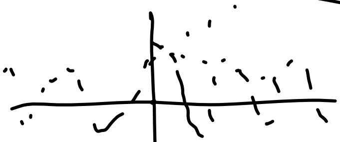
$$s = \alpha + j\omega$$

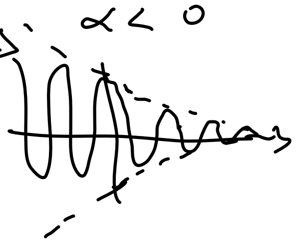
$$x(t) = e^{st} = e^{\alpha t} \cdot e^{j\omega t} = e^{\alpha t}(\cos \omega t + j \sin \omega t)$$

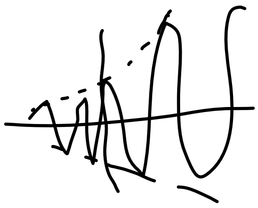
$$|x(t)| = e^{\alpha t} \qquad = \underbrace{e^{\alpha t} \cos \omega t} + j\, e^{\alpha t} \sin \omega t \qquad \omega = 2\pi f$$

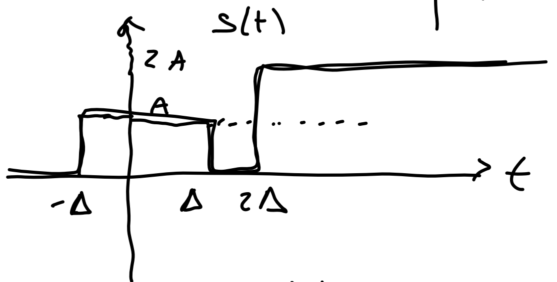
$$\angle x(t) = \omega t$$

$\alpha < 0$ $\qquad$ $\alpha > 0$



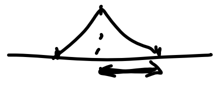


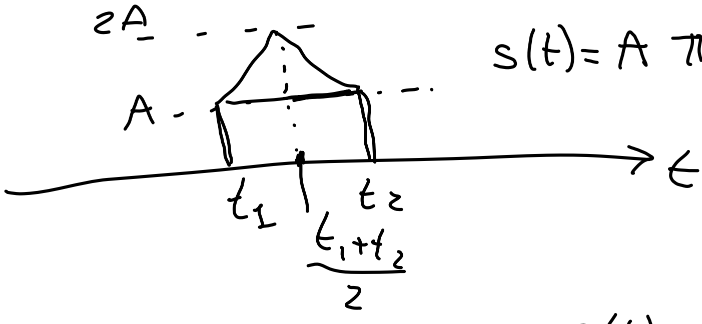




$$s(t) = A\,\Pi\left(\frac{t}{2\Delta}\right) + 2A\, u(t - 2\Delta)$$

$$= A u(t+\Delta) - A u(t - \Delta) + 2A u(t - 2\Delta)$$



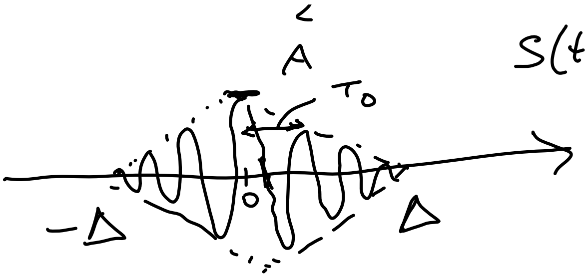


$$s(t) = A\,\Pi\left(\frac{t - \frac{t_1+t_2}{2}}{t_2 - t_1}\right) + A\Lambda\left(\frac{t - \frac{t_1+t_2}{2}}{\frac{t_2-t_1}{2}}\right)$$

$$s(t) = A\Lambda\left(\frac{t}{\Delta}\right)\cos \frac{2\pi}{T_0} t$$

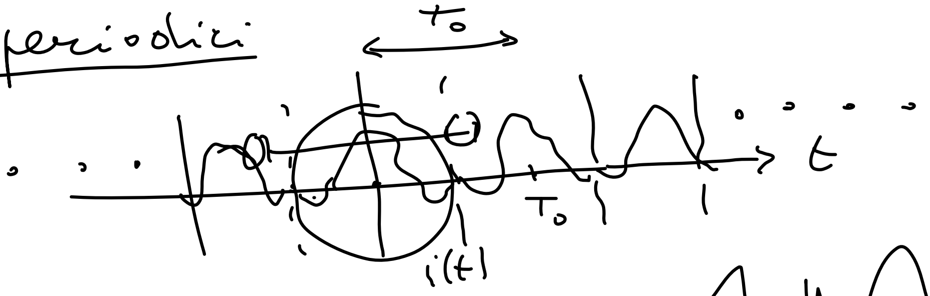
A

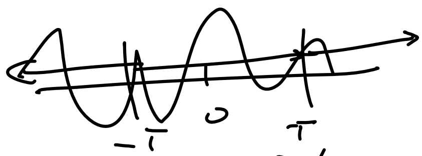
T_0



$$s(t) = A\Lambda\left(\frac{t}{\Delta}\right)\cos\frac{2\pi}{T_0}t$$

Segnali periodici

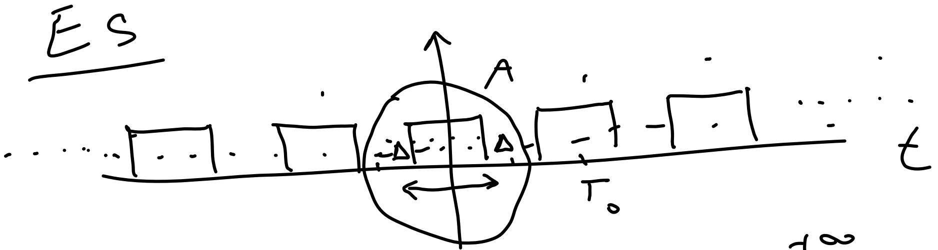




$$s(t) = \sum_{k=-\infty}^{+\infty} i(t-kT_0)$$

$$P_s = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}|s(t)|^2\,dt = \frac{\mathcal{E}_i}{T_0} = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2}|i(t)|^2\,dt$$

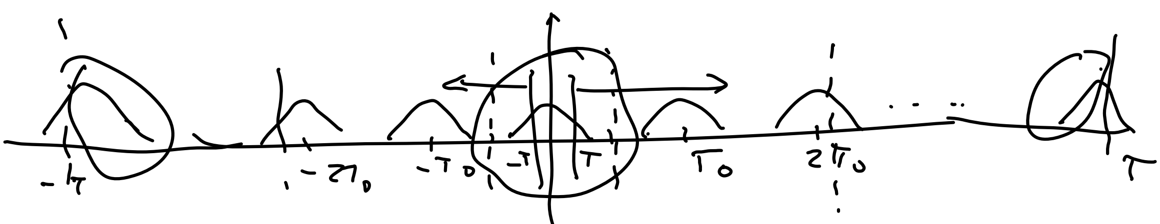
ES



$$i(t) = A\Pi\left(\frac{t}{2\Delta}\right) \qquad s(t) = A\sum_{k=-\infty}^{+\infty}\Pi\left(\frac{t-kT_0}{2\Delta}\right)$$

$$\mu_s = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} i(t)\,dt = \frac{1}{T_0}\int_{-\Delta}^{\Delta} A\,dt = \frac{A}{T_0}2\Delta$$

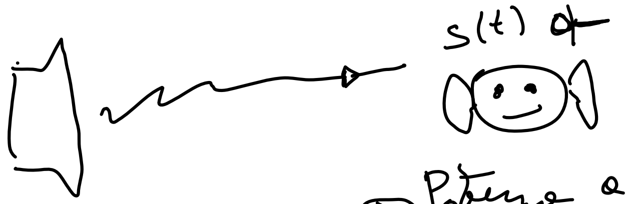
$$P_s = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2}|i(t)|^2\,dt = \frac{1}{T_0}\int_{-\Delta}^{\Delta} A^2\,dt = \frac{A^2}{T_0}2\Delta$$



$$P_s = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}|s(t)|^2\,dt = \frac{1}{NT_0}\int \#\text{intero di periodi} + \frac{1}{\Delta}\int \text{pezzettino}$$

$$P_s = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |s(t)|^2\, dt = \frac{1}{NT_0}\underbrace{\int \#\text{intero di periodi}} + \frac{1}{\Delta}\int \text{pezzettino}$$

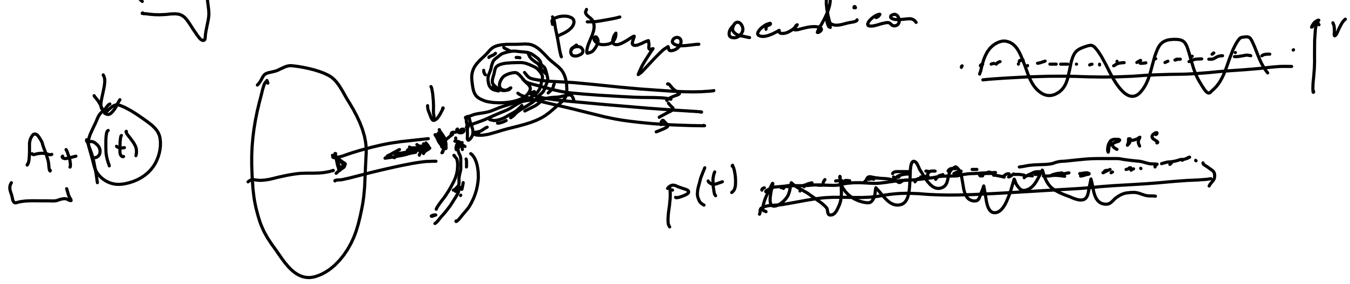
DECI $\underbrace{\text{BEL}}$

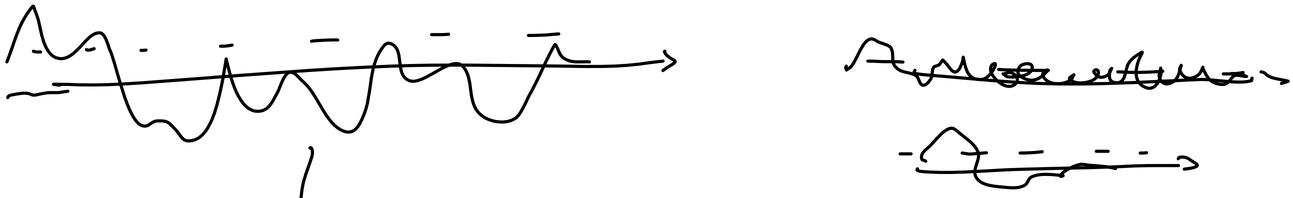


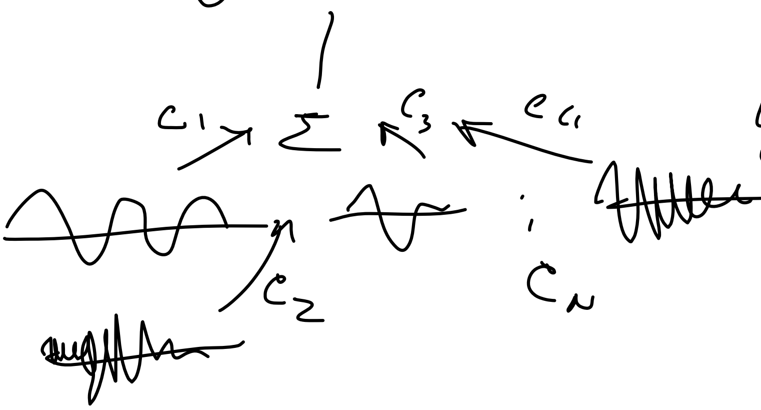
dB acustico

dB SPL ...

V^2





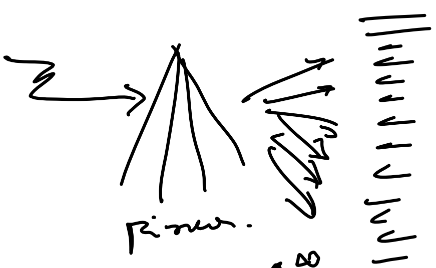


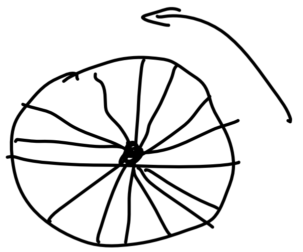
Quali funzioni?

Quante?

Appross. o rappr. esatta?

$$s(t) = \sum_i \overset{?}{c_i}\, \overset{?}{\varphi_i}(t)$$



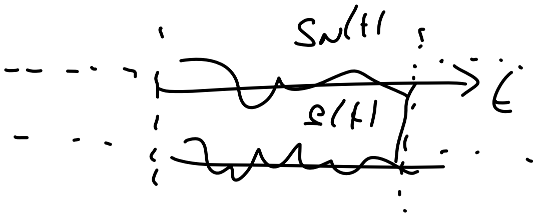


$$s(t) \Leftrightarrow S_N(t) = \sum_{i=1}^{N\to\infty} c_i\, \varphi_i(t)$$

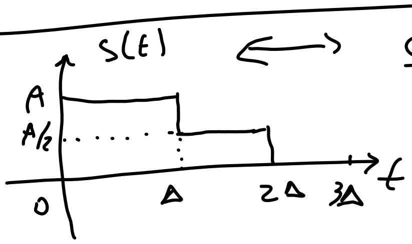
Dato un set di funzioni base $\{\varphi_1(t), \varphi_2(t), \ldots, \varphi_N(t)\}$

Trovare $c_1, c_2, \ldots, c_N \rightarrow$

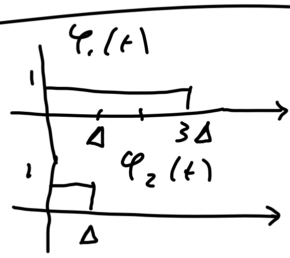
$$\varepsilon = \frac{1}{2T}\int_{-T}^{T} |s(t) - s_N(t)|^2\, dt$$

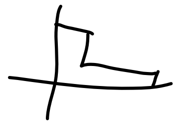


$$\frac{1}{2T}\int_{-T}^{T} |s(t)-s_N(t)|\, dt \qquad T \to \infty$$



$\longleftrightarrow$ $S_2(t) = c_1 \varphi_1(t) + c_2 \varphi_2(t)$





Hermitiana $A^{T*} = A$

$$\begin{bmatrix} \int |\varphi_1|^2 & \int \varphi_1 \varphi_2^* \\ \int \varphi_2 \varphi_1^* & \int |\varphi_2|^2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} \int s \varphi_1^* \\ \int s \varphi_2^* \end{bmatrix}$$

$$\int_0^{3\Delta} \varphi_1^2(t)\,dt = 3\Delta$$

$$\int_0^{3\Delta} \varphi_1(t)\varphi_2(t)\,dt = \Delta$$

$$\int_0^{3\Delta} \varphi_2^2(t)\,dt = \Delta$$

$$\int_0^{3\Delta} s(t)\,\overset{1}{\varphi_1(t)}\,dt = \int_0^{2\Delta} s(t)\,dt = A\Delta + \frac{A}{2}\Delta = \frac{A\Delta 3}{2}$$

$$\int_0^{3\Delta} s(t)\varphi_2(t)\,dt = A\Delta$$

$$\begin{bmatrix} 3\Delta & \Delta \\ \Delta & \Delta \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} \frac{A\Delta}{2}3 \\ A\Delta \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} \frac{3}{2}A \\ A \end{bmatrix}$$

$$\begin{cases} 3c_1 + c_2 = \frac{3}{2}A \\ c_1 + c_2 = A \end{cases}$$

$$c_1 = A - c_2$$

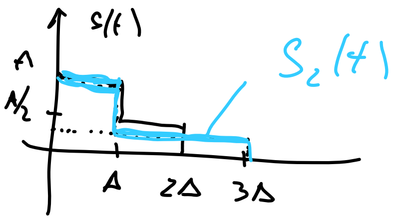
$$3(A - c_2) + c_2 = \frac{3}{2}A$$

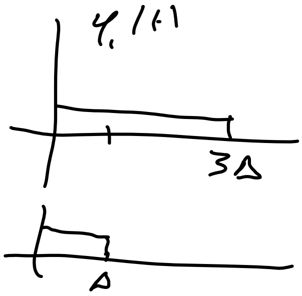
$$3A - 3c_2 + c_2 = \frac{3}{2}A$$

$$-2c_2 = \left(\frac{3}{2} - 3\right)A \implies c_2 = \frac{\left(3 - \frac{3}{2}\right)A}{2} = \frac{3}{4}A$$

$$c_1 = A - \frac{3}{4}A = \frac{A}{4}$$

$$S_2(t) = \frac{A}{4}\varphi_1(t) + \frac{3}{4}A\,\varphi_2(t)$$





$$\sum^{N} c_i \varphi_i(t) = S_N(t) \longrightarrow s(t)$$

$$\sum_{i=1}^{N} c_i \varphi_i(t) = S_N(t) \to s(t)$$

?

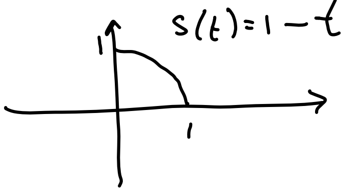
$$\begin{bmatrix} \int \varphi_1^2 & \cdots & \int \varphi_1 \varphi_N \\ \int \varphi_2 \varphi_1 & \int \varphi_2^2 \cdots & \int \varphi_2 \varphi_N \\ & \ddots & \\ & \cdots & \int \varphi_N^2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_N \end{bmatrix} = \begin{bmatrix} \int s \varphi_1 \\ \int s \varphi_2 \\ \vdots \\ \int s \varphi_N \end{bmatrix}$$

$$\int \varphi_i \varphi_j^* = 0 \quad i \neq j$$

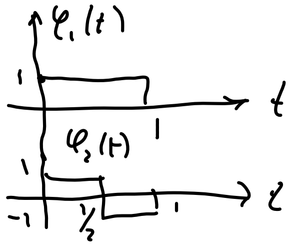
$$\begin{bmatrix} \int \varphi_1^2 & & 0 \\ & \int \varphi_2^2 \ddots & \\ 0 & & \int \varphi_N^2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_N \end{bmatrix} = \begin{bmatrix} \int s \varphi_1 \\ \int s \varphi_2 \\ \vdots \\ \int s \varphi_N \end{bmatrix} \Rightarrow \boxed{c_i = \frac{\int s \varphi_i}{\int \varphi_i^2} \quad i = 1, \dots, N}$$

ESEMPIO

$s(t) = 1 - t^2 \qquad t \in [0, 1]$



$$\int_0^1 \varphi_1(t)\varphi_2(t)\,dt = 0$$



$\Rightarrow$ ortogonali

$$S_2(t) = c_1 \varphi_1(t) + c_2 \varphi_2(t)$$

$$c_1 = \frac{\int_0^1 s(t)\varphi_1(t)\,dt}{\int_0^1 \varphi_1^2(t)\,dt} \qquad c_2 = \frac{\int_0^1 s(t)\varphi_2(t)\,dt}{\int_0^1 \varphi_2^2(t)\,dt}$$

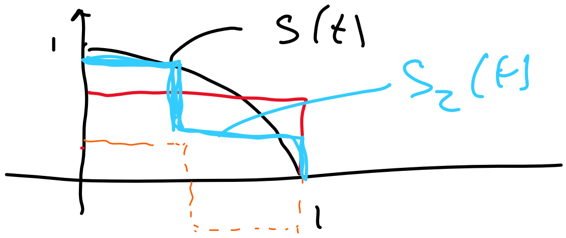
$$\int_0^1 \varphi_1^2(t)\,dt = 1$$

$$\int \varphi_2^2(t)\,dt = 1 \qquad \int_0^1 s(t)\varphi_1(t)\,dt = \int_0^1 (1-t^2)\,dt = 1 - \left.\frac{t^3}{3}\right]_0^1 = 1 - \frac{1}{3} = \frac{2}{3}$$

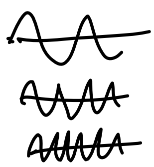
$$\int_0^1 s(t)\varphi_2(t)\,dt = \int_0^{1/2} (1-t^2)\,dt - \int_{1/2}^1 (1-t^2)\,dt = \cancel{\tfrac{1}{2}} - \left.\frac{t^3}{3}\right]_0^{1/2} - \cancel{\tfrac{1}{2}} + \left.\frac{t^3}{3}\right]_{1/2}^1 = -\frac{1}{8\cdot 3} + \frac{1}{3} - \frac{1}{8\cdot 3}$$

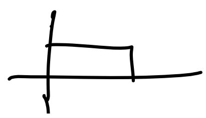
$$= \frac{1}{3} - \frac{1}{12} = \frac{4-1}{12} = \frac{3}{12} = \frac{1}{4}$$

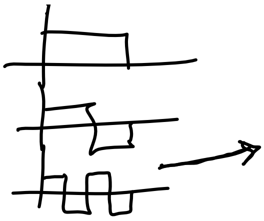
$$S_2(t) = \frac{2}{3}\varphi_1(t) + \frac{1}{4}\varphi_2(t)$$



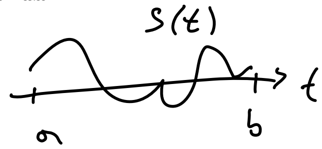
$s(t) \qquad \varphi_i(t) \qquad$ sinusoidi

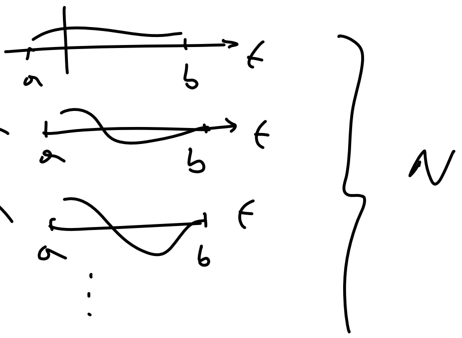






$N \to \infty$





$$\hat{s}_N(t) = \sum_{i=1}^{N} c_i \varphi_i(t)$$

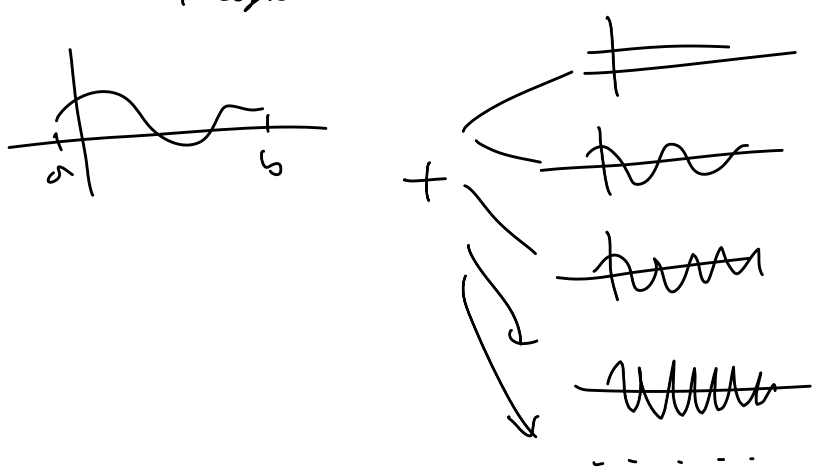
$\{\varphi_i(t)\}$ sistema di funzioni ortogonali

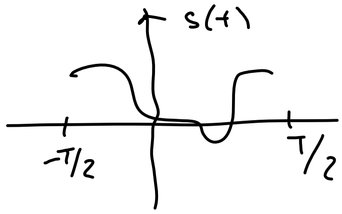
$$\int_a^b \varphi_i(t)\varphi_j^*(t)\,dt = \varepsilon_j \delta_{ij}$$

$$\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & \text{altrove} \end{cases}$$

$$c_{io} = \frac{\int_a^b s(t)\varphi_i^*(t)}{\int_a^b |\varphi_i(t)|^2 dt} \qquad i = 1, \dots, N$$

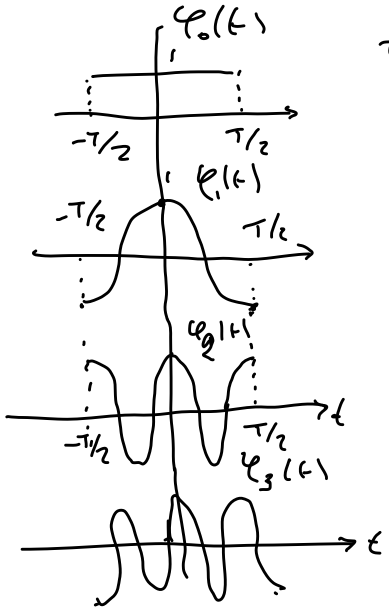
Funzioni di base : sinusoidali





$$\varphi_n(t) = \cos 2\pi \frac{n}{T} t \qquad n = 0, 1, 2, \dots$$

$$t \in \left[-\frac{T}{2}, \frac{T}{2}\right]$$



$n = 3 \quad \cos 2\pi \frac{3}{T} t$

$\cos 2\pi \frac{3}{\cancel{T}} \frac{\cancel{T}}{2}$

$$\cos\alpha \cos\beta = \frac{1}{2}\left[\cos(\alpha+\beta) + \cos(\alpha-\beta)\right]$$

$$\overset{n \neq m}{\int \varphi_n \varphi_m^*} = \int^{T/2} \cos 2\pi \frac{n}{T} t \cos 2\pi \frac{m}{T} t \, dt$$

$$\int \varphi_n \varphi_m^* = \int_{-T/2} \cos 2\pi \frac{n}{T} t \cos 2\pi \frac{m}{T} t \, dt$$

$$= \int_{-T/2}^{T/2} \frac{1}{2} \cos 2\pi \frac{(n+m)}{T} t \, dt + \int_{-T/2}^{T/2} \frac{1}{2} \cos 2\pi \frac{(n-m)}{T} t \, dt$$

$$= \frac{1}{2T} \left[\frac{\sin 2\pi \frac{(n+m)}{T} t}{\frac{2\pi (n+m)}{T}} \right]_{-T/2}^{T/2} + \frac{1}{2T} \left[\frac{\sin 2\pi \frac{(n-m)}{T} t}{\frac{2\pi (n-m)}{T}} \right]_{-T/2}^{T/2}$$

$$= \frac{1}{2T} \frac{\overset{0}{\cancel{\sin \pi (n+m)}} - \overset{0}{\cancel{\sin(-\pi(n+m))}}}{—} + \frac{1}{2T} \frac{\overset{0}{\cancel{\sin \pi (n-m)}} - \overset{0}{\cancel{\sin(-\pi(n-m))}}}{—} = 0$$

$$S_N(t) = \sum_{n=0}^{N-1} a_n \cos 2\pi \frac{n}{T} t \qquad a_0 = \frac{\int_{-T/2}^{T/2} s(t)\,dt}{\int_{-T/2}^{T/2} dt} = \frac{1}{T}\int_{-T/2}^{T/2} s(t)\,dt$$

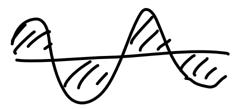
$$= a_0 +$$

$n > 0$

$$a_n = \frac{\int_{-T/2}^{T/2} s(t) \cos 2\pi \frac{n}{T} t \, dt}{\int_{-T/2}^{T/2} \cos^2 2\pi \frac{n}{T} t \, dt}$$

$$\int_{-T/2}^{T/2} \cos^2 2\pi \frac{n}{T} t \, dt = \int_{-T/2}^{T/2} \frac{1}{2} dt + \overset{0}{\cancel{\int_{-T/2}^{T/2} \frac{1}{2} \cos 4\pi \frac{n}{T} t \, dt}}$$

$$= \frac{T}{2}$$



$$\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$$

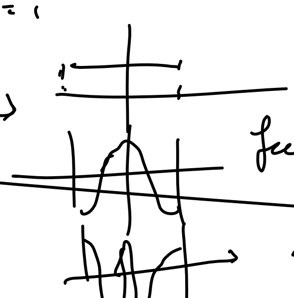
$$s(t) \leftarrow S_N(t) = a_0 + \sum_{n=1}^{N-1} a_n \cos 2\pi \frac{n}{T} t$$

$N \to \infty$

$$a_0 = \frac{1}{T} \int_{-T/2}^{T/2} s(t)\,dt$$

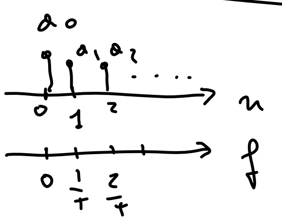
$n > 0$

$$a_n = \frac{2}{T} \int_{-T/2}^{T/2} s(t) \cos \frac{2\pi}{T} n t \, dt$$

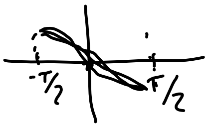


freq. $\frac{1}{T}$

freq $\frac{2}{T}$ freq. armonica $\frac{n}{T}$



SPETTRO



$S_p(t) = S_p(-t)$

$s(t) = S_p(t) + S_d(t)$ ... $s(t) + s(-t)$

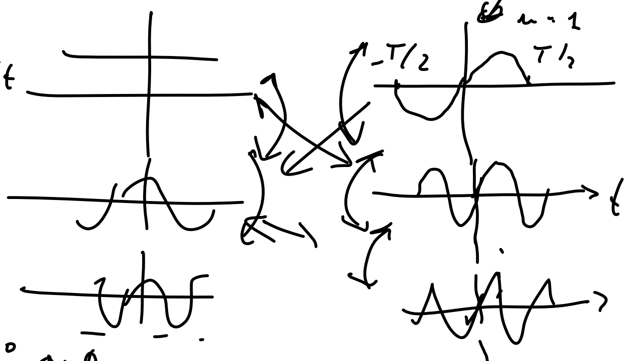
$$s(t) = \underbrace{s_p(t)} + \underbrace{s_d(t)}$$

$$\begin{cases} s_p(t) = \dfrac{s(t)+s(-t)}{2} & s_p(t) = s_p(-t) \\ s_d(t) = \dfrac{s(t)-s(-t)}{2} & s_d(t) = -s_d(-t) \end{cases}$$

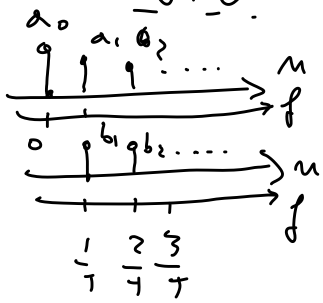
$$S_N(t) = a_0 + \sum_{n=1}^{N-1} \left(a_n \cos \frac{2\pi}{T} n t + b_n \sin \frac{2\pi}{T} n t \right)$$

$n=1$

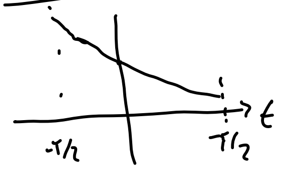
$$b_n = \frac{2}{T} \int_{-T/2}^{T/2} s(t) \sin \frac{2\pi}{T} n t \, dt$$



$$s(t) \to s_N(t)$$



Esempio: $e^{-\alpha t}$



$$S_N(t) = a_0 + \sum_{n=1}^{N-1} a_n \cos \frac{2\pi}{T} n t + b_n \sin \frac{2\pi}{T} n t$$

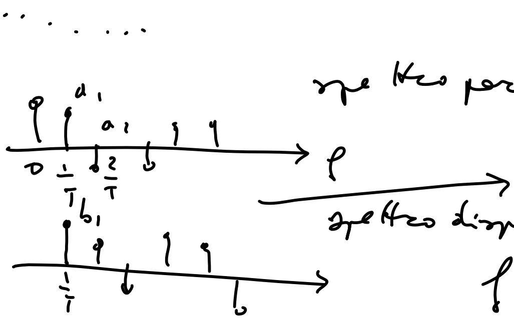
$$a_0 = \frac{1}{T} \int_{-T/2}^{T/2} e^{-\alpha t} dt = \frac{1}{T} \left. \frac{e^{-\alpha t}}{-\alpha} \right]_{-T/2}^{T/2} = \frac{1}{T} \frac{e^{-\alpha T/2} - e^{+\alpha T/2}}{-\alpha}$$

$$= \frac{e^{\alpha T/2} - e^{-\alpha T/2}}{\alpha T}$$

$n > 0$

$$a_n = \frac{1}{T} \int_{-T/2}^{T/2} e^{-\alpha t} \cos 2\pi \frac{n}{T} t \, dt = \frac{1}{T} \left. \frac{e^{-\alpha t}}{-\alpha} \cos \frac{2\pi n}{T} t \right|_{-T/2}^{T/2} - \int \frac{e^{-\alpha t}}{-\alpha} \frac{\sin \frac{2\pi n}{T} t}{\frac{2\pi n}{T}} dt$$

. . . .



spettro pari

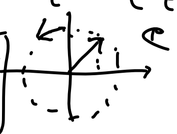
spettro dispari

RAPPR. ARMONICA

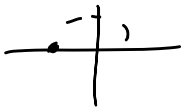
f. reali { cos, sin

$\varphi_n(t) = e^{j 2\pi \frac{n}{T} t}$ $t \in [-T/2, T/2]$

$\boxed{= \cos 2\pi \frac{n}{T} t + j \sin 2\pi \frac{n}{T} t}$



$$\int_{-T/2}^{T/2} \varphi_n(t)\,\varphi_m^*(t)\,dt = \int_{-T/2}^{T/2} e^{j 2\pi \frac{n}{T} t}\, e^{-j \frac{2\pi m}{T} t}\, dt \qquad n \neq m$$



$$= \int_{-T/2}^{T/2} e^{j 2\pi \frac{(n-m)}{T} t}\, dt$$

$$= \left. \frac{e^{j 2\pi \frac{(n-m)}{T} t}}{2\pi \frac{n-m}{T}} \right]_{-T/2}^{T/2}$$

$$= \frac{e^{j \cancel{2}\pi \frac{(n-m)}{\cancel{T}} \frac{\cancel{T}}{\cancel{2}}} - e^{-j 2\pi \frac{(n-m)}{\cancel{T}} \frac{\cancel{T}}{\cancel{2}}}}{2\pi \frac{(n-m)}{T}} = \frac{e^{j\pi(n-m)} - e^{-j\pi(n-m)}}{—} = \cancel{(-1)^{n-m} - (-1)^{n-m}} = 0$$

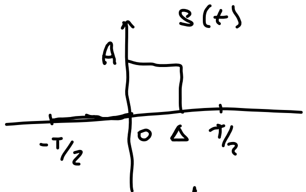
$$\int_{-T/2}^{T/2} |\varphi_n(t)|^2\, dt = \int_{-T/2}^{T/2} 1 \cdot dt = T$$

$\varphi_n(t) = e^{j 2\pi \frac{n}{T} t}$ $\int_{-T/2}^{T/2} |\varphi_n(t)|^2\, dt = T$

$$\boxed{S_N(t) = \sum_{n=-M}^{+M} c_n\, e^{j 2\pi \frac{n}{T} t}} \qquad \boxed{c_n = \frac{1}{T} \int_{-T/2}^{T/2} s(t)\, e^{-j \frac{2\pi n}{T} t}\, dt}$$

$M \to \infty$ serie di Fourier esponenziale.

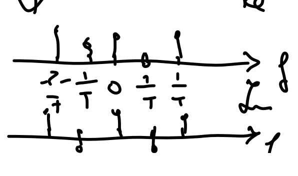
$\cos 2\pi \frac{n}{T} t + j \sin 2\pi \frac{n}{T} t$



$$S_N(t) = \sum_{n=-M}^{M} c_n\, e^{j 2\pi \frac{n}{T} t}$$

$$c_n = \frac{1}{T} \int_{-T/2}^{T/2} s(t)\, e^{-j 2\pi \frac{n}{T} t}\, dt$$

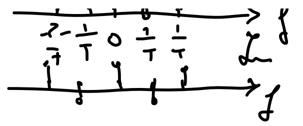
$$c_n = \frac{1}{T} \int_0^{\Delta} A\, e^{-j 2\pi \frac{n}{T} t}\, dt = \frac{A}{T} \left. \frac{e^{-j 2\pi \frac{n}{T} t}}{-j 2\pi \frac{n}{T}} \right]_0^{\Delta} = \frac{A}{\cancel{T}} \, \frac{e^{-j \frac{2\pi n}{T} \Delta} - 1}{-j 2\pi \frac{n}{\cancel{T}}}$$



$Re[c_n] =$

$Im[c_n] =$

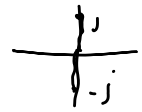
$j = e^{j\frac{\pi}{2}}$ e



$\mathrm{Re}[c_n] =$

$\mathrm{Im}[c_n] =$

$j = e^{j\frac{\pi}{2}}$ $\mathbb{C}$



$-j = e^{-j\pi/2} = \frac{1}{j}$

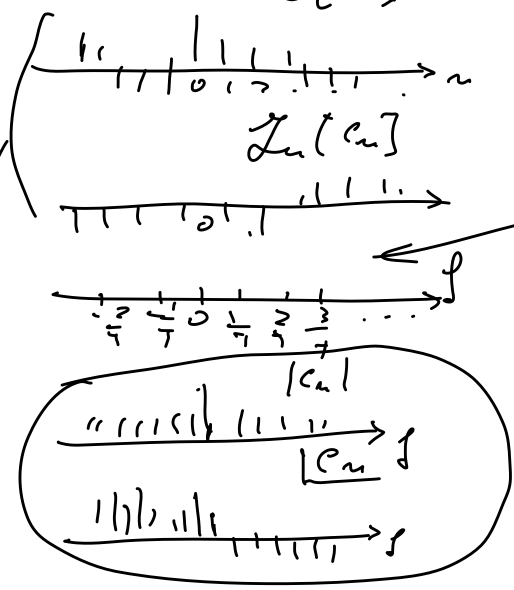
$$= \frac{A}{2\pi n} e^{+j\pi/2}\left(e^{-j\frac{2\pi n}{T}\Delta} - 1\right)$$

$$= \frac{A}{2\pi n}\, j \left(\cos 2\pi\frac{n}{T}\Delta - j\sin 2\pi\frac{n}{T}\Delta - 1\right)$$

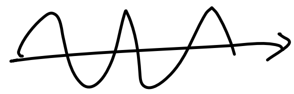
$$= \frac{A}{2\pi n}\left(j\cos 2\pi\frac{n}{T}\Delta + \sin 2\pi\frac{n}{T}\Delta - j\right)$$

$$= \underbrace{\frac{A}{2\pi n}\sin 2\pi\frac{n}{T}\Delta}_{\mathrm{Re}(c_n)} + j\underbrace{\frac{A}{2\pi n}\left(\cos 2\pi\frac{n}{T}\Delta - 1\right)}_{\mathrm{Im}(c_n)}$$

$S(t) \rightarrow$

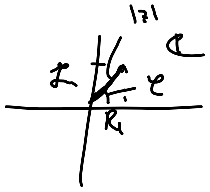


frequenze armoniche



$z = x + jy$

$= |z|\, e^{j\angle z}$



$N = 2M+1$

$$S_N(t) = \sum_{n=-M}^{M} c_n e^{j2\pi\frac{n}{T}t} \xrightarrow[M\to\infty]{} S(t) = \sum_{n=-\infty}^{+\infty} c_n e^{j2\pi\frac{n}{T}t}$$

SERIE DI FOURIER

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} s(t)\, e^{-j2\pi\frac{n}{T}t}\, dt \qquad S_N(t) \to S(t)$$

$$\int_{-T/2}^{T/2} |S_N(t) - S(t)|^2 dt \to 0 \qquad \text{MQ}$$

$s(t)$ reale $\longrightarrow$ $\boxed{c_n = c_{-n}^*}$ Hermitiana

$a + jb$

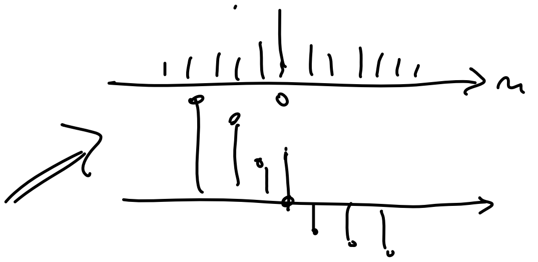
$a - jb$

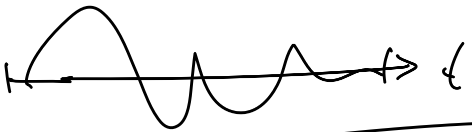
$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} s(t)\cos 2\pi\frac{n}{T}t\, dt - j\frac{1}{T}\int_{-T/2}^{T/2} s(t)\sin 2\pi\frac{n}{T}t\, dt$$

$$c_{-n} = \frac{1}{T}\int_{-T/2}^{T/2} s(t)\cos \frac{2\pi}{T}n t\, dt + j\frac{1}{T}\int_{-T/2}^{T/2} s(t)\sin 2\pi\frac{n}{T}t\, dt = c_n^*$$

$$\Big\{\ |c_n| = |c_{-n}| \quad \text{pari}$$

$$\begin{cases} |c_n| = |c_{-n}| & \text{pari} \\ \angle c_n = -\angle c_{-n} & \text{dispari} \end{cases}$$





$s(t) = s(-t)$ pari e reale

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} s(t)\cos 2\pi \frac{n}{T} t \, dt - j\frac{1}{T}\cancel{\int_{-T/2}^{T/2} s(t)\sin 2\pi \frac{n}{T} t \, dt} = 0$$

$\to$ c_n reali.

$s(t) = -s(-t)$ dispari $\longrightarrow$ c_n immaginari

$$\mathcal{E}_s = \int_{-T/2}^{T/2} |s(t)|^2 dt \qquad s(t) = \sum_{n=-\infty}^{+\infty} c_n e^{j 2\pi \frac{n}{T} t}$$

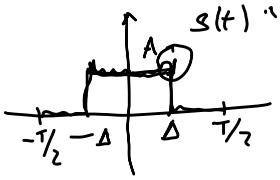
$$\int_{-T/2}^{T/2} \sum_{n_1=-\infty}^{+\infty} c_{n_1} e^{j 2\pi \frac{n_1}{T} t} \sum_{n_2=-\infty}^{+\infty} c^*_{n_2} e^{-j \frac{2\pi n_2}{T} t} \, dt$$

$$= \sum_{n_1}\sum_{n_2} c_{n_1} c^*_{n_2} \int_{-T/2}^{T/2} \cancel{e^{j\frac{2\pi}{T}(n_1-n_2)t} dt} = 0 \quad n_1 \neq n_2$$

$$= \sum_n c_n c_n^* T = T \sum_{n=-\infty}^{+\infty} |c_n|^2$$

$s_N(t) \to s(t)$

$\uparrow A$ $s(t)$: Gibbs. Δ $j 2\pi \frac{n}{T} t$ $j 2\pi n t$ T^Δ



~~Gi...~~

$$c_n = \frac{A}{T}\int_{-\Delta}^{\Delta} e^{-j2\pi\frac{n}{T}t}\,dt = \frac{A}{\cancel{T}}\left[\frac{e^{-j2\pi\frac{n}{T}t}}{-j2\pi\frac{n}{\cancel{T}}}\right]_{-\Delta}^{\Delta}$$

$n \neq 0$

$\{c_n\}$ Hermitiana.

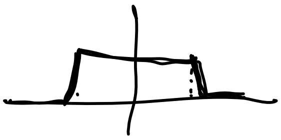
Reale.

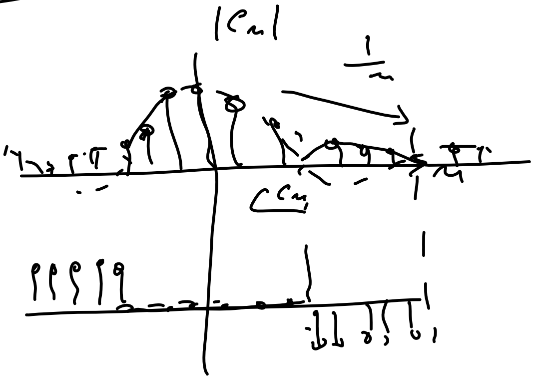
$$= \frac{A}{j2\pi n}\left(e^{j2\pi\frac{n\Delta}{T}} - e^{-j2\pi\frac{n\Delta}{T}}\right)$$

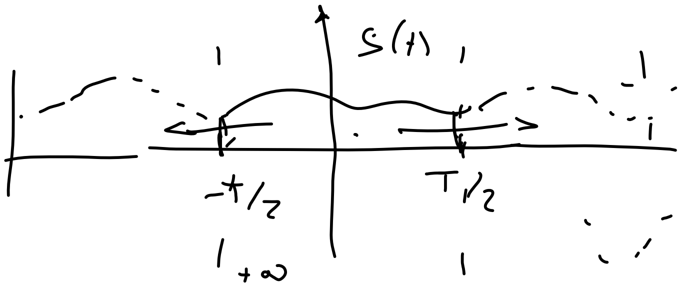
$$\cos x = \frac{e^{jx}+e^{-jx}}{2} \qquad \sin x = \frac{e^{jx}-e^{-jx}}{2j}$$

$$c_n = c_{-n}^*$$

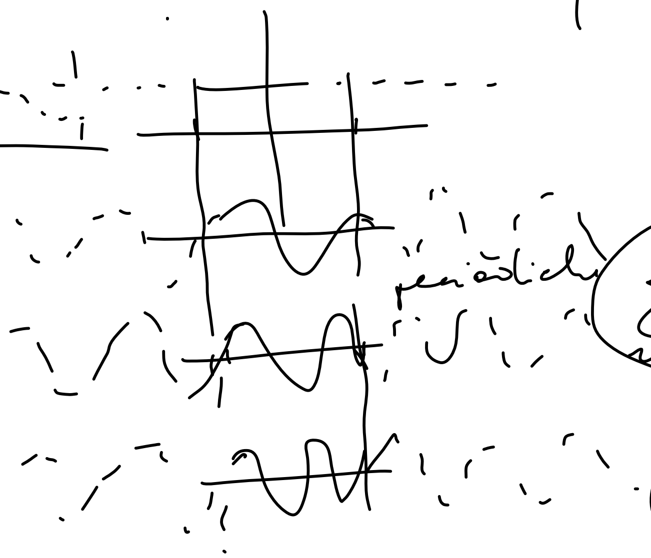
$$= \frac{A}{\pi n}\sin\frac{2\pi n\Delta}{T}$$







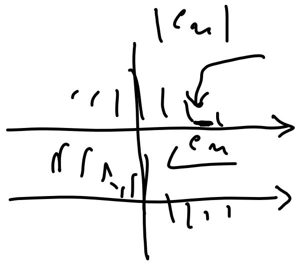
$$S_p(t) = \sum_{k=-\infty}^{+\infty} s(t-kT)$$



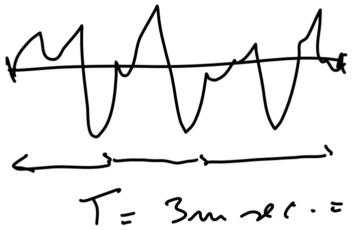
periodiche

$$\sum_{n=-\infty}^{\infty} c_n e^{j\frac{2\pi n}{T}t}$$

$$\frac{n}{T}$$



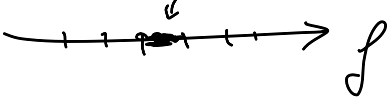
$T = 1\,\mu sec. = 10^{-6}$ $\quad \frac{1}{T} = 10^6$ 1MHz.



$$\frac{1}{T} = \frac{1}{3}10^3 = 0.333\ \text{KHz}$$

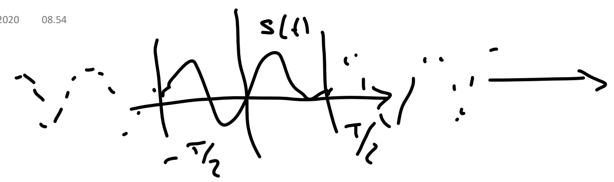
$$\frac{K}{T} = K \cdot 0.333\ \text{KHz}.$$

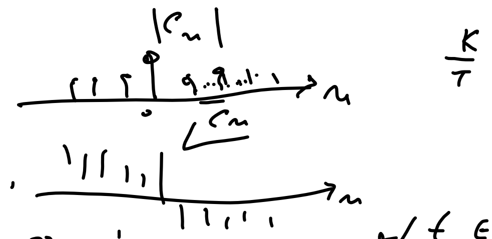
$T = 3$msec. $= 3\cdot 10^{-3}$ sec.



→ si TF

$$\sum c_n e^{j2\pi\frac{n}{T}t} \rightarrow \int$$





$$e^{j2\pi\frac{n}{T}t} = \cos 2\pi\frac{n}{T}t + j\sin 2\pi\frac{n}{T}t$$

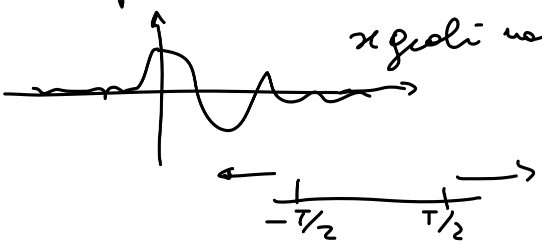
$$s(t) = \sum_{n=-\infty}^{+\infty} c_n e^{j2\pi\frac{n}{T}t} \qquad \forall t \in \left[-\frac{T}{2}, \frac{T}{2}\right] \qquad \forall t$$

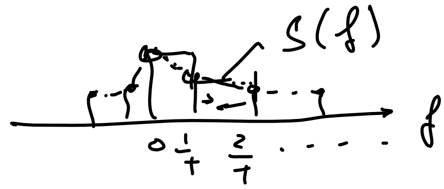
$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} s(t)e^{-j2\pi\frac{n}{T}t}\,dt$$

funzione limitata a $\left[-\frac{T}{2}, \frac{T}{2}\right]$

funzione periodica

segnali non limitati





$$c_n = \frac{1}{T}S\left(\frac{n}{T}\right) = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} s(t)e^{-j2\pi\frac{n}{T}t}dt.$$

Quindi il segnale $s(t)$ può essere riscritto come

$$s(t) = \sum_{n=-\infty}^{\infty} \frac{1}{T}S\left(\frac{n}{T}\right)e^{j2\pi\frac{n}{T}t}.$$

$T \to \infty \qquad \frac{1}{T} \to 0$

$$\boxed{\begin{aligned} s(t) &= \int_{-\infty}^{+\infty} S(f)\,e^{j2\pi ft}\,df \\ S(f) &= \int_{-\infty}^{+\infty} s(t)\,e^{-j2\pi ft}\,dt \end{aligned}}$$

INTEGRALE DI FOURIER

TRASFORMATA DI FOURIER

$s(t)$ — t

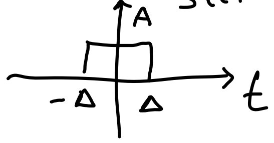
DOMINIO DEL TEMPO (continuo)

$\longleftrightarrow$

$S(f)$ — f

DOMINIO DELLA FREQUENZA

Esempio

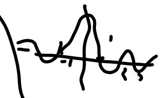


TDF

$$S(f) = A\int_{-\Delta}^{\Delta} e^{-j2\pi ft}\,dt = A\left.\frac{e^{-j2\pi ft}}{-j2\pi f}\right|_{-\Delta}^{\Delta} = A\,\frac{e^{-j2\pi f\Delta} - e^{j2\pi f\Delta}}{-j2\pi f}$$

$$\sin x = \frac{e^{jx} - e^{-jx}}{2j}$$

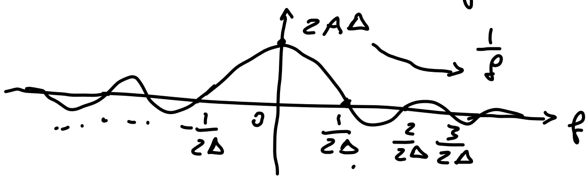
$$\mathrm{sinc}\, x = \frac{\sin \pi x}{\pi x}$$



$$\ldots \frac{\ldots}{-j2\pi f}\Big|_{-\Delta} = \ldots \frac{\ldots}{-j2\pi f}$$

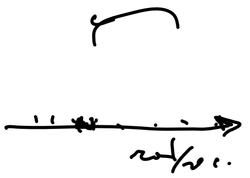
$$= A\,\frac{e^{j2\pi f\Delta} - e^{-j2\pi f\Delta}}{2j\pi f} = 2A\Delta\,\frac{\sin 2\pi\Delta f}{2\Delta\pi f} = 2A\Delta\,\mathrm{sinc}\,2\Delta f$$

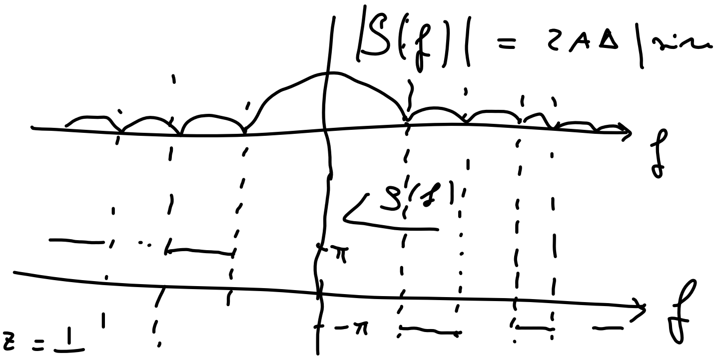
$2\Delta f = 1$



$Re(S(f))$ $\quad Im(S(f)) = 0$

$|S(f)| = 2A\Delta\,|\mathrm{sinc}\,2\Delta f|$





$x = |x|e^{j\angle x}$

-3

$|3|$

$-1 = e^{j\pi}$

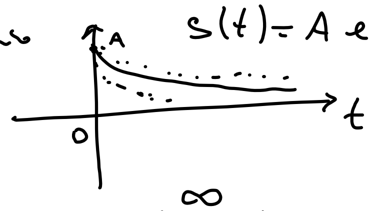
rad/sec.

$\downarrow$ $\quad$ $Hz = \frac{1}{sec}$

$\omega = 2\pi f$

$f = \frac{\omega}{2\pi}$

Esempio $\quad s(t) = A e^{-\alpha t} u(t)$



$x = x_R + j x_I$

$|x| = \sqrt{x_R^2 + x_I^2}$

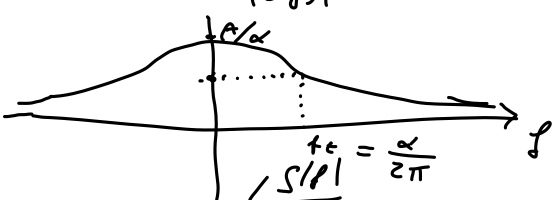
$\angle x = tg^{-1}\frac{x_I}{x_R}$

$$S(f) = A\int_0^\infty e^{-\alpha t} e^{-j2\pi f t}\,dt = A\int_0^\infty e^{-(\alpha + j2\pi f)t}\,dt$$

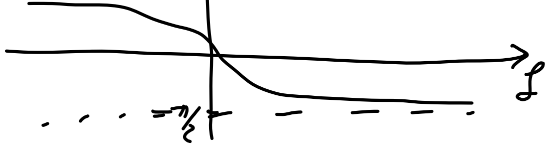
$$= A\,\frac{e^{-(\alpha+j2\pi f)t}}{-(\alpha+j2\pi f)}\Bigg]_0^\infty = A\,\frac{-1}{-(\alpha+j2\pi f)} = \frac{A}{\alpha + j2\pi f}$$

$$\begin{cases} |S(f)| = \dfrac{A}{|\alpha + j2\pi f|} = \dfrac{A}{\sqrt{\alpha^2 + 4\pi^2 f^2}} \\ \angle S(f) = -tg^{-1}\dfrac{2\pi f}{\alpha} \end{cases}$$

$|S(f)|$



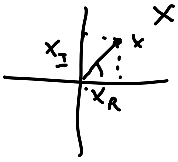
$\angle S(f)$



$$\to \frac{A}{\alpha}\,\frac{1}{\sqrt{1 + \frac{4\pi^2 f^2}{\alpha^2}}} = \frac{A}{\alpha}\,\frac{1}{\sqrt{1 + \left(\frac{2\pi f}{\alpha}\right)^2}}$$

$$= \frac{A}{\alpha}\,\frac{1}{\sqrt{1 + (f\ldots)^2}}$$

$$= \frac{\Gamma}{\alpha} \frac{1}{\sqrt{1+\left(\frac{f}{f_t}\right)^2}}$$



$x = x_R + j x_I \qquad |x| = \sqrt{x_R^2 + x_I^2} \qquad \boxed{\angle x = tg^{-1} \frac{x_I}{x_R}} \qquad x^* = x_R - j x_I = |x| e^{-\angle x}$

$= \rho e^{j\theta} = |x| e^{j\angle x} \qquad x_R = |x| \cos \angle x \qquad x_I = |x| \sin \angle x$

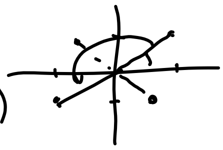
$z = x\, y \qquad |z| = |x|\,|y| \qquad \angle z = \angle x + \angle y \qquad \boxed{e^{jx} = \cos x + j \sin x}$

$|x| e^{j\angle x} |y| e^{j\angle y}$

$z = \frac{x}{y} = \frac{|x| e^{j\angle x}}{|y| e^{j\angle y}} \qquad |z| = \frac{|x|}{|y|} \qquad \angle z = \angle x - \angle y$

$\cos x = \frac{e^{jx} + e^{-jx}}{2} \qquad \sin x = \frac{e^{jx} - e^{-jx}}{2j}$

$\angle x = tg^{-1} \frac{x_I}{x_R} = \text{atan2}(x_R, x_I)$ (*) WARNING



PROPRIETA' DELLA TRASFORMATA DI FOURIER

$$s(t) \longleftrightarrow S(f)$$

$$s(t) = \int_{-\infty}^{+\infty} S(f) e^{j2\pi f t} df \qquad S(f) = \int_{-\infty}^{+\infty} s(t) e^{-j2\pi f t} dt$$

LINEARITA'

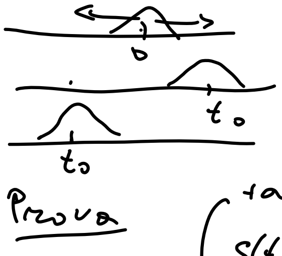
$$c_1 s_1(t) + c_2 s_2(t) \longleftrightarrow c_1 S_1(f) + c_2 S_2(f)$$

SCALA

$$s(t) \qquad s(\alpha t) \longleftrightarrow \frac{1}{|\alpha|} S\left(\frac{f}{\alpha}\right)$$

TRASLAZIONE TEMPORALE

$$s(t - t_0) \longleftrightarrow e^{-j2\pi f t_0} S(f)$$



Prova

$$\xi = t - t_0$$

$$\int_{-\infty}^{+\infty} s(t-t_0) e^{-j2\pi f t} dt = \int_{-\infty}^{+\infty} s(\xi)\, e^{-j2\pi f(\xi + t_0)} d\xi$$

$$= e^{-j2\pi f t_0} \int_{-\infty}^{+\infty} s(\xi) e^{-j2\pi f \xi} d\xi, \dots$$

$-\infty$

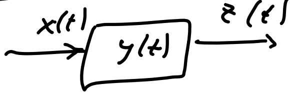
$$= e^{-j2\pi f t_0} \underbrace{\int_{-\infty}^{+\infty} s(\xi)\, e^{-j2\pi f \xi}\, d\xi}_{S(f)}$$

TRASLAZIONE IN FREQUENZA
(O MODULAZIONE)

$$e^{j2\pi f_0 t} s(t) \longleftrightarrow S(f-f_0)$$

$$\int_{-\infty}^{+\infty} e^{j2\pi f_0 t} s(t)\, e^{-j2\pi f t}\, dt = \int_{-\infty}^{+\infty} s(t)\, e^{-j2\pi (f-f_0) t}\, dt$$

CONVOLUZIONE



$$z(t) = (x*y)(t) = \int_{-\infty}^{+\infty} x(\tau)\, y(t-\tau)\, d\tau \iff Z(f) = X(f)\, Y(f)$$

Prova:

$$Z(f) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} x(\tau) y(t-\tau) d\tau \; e^{-j2\pi f t} dt$$

$$= \int_{-\infty}^{+\infty} x(\tau) \left(\int_{-\infty}^{+\infty} y(t-\tau) e^{-j2\pi f t} dt \right) d\tau \qquad \underbrace{\int_{-\infty}^{+\infty} x(\tau)\, e^{-j2\pi f \tau}}_{X(f)} Y(f)\, d\tau = X(f) Y(f)$$

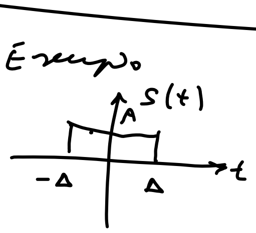
PRODOTTO

$$z(t) = x(t)\, y(t) \longleftrightarrow Z(f) = (X*Y)(f)$$

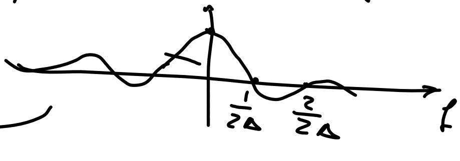
DUALITA'

$$s(t) \leftrightarrow S(f) \qquad \mathcal{F}[S(t)] = s(-f)$$

Esempio



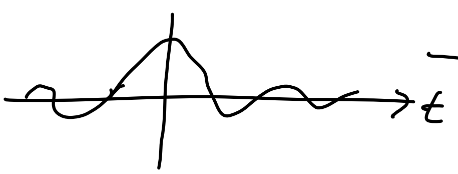
$\longrightarrow$ $S(f) = A 2\Delta \,\mathrm{sinc}\, 2\Delta f$



DT

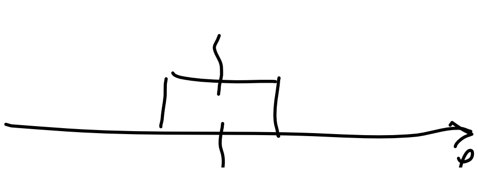
DF

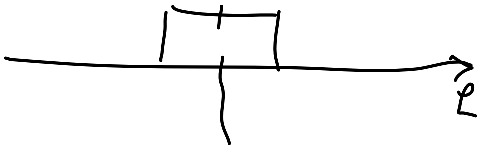
$\hookrightarrow S(t)$



$\longrightarrow$

s





<u>DERIVATA</u>

$$\frac{d^n s(t)}{dt^n} \longrightarrow (j2\pi f)^n S(f)$$

$$\mathcal{E}_s = \int_{-\infty}^{+\infty} |s(t)|^2 dt = \int_{-\infty}^{+\infty} |S(f)|^2 df$$

$s(t)$ reale $\Longrightarrow$ $S(f)$ Hermitiana $\left(S(f) = S^*(-f) \right)$

$$S(f) = \int_{-\infty}^{+\infty} s(t)\, e^{-j2\pi f t} dt = \int_{-\infty}^{+\infty} s(t) \cos 2\pi f t \, dt - j \int_{-\infty}^{+\infty} s(t) \sin 2\pi f t \, dt$$

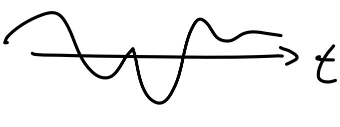
$$S^*(f) = \int_{-\infty}^{+\infty} s(t) \cos 2\pi f t \, dt + j \int_{-\infty}^{+\infty} s(t) \sin 2\pi f t \, dt$$

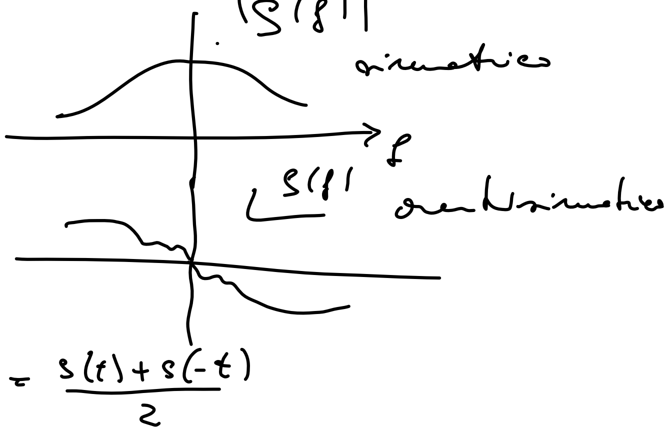
$$S(f) = S^*(-f)$$

$$|S(f)| = |S(-f)| \quad \text{modulo è pari}$$

$$\angle S(f) = -\angle S(-f)$$

$s(t)$ reale





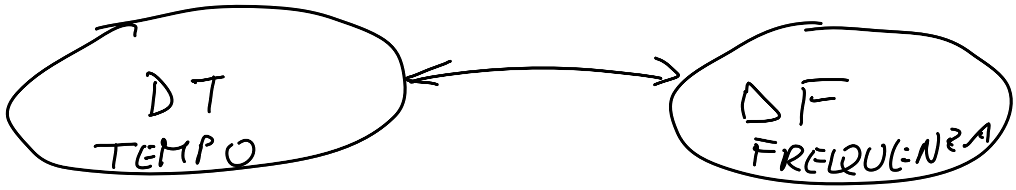
$$s(t) = s_p(t) + s_d(t) \qquad s_p(t) = \frac{s(t) + s(-t)}{2}$$

$$s_p(t) \longrightarrow \text{puramente reale} \qquad s_d(t) = \frac{s(t) - s(-t)}{2}$$

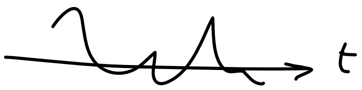
$S_p(t) \longrightarrow$ puramente reale

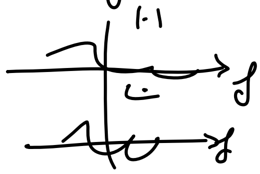
$S_d(t) \longrightarrow$ puramente immaginario

$$S_d(t) = \frac{s(t) - s(-t)}{2}$$



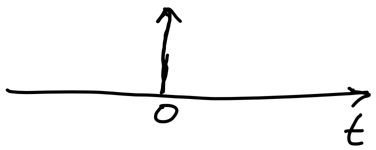
$s(t)$ $\qquad\qquad$ $S(f)$

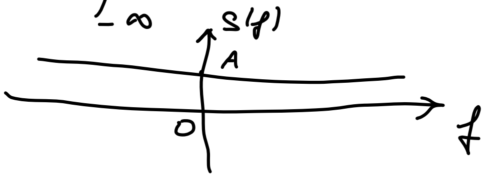




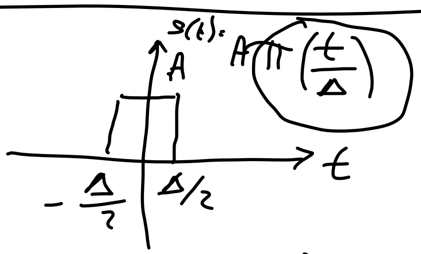
SEGNALI CANONICI

$$s(t) = A\,\delta(t) \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad S(f) = \int_{-\infty}^{+\infty} A\,\delta(t)\, e^{-j2\pi f t}\, dt = A$$





$$s(t) = A\,\Pi\!\left(\frac{t}{\Delta}\right) \longleftrightarrow S(f) = A\int_{-\Delta/2}^{\Delta/2} e^{-j2\pi f t}\, dt = A \left[\frac{e^{-j2\pi f t}}{-j2\pi f}\right]_{-\Delta/2}^{\Delta/2}$$

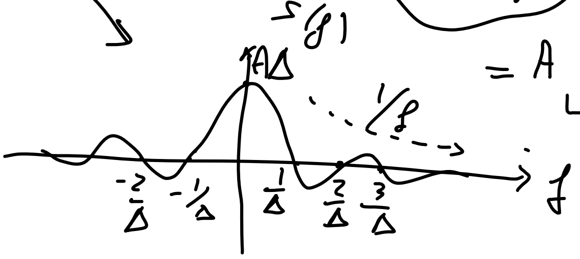


$$= A\,\frac{e^{-j2\pi f\frac{\Delta}{2}} - e^{j2\pi f\frac{\Delta}{2}}}{-j2\pi f} = A\Delta\,\frac{\sin \pi\Delta f}{\pi f \Delta}$$

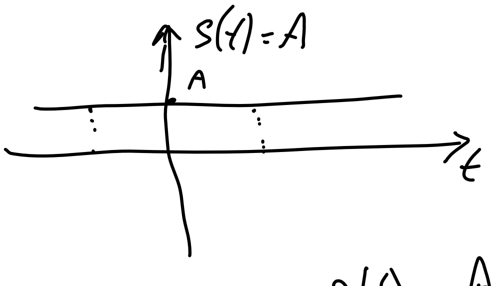
$$\operatorname{sinc} x = \frac{\sin \pi x}{\pi x}$$

$$= A\,\Delta\, \underbrace{\operatorname{sinc} \Delta f}$$

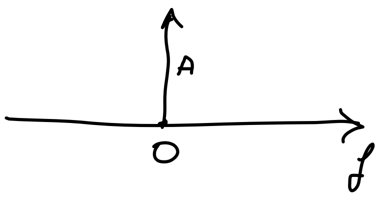
$$\Delta f = K$$



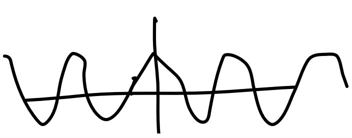
$s(t) = A$



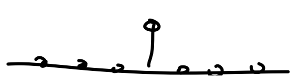
$$S(f) = A\,\delta(f)$$

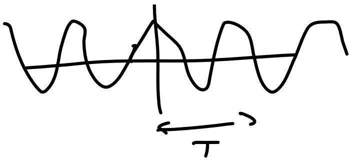


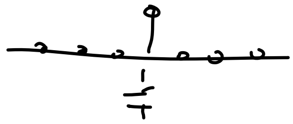
$$s(t) = A\int_{-\infty}^{+\infty} \underset{\delta(f)}{S(f)}\, e^{j2\pi f t}\, df$$



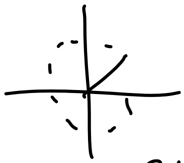
$\longrightarrow$ serie di Fourier







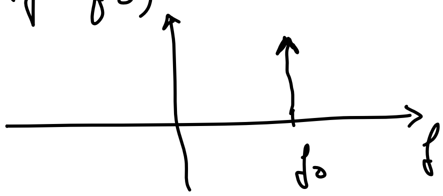
$\int |\cdot| < \infty$



$$s(t) = e^{j2\pi f_0 t} = \cos 2\pi f_0 t + j \sin 2\pi f_0 t$$

$$S(f) = \int_{-\infty}^{+\infty} e^{j2\pi f_0 t} e^{-j2\pi f t} dt = \int_{-\infty}^{+\infty} e^{-j2\pi (f-f_0)t} dt$$

$$\ldots\ldots = \delta(f - f_0)$$



$$\boxed{\begin{array}{c} \text{T. in freq.} \\ s(t) \leftrightarrow S(f) \\ e^{j2\pi f_0 t} s(t) \leftrightarrow S(f-f_0) \end{array}}$$

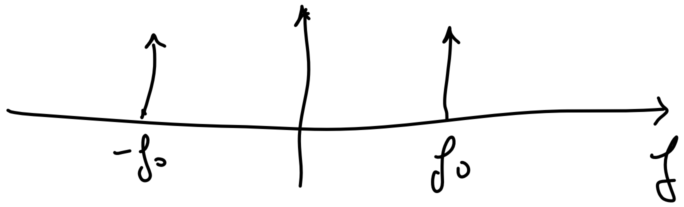
$$s(t) = A \cos 2\pi f_0 t \xleftrightarrow{\mathcal{F}} S(f)$$

$$\cos x = \frac{e^{jx} + e^{-jx}}{2}$$

$$= A \, \frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2}$$

$$= \mathcal{F}\left[\frac{A}{2} e^{j2\pi f_0 t}\right] + \mathcal{F}\left[\frac{A}{2} e^{-j2\pi f_0 t}\right]$$

$$= \frac{A}{2} \delta(f - f_0) + \frac{A}{2} \delta(f + f_0)$$



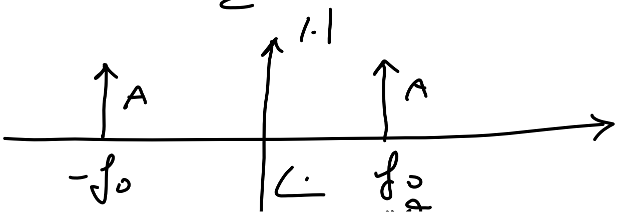
$s(t)$ reale
$\downarrow$
$S(f)$ Hermit.

$$s(t) = A \cos(2\pi f_0 t + \theta) = \frac{A}{2} e^{j(2\pi f_0 t + \theta)} + \frac{A}{2} e^{-j(2\pi f_0 t + \theta)}$$

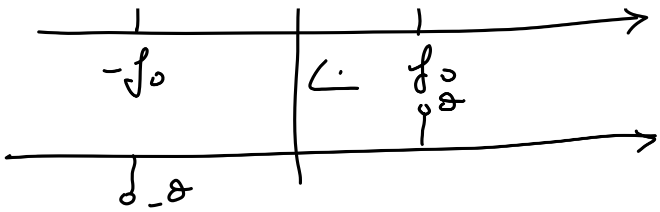
$$= \frac{A}{2} e^{j\theta} e^{j2\pi f_0 t} + \frac{A}{2} e^{-j\theta} e^{-j2\pi f_0 t}$$

$$\Updownarrow \mathcal{F}$$

$$\frac{A}{2} e^{j\theta} \delta(f - f_0) + \frac{A}{2} e^{-j\theta} \delta(f + f_0)$$



$j^x \quad -j^x$



$$\sin x = \frac{e^{jx} - e^{-jx}}{2j}$$

$$s(t) = A \sin(2\pi f_0 t + \theta) = \frac{A}{2j} e^{j(2\pi f_0 t + \theta)} - \frac{A}{2j} e^{-j(2\pi f_0 t + \theta)}$$

$$\Updownarrow \mathcal{F}$$

$$S(f) = \frac{A e^{j\theta}}{2j} \delta(f - f_0) - \frac{A e^{-j\theta}}{2j} \delta(f + f_0)$$

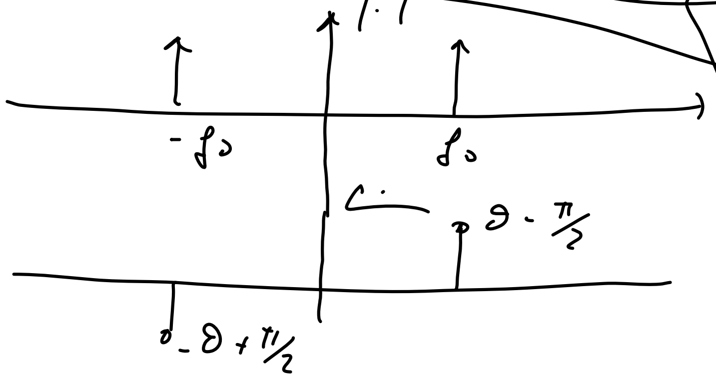
$$\frac{1}{j} = -j \qquad = -\frac{\pi}{2}$$

$|\cdot|$

$\angle\ \theta - \frac{\pi}{2}$

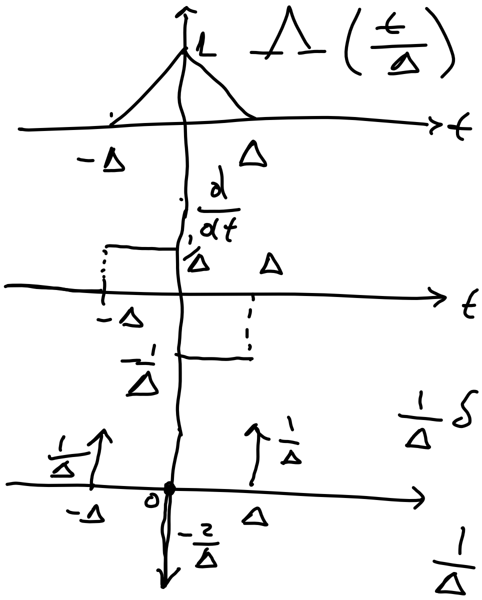
$-\frac{1}{j} = j$

$-\theta + \frac{\pi}{2}$



$$\frac{d^2}{dt^2} \rightarrow (j2\pi f)^2$$

$$\int \rightarrow \frac{1}{j2\pi f}$$



$$S(f) = \frac{1}{(j2\pi f)^2} \frac{\left(e^{j\pi\Delta f} - e^{-j\pi f\Delta}\right)^2}{\Delta}$$

$$\frac{1}{(j2\pi f)^2}$$

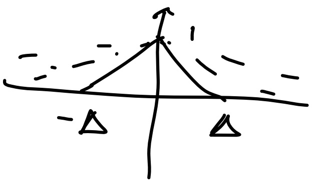
$$\frac{1}{\Delta}\delta(t+\Delta) - \frac{2}{\Delta}\delta(t) + \frac{1}{\Delta}\delta(t-\Delta)$$

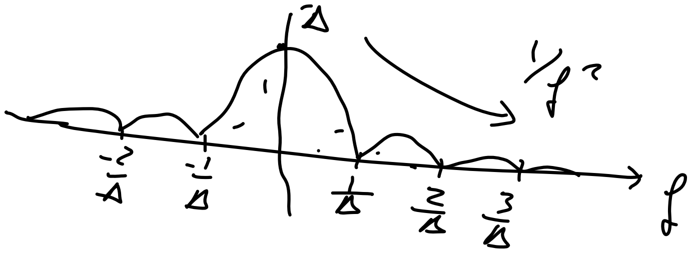
$$\Updownarrow$$

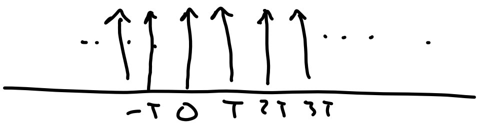
$$\frac{1}{\Delta} e^{j2\pi\Delta f} - \frac{2}{\Delta} + \frac{1}{\Delta} e^{-j2\pi\Delta f} = \frac{1}{\Delta}\left(e^{j\pi\Delta f} - e^{-j\pi\Delta f}\right)^2$$

$$S(f) = \frac{1}{\Delta}\left(\frac{e^{j\pi f\Delta} - e^{-j\pi f\Delta}}{j2\pi f}\right)^2 = \Delta \frac{\sin^2 \pi\Delta f}{\Delta^2 \pi^2 f^2} = \Delta\, \mathrm{sinc}^2 \Delta f$$

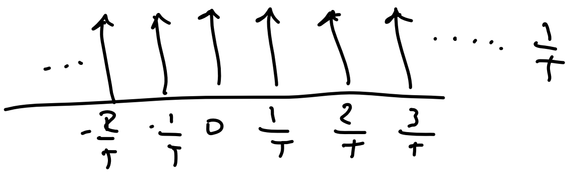
$$\Lambda\left(\frac{t}{\Delta}\right) \longleftrightarrow \Delta\, \mathrm{sinc}^2 \Delta f$$







$$\xleftrightarrow{\mathcal{F}}$$

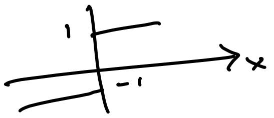


$$S(t) = \sum_{k=-\infty}^{+\infty} \delta(t - kT)$$

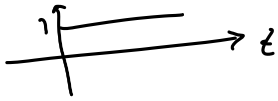
$$S(f) = \frac{1}{T} \sum_{k=-\infty}^{+\infty} \delta\left(f - \frac{k}{T}\right)$$

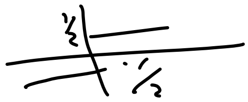
Prova: . . . .

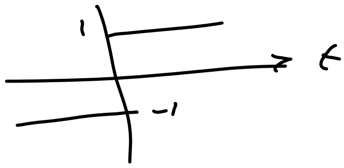
sgn x =



u(t)

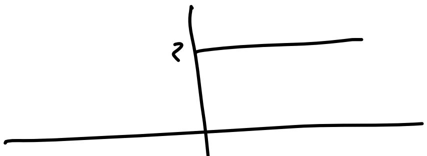




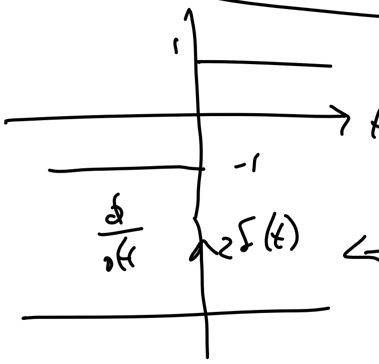


$$\operatorname{sgn} t = \left(u(t) - \frac{1}{2}\right) 2 = 2u(t) - 1$$

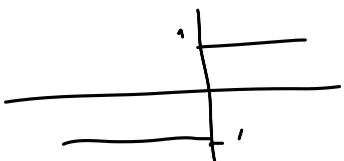
$$\boxed{\mathcal{F}[\operatorname{sgn}(t)] = \frac{1}{j\pi f}}$$



$$\frac{1}{j\pi f}$$



$$\frac{d}{dt} \quad 2\delta(t) \xleftrightarrow{\mathcal{F}} 2 \cdot \frac{1}{j2\pi f}$$



$$u(t) = \frac{\operatorname{sgn}(t) + 1}{2}$$

$$u(t) = \frac{1}{2} \operatorname{sgn}(t) + \frac{1}{2} \longleftrightarrow \frac{1}{j2\pi f} + \frac{1}{2}\delta(f)$$

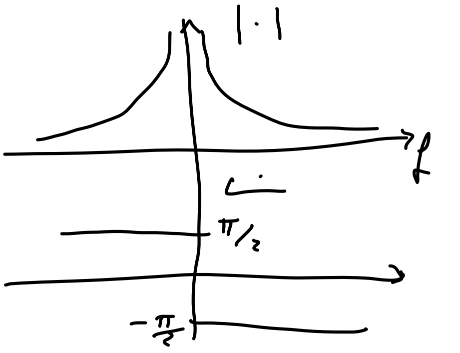
$$\operatorname{sgn}(t) \longleftrightarrow \frac{1}{j\pi f}$$

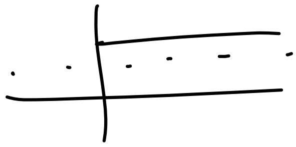
$$u(t) \longleftrightarrow \frac{1}{j2\pi f} + \frac{1}{2}\delta(f)$$

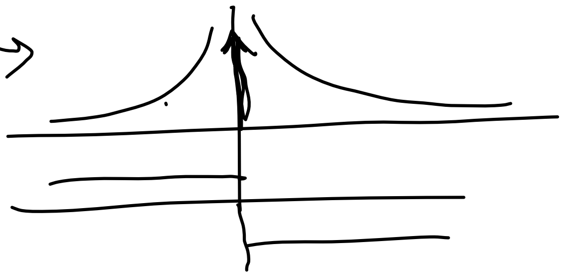
$$u(t) \longleftrightarrow \frac{1}{j2\pi f} + \frac{1}{2}\delta(f)$$

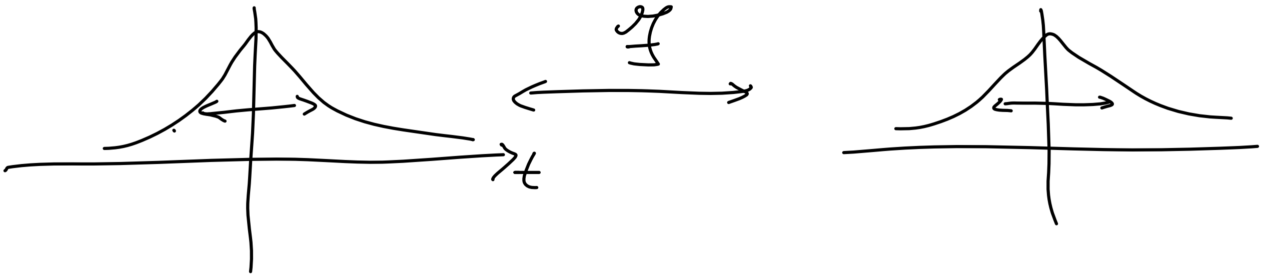
$$|\cdot| = \frac{1}{\pi |f|}$$

$$\angle \cdot = \begin{cases} f>0 & -\frac{\pi}{2} \\ f<0 & \frac{\pi}{2} \end{cases}$$





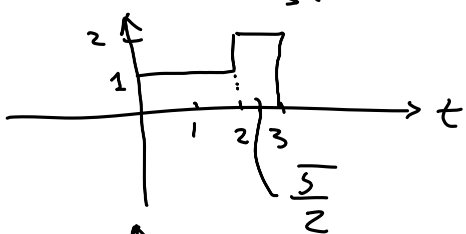




$\mathcal{F}$

Esercizi

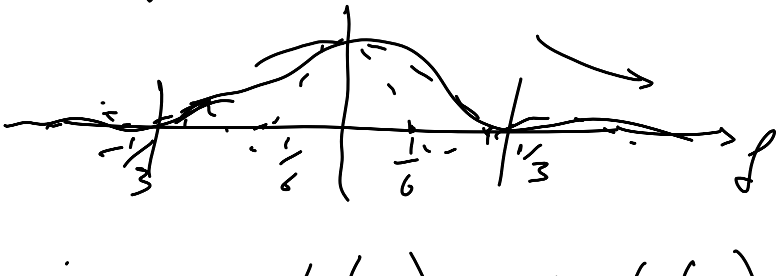
$$s(t) = \Pi\left(\frac{t-1}{2}\right) + 2\,\Pi\left(t - \frac{5}{2}\right)$$



$$S(f) = e^{-j2\pi f}\, 2\,\mathrm{sinc}\, 2f + 2\, e^{-j2\pi f \frac{5}{2}}\,\mathrm{sinc}\, f$$

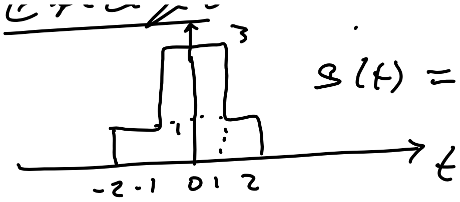
$$s(t) = \Pi\left(\frac{t}{6}\right) + \Lambda\left(\frac{t}{3}\right)$$

$$S(f) = 6\,\mathrm{sinc}\, 6f + 3\,\mathrm{sinc}^2 3f$$



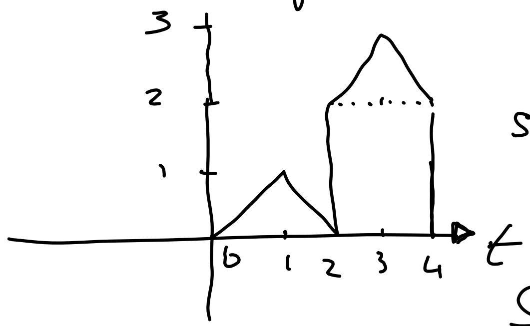
Esercizio

$$s(t) = \Pi\left(\frac{t}{\ }\right) + 2\Pi\left(\frac{t}{\ }\right)$$



$$s(t) = \Pi\left(\frac{t}{4}\right) + 2\Pi\left(\frac{t}{2}\right)$$

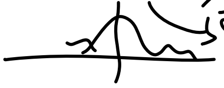
$$S(f) = 4\,\text{sinc}\,4f + 2 \cdot 2\,\text{sinc}\,2f$$



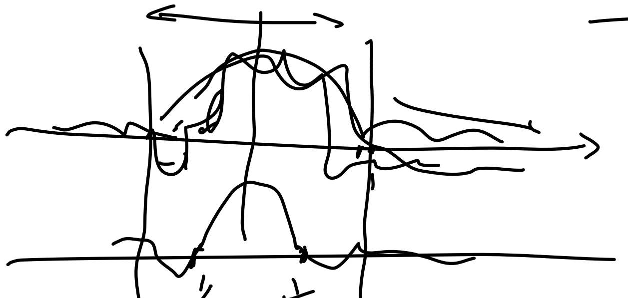
$$s(t) = \Lambda(t-1) + 2\,\Pi\left(\frac{t-3}{2}\right) + \Lambda(t-3)$$

$$\Updownarrow \mathcal{F}$$

$$S(f) = e^{-j2\pi f}\,\text{sinc}^2 f + 2 \cdot 2\, e^{-j2\pi f 3}\,\text{sinc}\,2f + e^{-j2\pi f 3}\,\text{sinc}^2 f$$



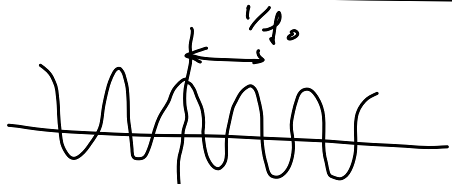
$1/f \quad \frac{1}{f^2}$

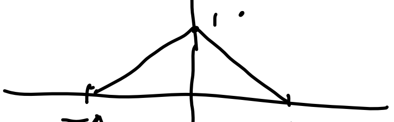


f

$-1/2 \quad 1/2$

$1/f_0$



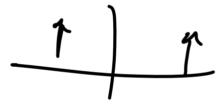


$-\Delta \quad \Delta$

$$\Delta >> \frac{1}{f_0}$$

$$x(t)\,y(t) \longleftrightarrow (X * Y)(f)$$

$\cos \longrightarrow$

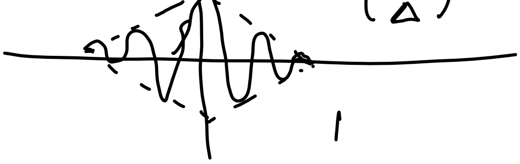


$\Lambda \longrightarrow$

sinc^2

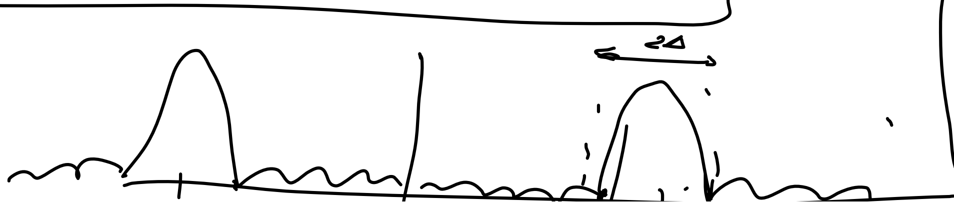
=

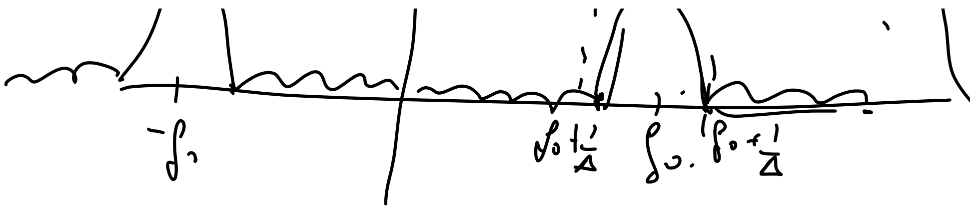
$$s(t) = \Lambda\left(\frac{t}{\Delta}\right)\cos 2\pi f_0 t \overset{\mathcal{F}}{\longleftrightarrow} S(f) = \Delta\,\text{sinc}^2\Delta f * \left(\frac{1}{2}\delta(f-f_0) + \frac{1}{2}\delta(f+f_0)\right)$$

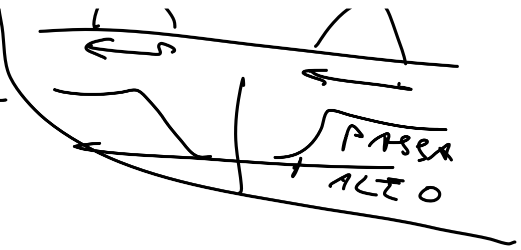


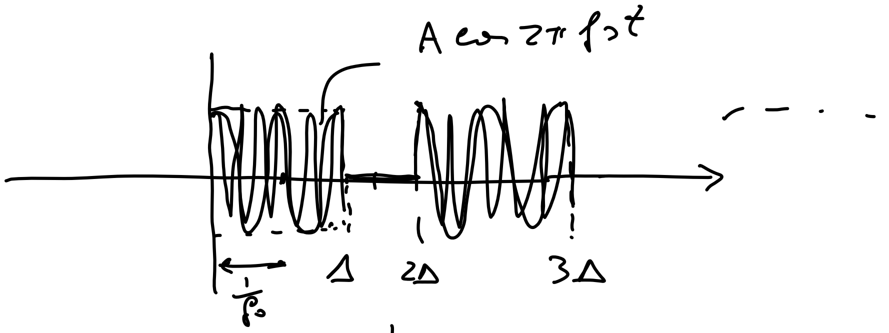
$$= \frac{\Delta}{2}\,\text{sinc}^2\Delta(f-f_0) + \frac{\Delta}{2}\,\text{sinc}^2\Delta(f+f_0)$$

$$\boxed{f(x) * \delta(x-x_0) = f(x-x_0)}$$









$$\cos x = \frac{e^{jx}+e^{-jx}}{2}$$

$$\Delta \gg \frac{1}{f_0}$$

$$s(t) = A\cos 2\pi f_0 t \cdot \left[\Pi\left(\frac{t-\Delta/2}{\Delta}\right) + \Pi\left(\frac{t-\frac{5}{2}\Delta}{\Delta}\right)\right]$$

$$S(f) = \left[\frac{A}{2}\delta(f-f_0) + \frac{A}{2}\delta(f+f_0)\right] * \left[e^{-j2\pi f\frac{\Delta}{2}}\Delta\,\mathrm{sinc}\,\Delta f + e^{-j2\pi f\frac{5}{2}\Delta}\Delta\,\mathrm{sinc}\,\Delta f\right]$$

$$= \frac{A}{2}\left[\delta(f-f_0)+\delta(f+f_0)\right] * \Delta\,\mathrm{sinc}\,\Delta f\left[e^{-j\pi f\Delta} + e^{-j\pi f 5\Delta}\right]$$

$$e^{-j\pi f 3\Delta}\, 2\left[\frac{e^{+j\pi f2\Delta} + e^{-j\pi f 2\Delta}}{2}\right]$$

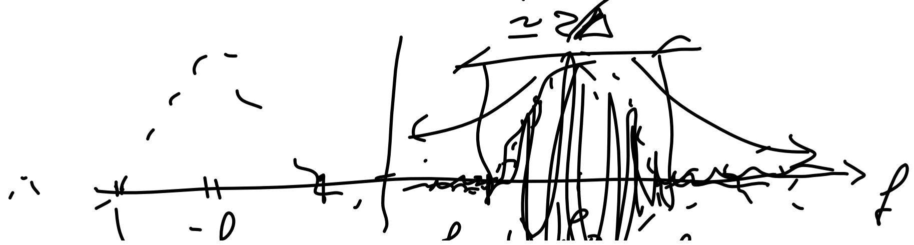
$$= \frac{A}{\cancel{2}}\left[\delta(f-f_0)+\delta(f+f_0)\right] * \Delta\,\cancel{2}\,e^{-j\pi f3\Delta}\cos\pi 2\Delta f\;\mathrm{sinc}\,\Delta f$$

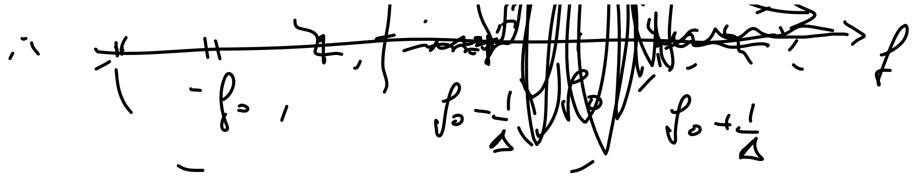
$$f(t) * \delta(t-t_0) = f(t-t_0)$$

$$\int f(\xi)\,\delta(t-\xi-t_0)\,d\xi = f(t-t_0)$$

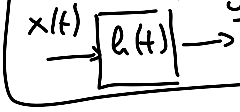
$$A\Delta\left[e^{-j\pi(f-f_0)3\Delta}\cos\pi 2\Delta(f-f_0)\,\mathrm{sinc}\,\Delta(f-f_0) + e^{-j\pi(f+f_0)3\Delta}\cos\pi 2\Delta(f+f_0)\,\mathrm{sinc}\,\Delta(f+f_0)\right]$$

sinc x




$-f_0$
$f_0 - \frac{1}{\Delta}$
f_0
$f_0 + \frac{1}{\Delta}$
f

Motivazione



$y(t) = (h * x)(t)$

CONVOLUZIONE

$$x(t) * y(t) = z(t)$$

$$(x * y)(t)$$

$$\int_{-\infty}^{+\infty} x(\tau)\, y(t-\tau)\, d\tau = \int_{-\infty}^{+\infty} x(t-\eta)\, y(\eta)\, d\eta$$

$\eta = t - \tau$

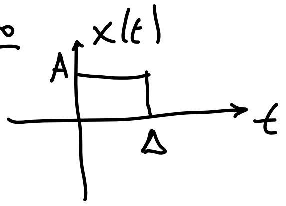
$$(x * y)(t) = (y * x)(t)$$

$$\int_{-\infty}^{+\infty} x(\tau)\, y(t-\tau)\, d\tau$$

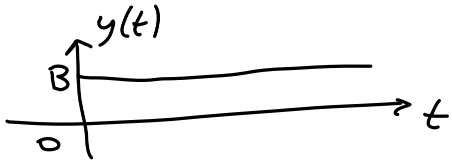
$y(-(\tau - t))$

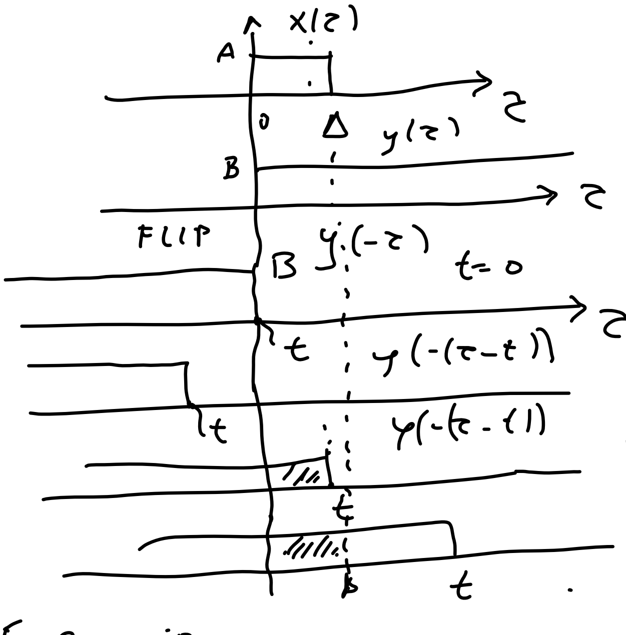
flip & shift

Esempio

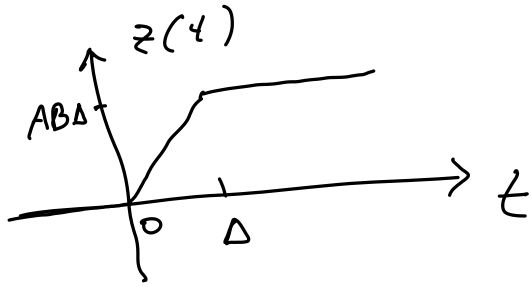


$*$





$\Longrightarrow$



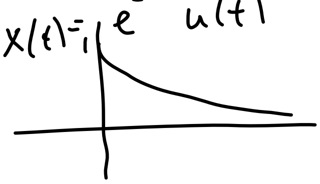
$t < 0 \Rightarrow z(t) = 0$

$0 < t < \Delta \Rightarrow z(t) = ABt$

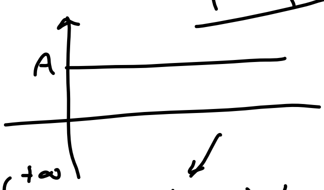
$t > \Delta \Rightarrow z(t) = AB\Delta$

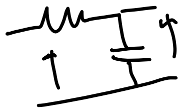
Esempio:

$x(t) = e^{-\alpha t}\, u(t)$

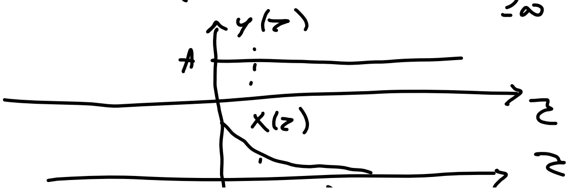


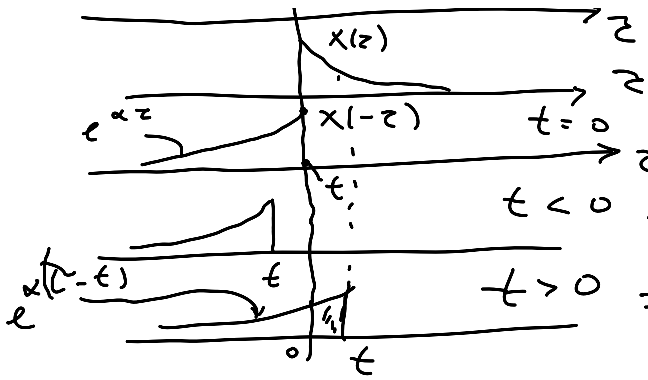
$y(t) = A\, u(t)$





$$z(t) = (x * y)(t) = \int_{-\infty}^{+\infty} y(\tau)\, x(t-\tau)\, d\tau$$





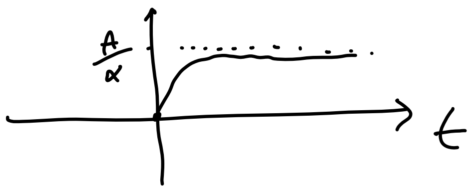
$t<0 \Rightarrow z(t)=0$

$$t>0 \Rightarrow z(t)=A\int_0^t e^{\alpha(\tau-t)}\,d\tau$$

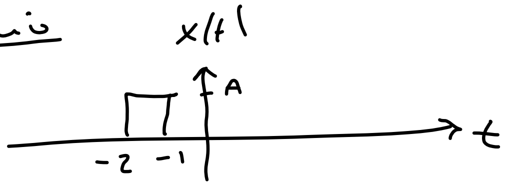
$$= A e^{-\alpha t}\int_0^t e^{\alpha\tau}\,d\tau = A e^{-\alpha t}\left[\frac{e^{\alpha\tau}}{\alpha}\right]_0^t$$

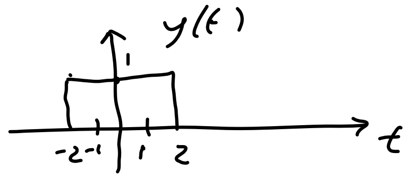
$$= \frac{A}{\alpha} e^{-\alpha t}\left(e^{\alpha t}-1\right)$$

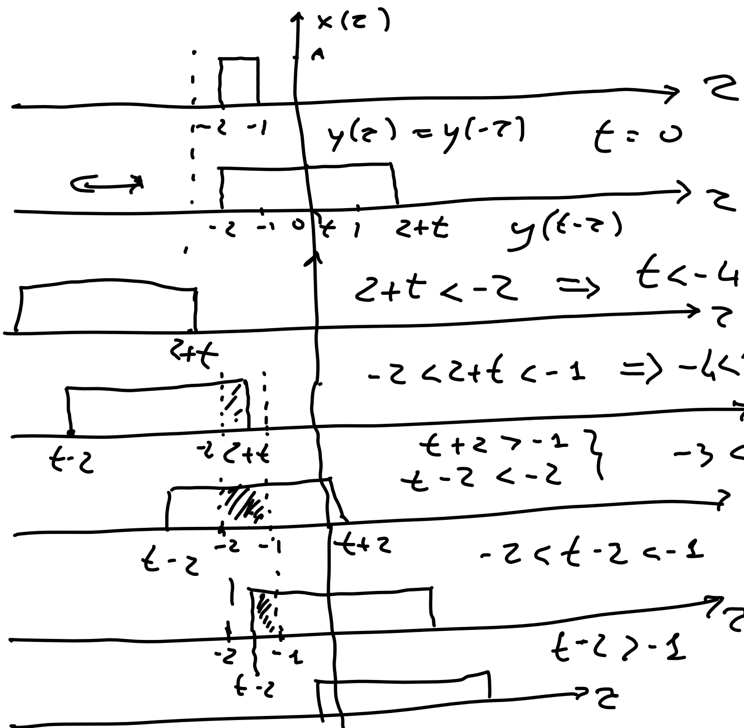
$$= \frac{A}{\alpha}\left(1-e^{-\alpha t}\right)$$



Esempio







$y(\tau)=y(-\tau)$ $\quad t=0$

$y(t-\tau)$

$2+t<-2 \Rightarrow t<-4 \quad z(t)=0$

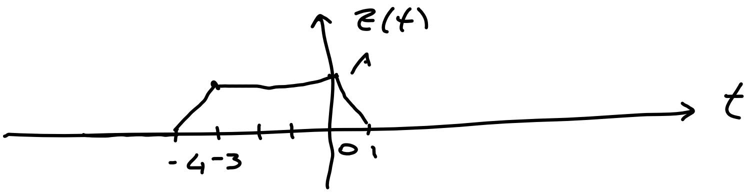
$-2<2+t<-1 \Rightarrow -4<t<-3 \quad z(t)=\int_{-2}^{2+t} A\,d\tau = A(2+t+2) = A(t+4)$

$\left.\begin{array}{l} t+2>-1 \\ t-2<-2 \end{array}\right\}$ $\quad -3<t<0 \Rightarrow z(t)=\int_{-2}^{-1} A\,d\tau = A(-1+2)=A$

$-2<t-2<-1 \quad 0<t<1 \Rightarrow z(t)=\int_{t-2}^{-1} A\,d\tau = A(-1-t+2) = A(1-t)$

$t-2>-1 \quad t>1$

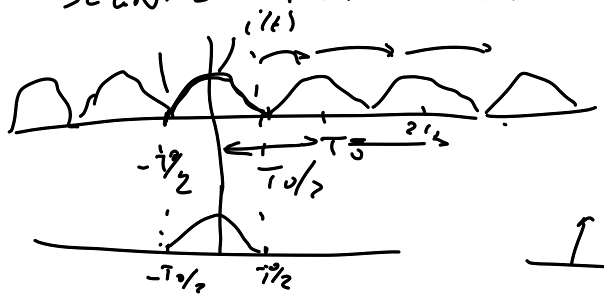
$z(t)=0$



SEGNALI PERIODICI

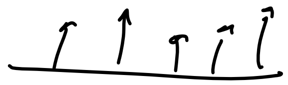
$$s_p(t)=\sum_{n=-\infty}^{+\infty} c_n\, e^{j2\pi \frac{n}{T_0}t}$$

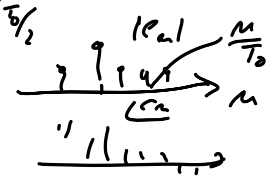
SEGNALI PERIODICI



$$s_p(t) = \sum_{n=-\infty} c_n e^{j2\pi n \frac{t}{T_0}}$$

$$c_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} s(t) e^{-j2\pi \frac{n}{T_0} t}\, dt$$





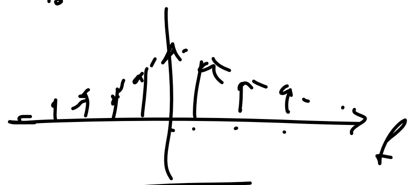
$$s(t) = \sum_{k=-\infty}^{+\infty} i(t - kT_0)$$

$$S(f) = \sum_{k=-\infty}^{+\infty} I(f) e^{-j2\pi f k T_0} = I(f) \sum_{k=-\infty}^{+\infty} e^{-j2\pi f k T_0} \longrightarrow \text{Formula di Poisson.}$$

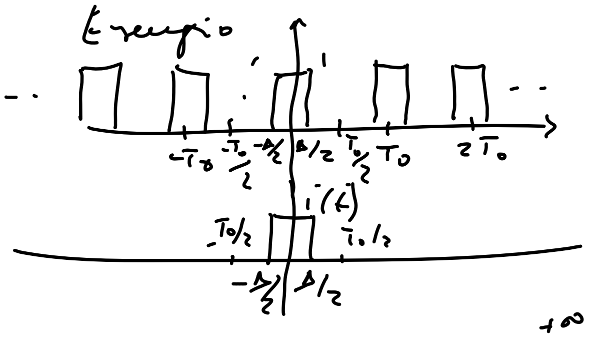
$$s(t) = i(t) * \sum_{k=-\infty}^{+\infty} \delta(t - kT_0) \longrightarrow \frac{1}{T_0}\sum_{n=-\infty} \delta\left(f - \frac{n}{T_0}\right)$$

$$S(f) = I(f)\cdot \frac{1}{T_0}\sum_{k=-\infty}^{+\infty} \delta\left(f - \frac{k}{T_0}\right)$$

$$= \frac{1}{T_0}\sum_{k=-\infty}^{+\infty} I\left(\frac{k}{T_0}\right) \delta\left(f - \frac{k}{T_0}\right)$$

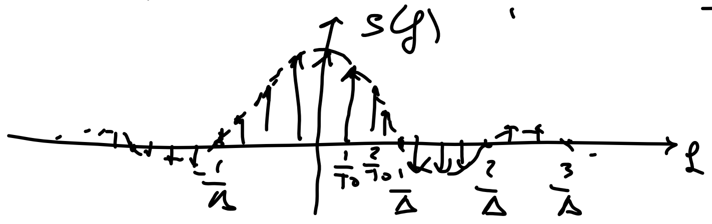


Esempio



$$S(f) = \frac{\Delta}{T_0} \operatorname{sinc} \Delta f \sum_{k=-\infty} \delta\left(f - \frac{k}{T_0}\right)$$

$$= \frac{\Delta}{T_0} \sum_{k=-\infty}^{+\infty} \operatorname{sinc} \frac{\Delta k}{T_0}\, \delta\left(f - \frac{k}{T_0}\right)$$



$T_0 > \Delta$

$\frac{1}{T_0} < \frac{1}{\Delta}$

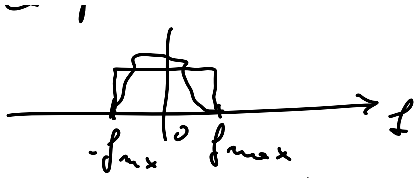
Segnali a banda definita.

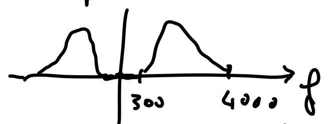
Segnale passa-basso

segnale valido

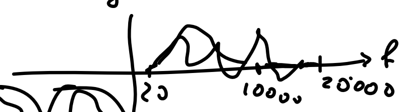
segnale ...

segnale vocale

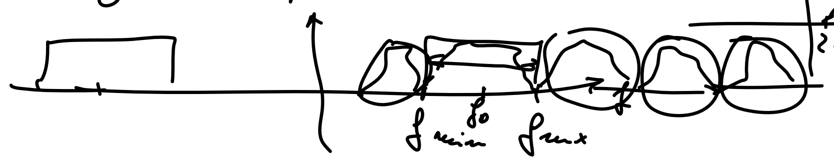




segnale audio

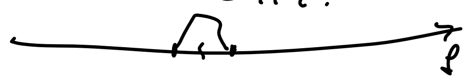


Segnale passa banda

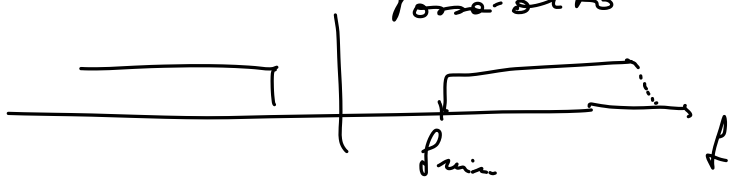


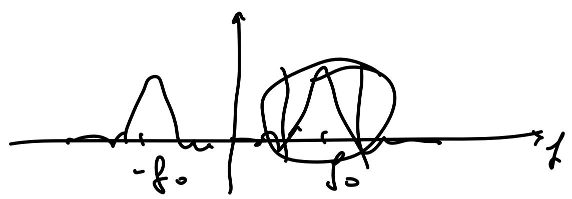
10.8 MHz.

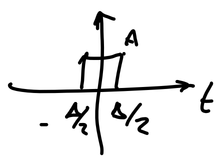
GHz.



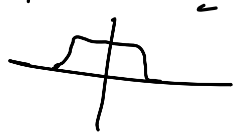
Passa-alto

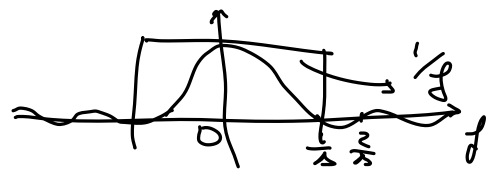






$\Rightarrow S(f) = \Delta \, \mathrm{sinc}\, \Delta f$





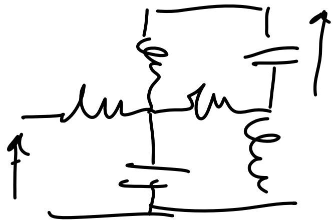
Lim. $X(f) = $ —

Sistemi lineari (tempo-invariante)

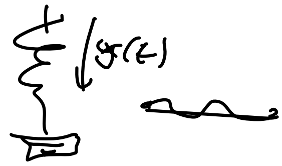
C.I. nulle

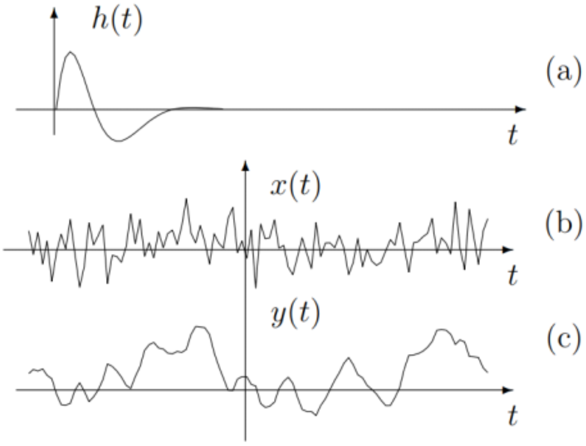
$$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$$

$$y(t) = \int_{-\infty}^{+\infty} h(\tau)\, x(t-\tau)\, d\tau$$



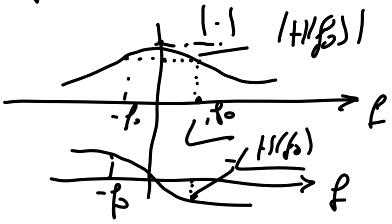
$$Y(f) = H(f)\, X(f)$$





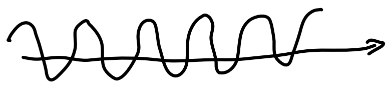
$H(f)$ funzione di risposta armonica

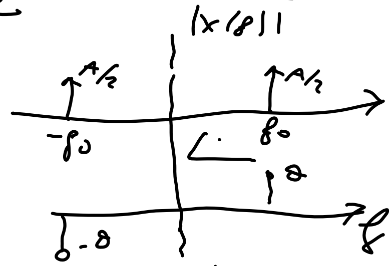
$$H(f) = |H(f)|\, e^{j\angle H(f)} = H_R(f) + j\, H_I(f)$$



$$x(t) \rightarrow \boxed{H(f)} \rightarrow y(t)$$

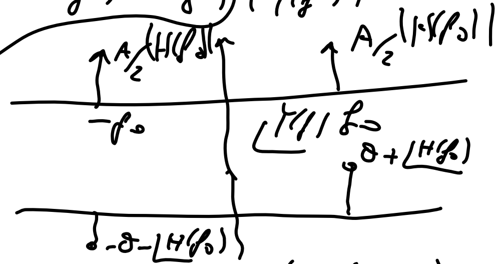
$$x(t) = A\cos(2\pi f_0 t + \theta) \qquad X(f) = \frac{A}{2} e^{j\theta}\delta(f-f_0) + \frac{A}{2} e^{-j\theta}\delta(f+f_0)$$





$$Y(f) = |Y(f)|\, e^{j\angle Y(f)}$$

$$\begin{cases} |Y(f)| = |H(f)|\,|X(f)| \\ \angle Y(f) = \angle H(f) + \angle X(f) \end{cases} ; \quad Y(f) = H(f)\, X(f)$$



$$Y(f) = \frac{A}{2} e^{j(\theta + \angle H(f_0))} |H(f_0)|\, \delta(f-f_0) + \frac{A}{2} |H(f_0)|\, e^{-j(\theta + \angle H(f_0))} \ldots$$

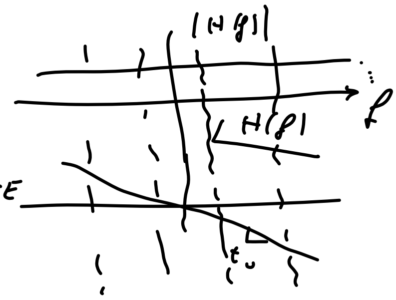
$$Y(f) = \frac{A}{2} e^{j(\theta + \angle H(f_0))} |H(f_0)| \, \delta(f - f_0) + \frac{A}{2} |H(f_0)| e^{-j(\theta + \angle H(f_0))} \delta(f + f_0)$$

$$y(t) = A|H(f_0)| \cos(2\pi f_0 t + \theta + \angle H(f_0))$$

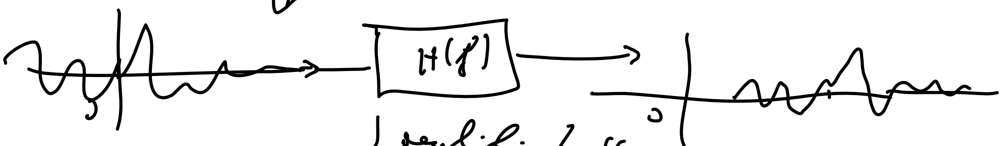
Sistema è non distorcente

DEF.
$$\begin{cases} |H(f)| = K \\ \angle H(f) = -2\pi t_0 f \end{cases} \quad f \in I_x(f)$$

BANDA DI INTERESSE



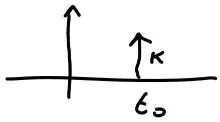
PASSA TUTTO

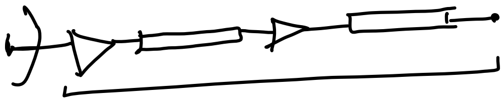


amplifico/attenuo
ritardo

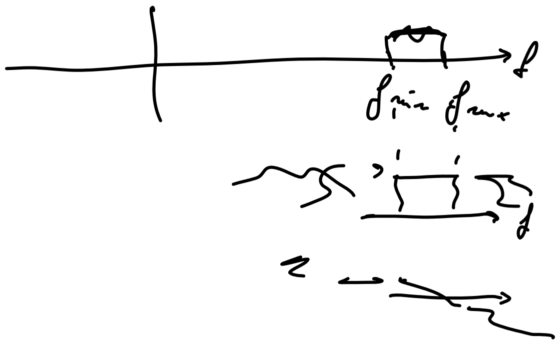
$$y(t) = K\,x(t - t_0) \quad \text{sist. non distorcente}$$

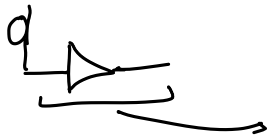
$$Y(f) = \underbrace{K e^{-j2\pi f t_0}} X(f) \qquad h(t) = K\,\delta(t - t_0)$$



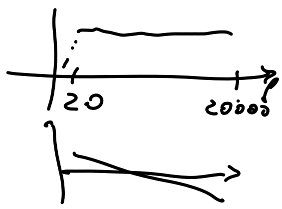


Sistema audio



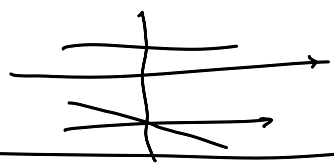


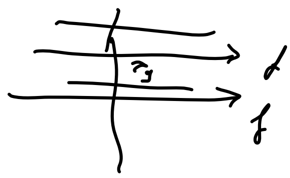
Banda audio



DEF: Sist non distorcente in ampiezza $|H(f)| = K \quad f \in I_x(f)$

" " " in fase $\angle X(f) = -2\pi f t_0 \quad f \in I_x(f)$





$$\boxed{\text{ritardo di gruppo} \quad \tau_g(f) = -\frac{1}{2\pi} \frac{d\,\angle H(f)}{\partial f}}$$

$(-2\pi f t_0$

$$\boxed{\text{ritardo di gruppo} \quad \tau_g(f) = -\frac{1}{2\pi}\frac{d\angle H(f)}{df}}$$

$-2\pi f t_0$

dist. non distorcente in fase.

$\tau_g(f) = t_0$

$x(t) = A\cos(2\pi f_0 t + \theta)$

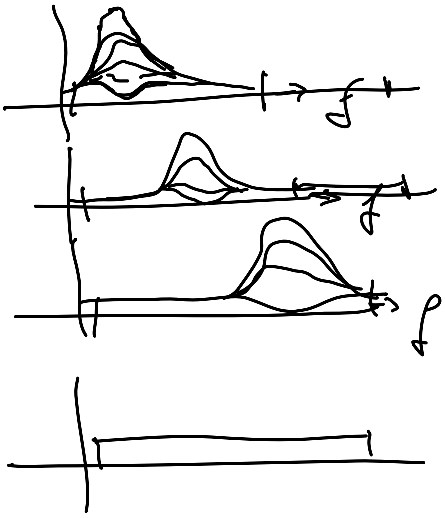
$y(t) = A|H(f_0)|\cos\left(2\pi f_0 t + \theta + \angle H(f_0)\right)$

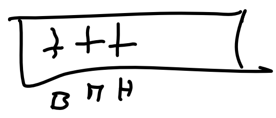
$= A|H(f_0)|\cos\left(2\pi f_0\left(t + \frac{\angle H(f_0)}{2\pi f_0}\right) + \theta\right)$

$$\boxed{\text{ritardo di fase} \quad \tau_\phi(f) = -\frac{1}{2\pi}\frac{\angle H(f)}{f}}$$

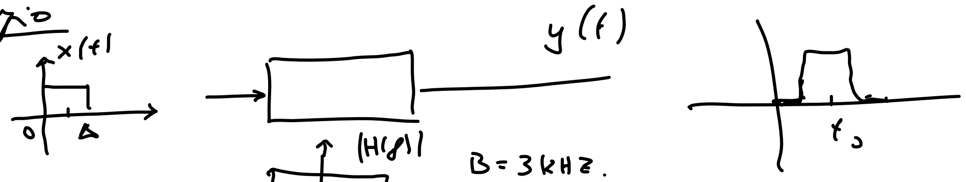
$\tau_\phi(f) = t_0$

Filtri

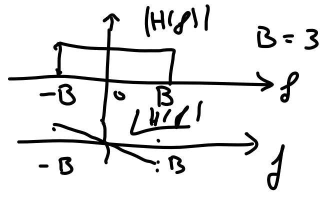




Esempio



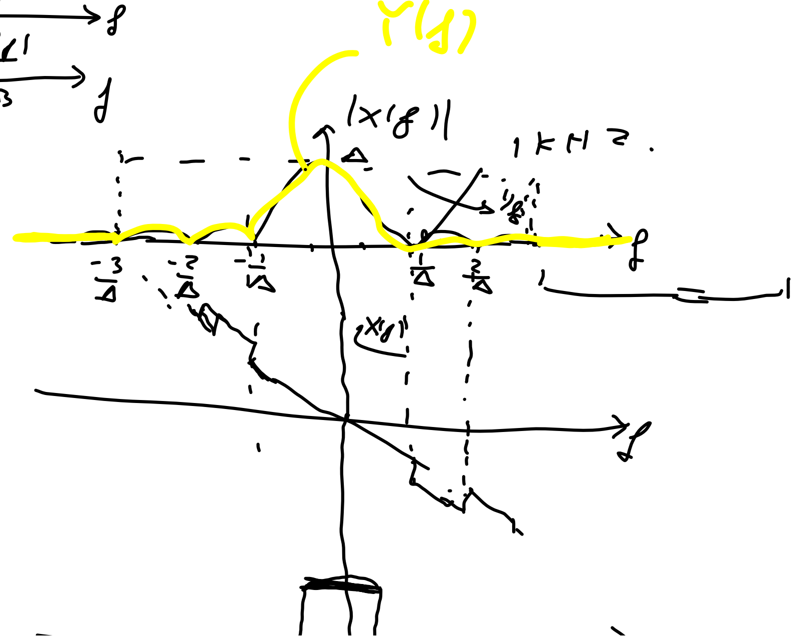
$\Delta = 1$ msec



B = 3 kHz.

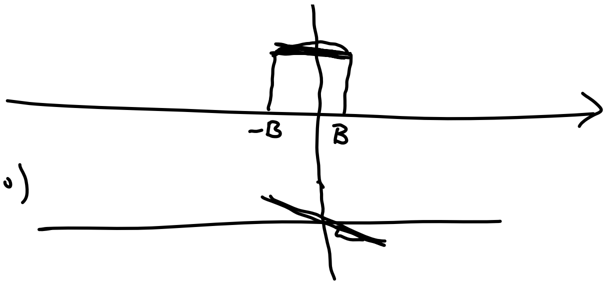
$X(f) = e^{-j2\pi f\frac{\Delta}{2}}\Delta\,\mathrm{sinc}\,\Delta f$

$\frac{1}{\Delta} = 10^3 = 1\,kHz$

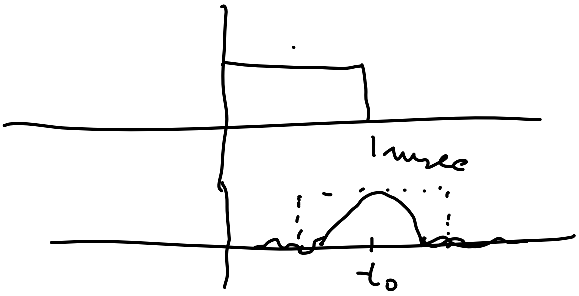


B = 300 Hz.

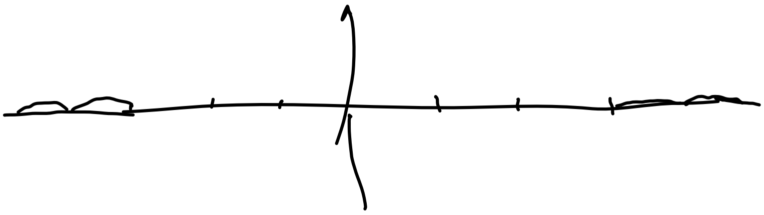
B = 3000 Hz.



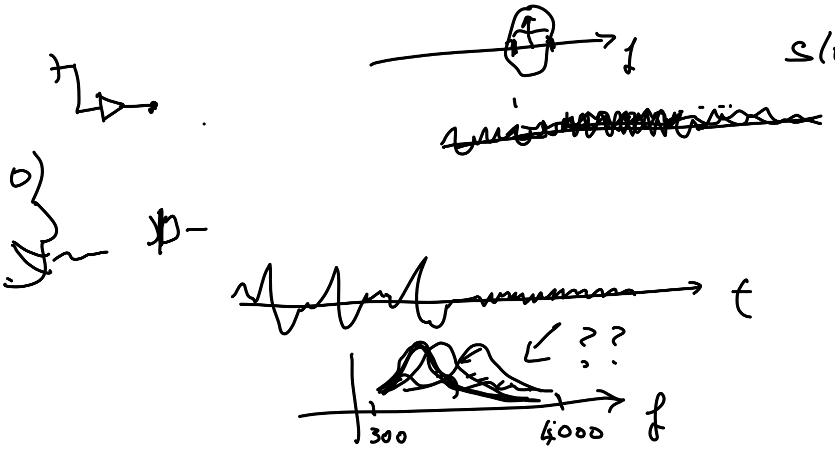
$y(t) \simeq 2B\,\mathrm{sinc}\,2B(t-t_0)$

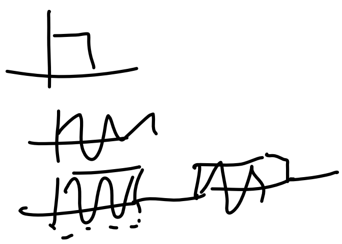


PASSA BANDA [3, 4] KHz.

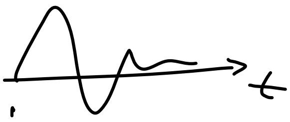


$s(t) \longleftrightarrow S(f)$



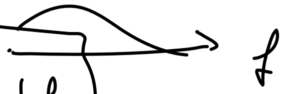


Segnali di energia



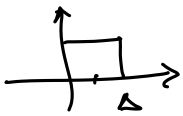
$$\mathcal{E}_s = \int_{-\infty}^{+\infty} |s(t)|^2\,dt < \infty$$

$$= \int_{-\infty}^{+\infty} |S(f)|^2\,dt = \boxed{\int_{-\infty}^{+\infty} \underbrace{\mathcal{E}_s(f)}_{\text{densità spettrale di energia}}\,df}$$

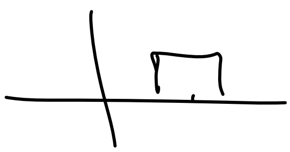


Esempio

$$s(t) = \Pi\left(\frac{t-\Delta/2}{\Delta}\right)$$

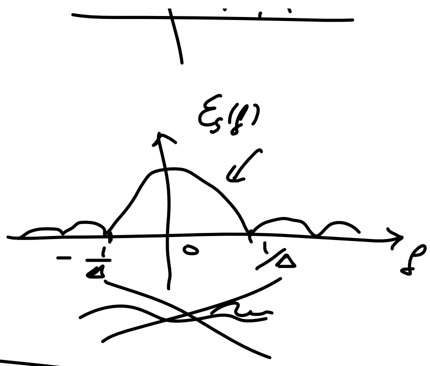


$$S(f) = e^{-j2\pi f\Delta/2}\,\Delta\,\mathrm{sinc}\,\Delta f$$



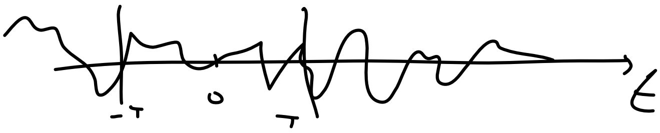
$$S(f) = e^{-j2\pi f\Delta/2}\,\Delta\,\mathrm{sinc}\,\Delta f$$

$$|S(f)|^2 = \Delta^2\,|\mathrm{sinc}\,\Delta f|^2$$

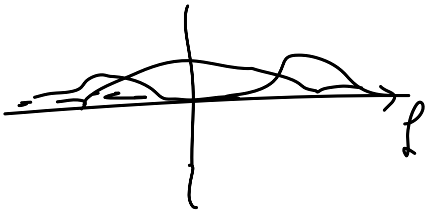


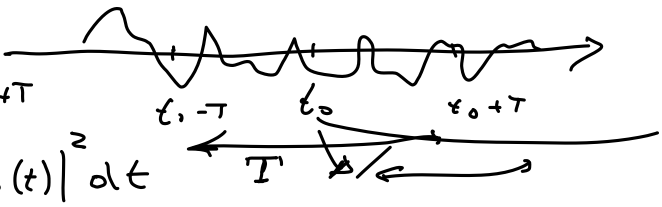
Segnali di potenza

$$P_s = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}|s(t)|^2\,dt$$

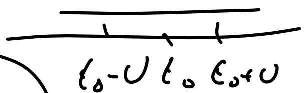


$$\boxed{P_s = \int_{-\infty}^{+\infty}\underbrace{P_s(f)}_{\text{spettro di potenza}}\,df}$$

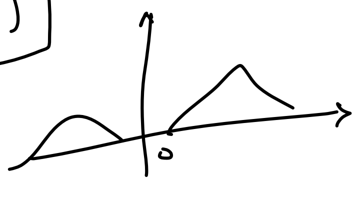




$$\frac{1}{2T}\int_{t_0-T}^{t_0+T}|s(t)|^2\,dt$$

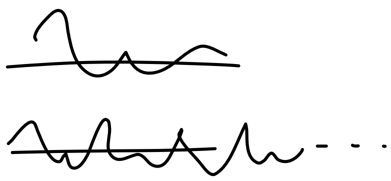


$$\lim_{T\to\infty}\frac{1}{2T}\lim_{U\to\infty}\frac{1}{2U}\int_{-U}^{U}|S_T(f;t_0)|^2\,dt_0 = P_s(f)$$



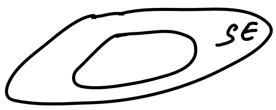
Segnali di energia

Segnali di potenza ...



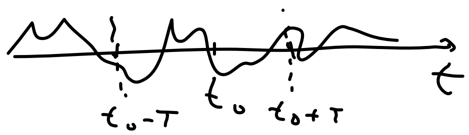
SPETTRO DI ENERGIA

$$\mathcal{E}_s(f) = |S(f)|^2 \qquad \mathcal{E}_s = \int_{-\infty}^{+\infty} |s(t)|^2 dt = \int_{-\infty}^{+\infty} \mathcal{E}_s(f) df$$



SPETTRO DI POTENZA

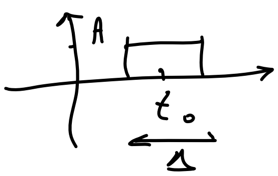
$$\int_{-\infty}^{+\infty} P_s(f)\, df = P_s$$



$$\lim_{T\to\infty}\frac{1}{2T}\lim_{U\to\infty}\frac{1}{2U}\int_{-U}^{U}|S_T(f;t_0)|^2\, dt_0 = P_s(f)$$

$$\cancel{\lim_{T\to\infty}\frac{|S_T(f)|^2}{2T}} = P_s(f)$$

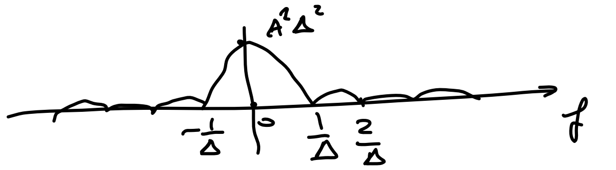
Esempio Segnali di energia



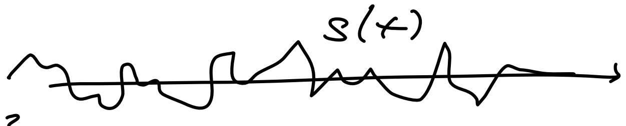
$$s(t) = A\,\Pi\left(\frac{t-t_0}{\Delta}\right)$$

$$S(f) = A\, e^{-j2\pi f t_0}\, \Delta\, \text{sinc}\, \Delta f$$

$$\mathcal{E}_s(f) = |S(f)|^2 = A^2\Delta^2\, \text{sinc}^2 \Delta f$$



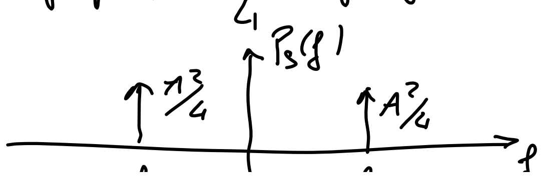
Esempio Segnali di potenza

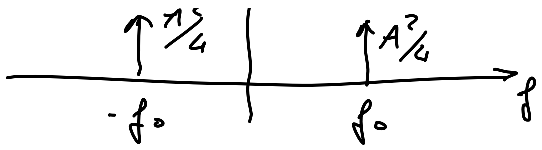


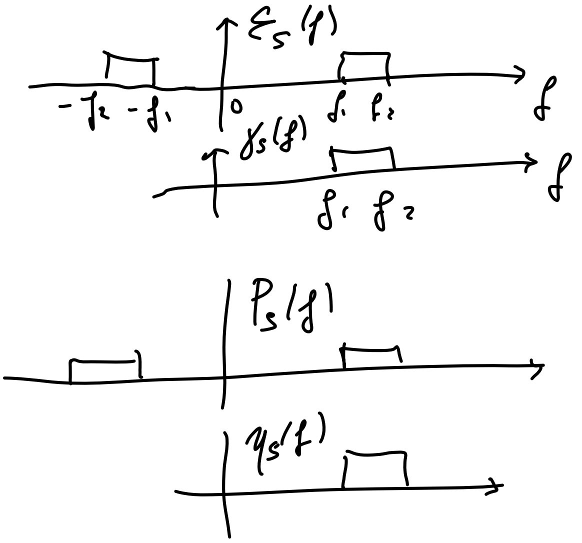
$$s(t) = A\cos(2\pi f_0 t + \theta) \qquad P_s = \frac{A^2}{2} \qquad P_s^{RMS} = \frac{A}{\sqrt{2}}$$

$\vdots$

$$P_s(f) = \frac{A^2}{4}\delta(f-f_0) + \frac{A^2}{4}\delta(f+f_0)$$







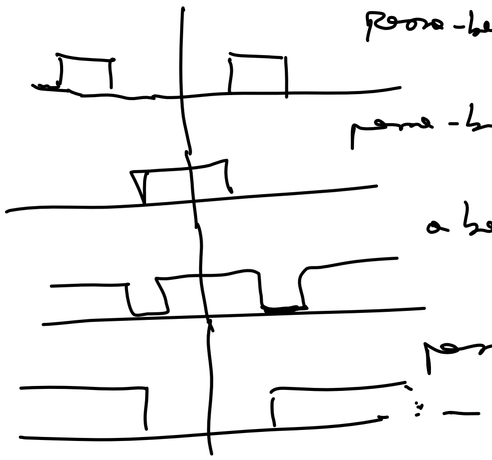
$$\mathcal{E}_s(f) = |S(f)|^2$$

$$\gamma_s(f) = \begin{cases} 2\,\mathcal{E}_s(f) & f > 0 \\ 0 & f < 0 \end{cases}$$

$$\int_0^\infty \gamma_s(f)\,df = \mathcal{E}_s$$

$$\eta_s(f) = \begin{cases} 2\,P_s(f) & f > 0 \\ 0 & f < 0 \end{cases}$$

$$\int_0^\infty \eta_s(f)\,df = P_s$$



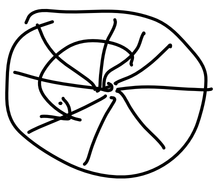
passa-banda

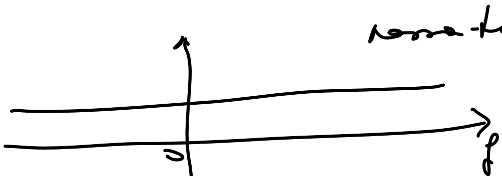
passa-basso

a banda oscura

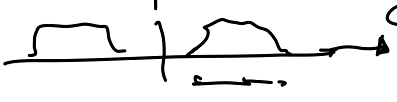
passa-alto

passa-tutto BIANCO

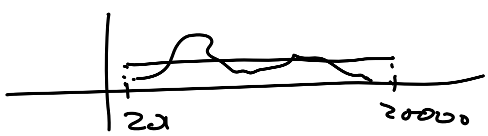




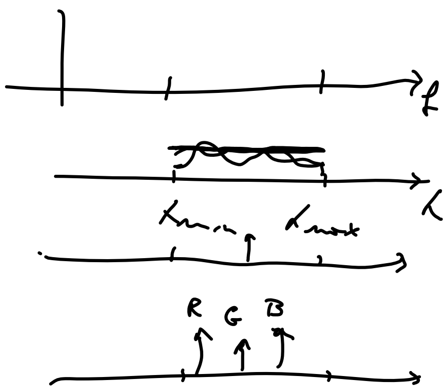
COLORATO



segnale bianco audio

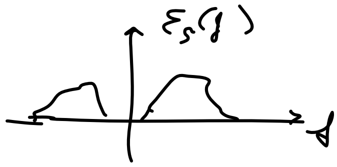


OTTICA



Segnali di energia

$$s(t) \overset{\mathcal{F}}{\longleftrightarrow} S(f) \xrightarrow{|\cdot|^2} |S(f)|^2 = \mathcal{E}_s(f)$$



$\mathcal{F}^{-1}$

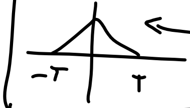
funzione di autocorrelazione.

$$\mathcal{F}^{-1}\left[|S(f)|^2\right](\tau) = \int_{-\infty}^{+\infty} \underbrace{S(f)}\,\underbrace{S^*(f)\,e^{j2\pi f\tau}}\,df$$

$$\boxed{r_s(\tau) = \int_{-\infty}^{+\infty} s(t)\,s^*(t-\tau)\,dt}$$

FUNZIONE DI AUTOCORRELAZIONE

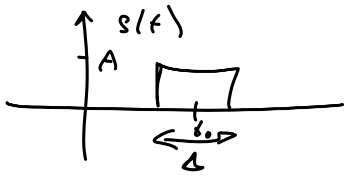
$\Lambda\left(\frac{t}{T}\right) \longleftrightarrow T\,\text{sinc}^2 Tf$



$s(t) \longleftrightarrow S(f)$

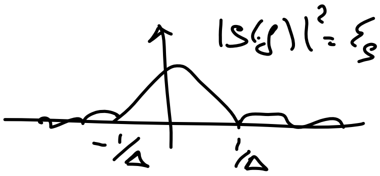
$\boxed{s(0) = \int S(f)\,df}$

$s(t) = \int_{-\infty}^{+\infty} S(f)\,e^{j2\pi f t}\,df$

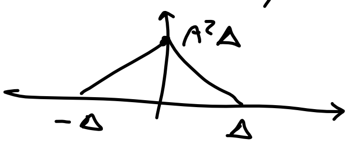


$\longrightarrow$

$|S(f)|^2 = \mathcal{E}_s(f) = A^2\Delta^2\,\text{sinc}^2\Delta f$



$$r_s(\tau) = A^2\Delta\,\Lambda\left(\frac{\tau}{\Delta}\right) \xleftarrow{\mathcal{F}^{-1}}$$



$$\Rightarrow \boxed{\mathcal{E}_s = r_s(0)}$$

Segnali di potenza

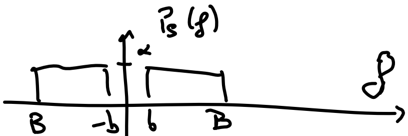
$$s(t) \longrightarrow P_s(f)$$

$\mathcal{F}^{-1}$

$r_s(\tau)$

T. di Wiener Khinchine.

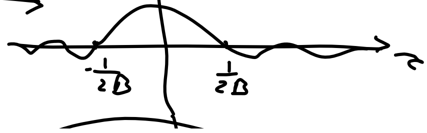
$$r_s(\tau) \triangleq \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} s(t)\,s^*(t-\tau)\,dt$$



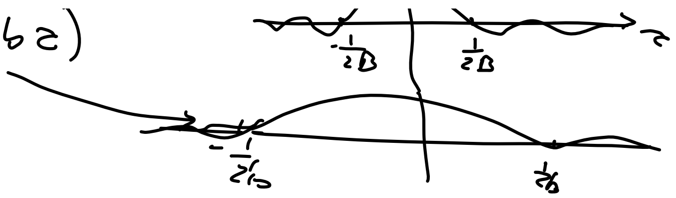
$$P_s(f) = \alpha\left(\Pi\left(\frac{f}{2B}\right) - \Pi\left(\frac{f}{2b}\right)\right)$$

$$r_s(\tau) = \mathcal{F}^{-1}\left[P_s(f)\right] = \int_{-\infty}^{+\infty} P_s(f)\,e^{j2\pi f\tau}\,df$$

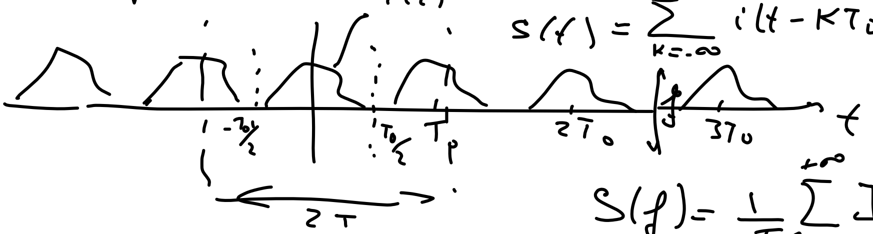
$$= \alpha\left(2B\,\text{sinc}\,2B\tau - 2b\,\text{sinc}\,2b\tau\right)$$



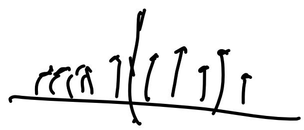
$$= \alpha \left(2B \,\mathrm{sinc}\, 2B\tau - 2b \,\mathrm{sinc}\, 2b\tau \right)$$



Segnale periodico



$$s(t) = \sum_{k=-\infty}^{+\infty} i(t - kT_0) \qquad i(t) \overset{\mathcal{F}}{\leftrightarrow} I(f)$$



$$S(f) = \frac{1}{T_0} \sum_{n=-\infty}^{+\infty} I\left(\frac{n}{T_0}\right) \delta\left(f - \frac{n}{T_0}\right)$$

$$r_s(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} s(t)\, s^*(t-\tau)\, dt \qquad P_s(f) = ?$$

definizione (troppo complicata)

passare per la autocorrelaz.

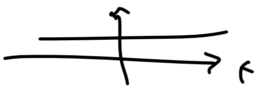
$$= \int_{-T}^{T} \sum_{k_1=-\infty}^{+\infty} i(t - k_1T_0) \sum_{k_2=-\infty}^{+\infty} i^*(t - \tau - k_2T_0)\, dt$$

$$= \sum_{k_1} \sum_{k_2} \int_{-T}^{T} i(t - k_1T_0)\, i^*(t - \tau - k_2T_0)\, dt$$

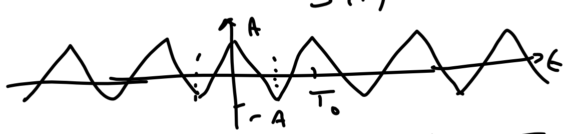
$$\sum_{k_1}\sum_{k_2} \lim \frac{1}{2T} \int_{-T}^{T} \ldots\ldots \, dt$$

$$r_s(\tau) = \sum_{k=-\infty}^{\infty} \frac{1}{T_0^2} \left| I\left(\frac{k}{T_0}\right) \right|^2 e^{j2\pi \frac{k}{T_0}\tau}.$$

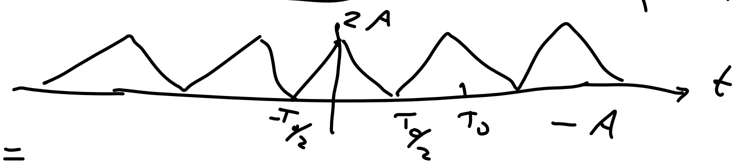
$$\boxed{P_s(f) = \sum_{k=-\infty}^{+\infty} \frac{1}{T_0^2} \left| I\left(\frac{k}{T_0}\right) \right|^2 \delta\left(f - \frac{k}{T_0}\right)}$$



Esempio:

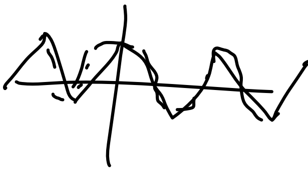


$=$



$$s(t) = \underbrace{2A \sum_{k=-\infty}^{+\infty} \Lambda\left(\frac{t - kT_0}{T_0/2}\right)}_{v(t)} - A$$

$$\Lambda\left(\frac{t}{T_0/2}\right) \longleftrightarrow \frac{T_0}{2}\,\mathrm{sinc}^2 \frac{T_0}{2} f$$



$$P_v(f) = \sum_k \frac{1}{\cancel{T_0^2}} \frac{\cancel{4}A^2\cancel{T_0^2}}{\cancel{4}}\, \mathrm{sinc}^4 \frac{\cancel{T_0}}{2}\frac{k}{\cancel{T_0}}\, \delta\left(f - \frac{k}{T_0}\right)$$

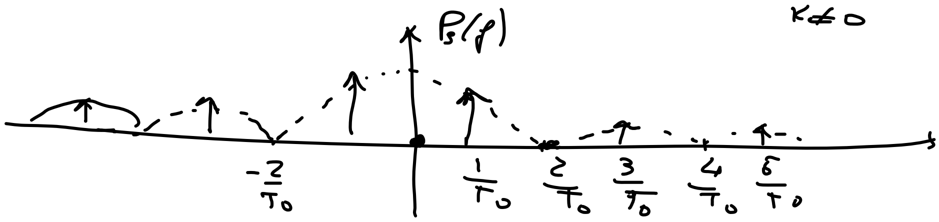
$$= \sum_k A^2\, \mathrm{sinc}^4 \frac{k}{2}\, \delta\left(f - \frac{k}{T_0}\right)$$

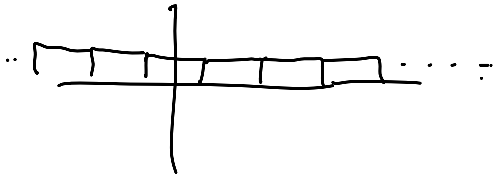
$$P_s(f) = P_v(f) - A^2\delta(f) = \sum^{+\infty} A^2 \,\mathrm{sinc}^4 k\, \delta(f - k \ldots)$$

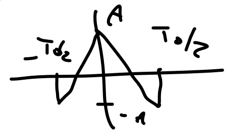
$$= \sum_k A^2 \,\mathrm{sinc}\,\frac{k}{2}\,\delta\left(f - \frac{k}{T_0}\right)$$

$$P_s(f) = P_v(f) - A^2\delta(f) = \sum_{\substack{k=-\infty \\ k\neq 0}}^{+\infty} A^2 \underbrace{\mathrm{sinc}^4\frac{k}{2}}\,\delta\left(f - \frac{k}{T_0}\right)$$

$\frac{k}{2} = n$ nulla per k pari







$$i(t) = 2A\Lambda\left(\frac{t}{T_0/2}\right) - A\Pi\left(\frac{t}{T_0}\right)$$

$$P_s(f) = \sum_{k=-\infty}^{+\infty} \frac{1}{T_0^2}\left|I\left(\frac{k}{T_0}\right)\right|^2 \delta\left(f - \frac{k}{T_0}\right) = \frac{1}{T_0^2}|I(f)|^2 \sum_{k=-\infty}^{+\infty}\delta\left(f - \frac{k}{T_0}\right)$$

$$R_s(\tau) = \sum_{k=-\infty}^{+\infty} \frac{1}{T_0^2}\left|I\left(\frac{k}{T_0}\right)\right|^2 e^{+j2\pi\frac{k}{T_0}\tau} \Rightarrow \text{periodica.}$$

$$= \frac{1}{T_0^2}\mathcal{F}^{-1}\left(|I(f)|^2\right) * T_0\sum_{n=-\infty}^{+\infty}\delta(\tau - nT_0)$$

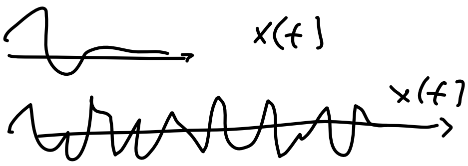
$$= \frac{1}{T_0} R_i(\tau) * \sum_{n=-\infty}^{+\infty}\delta(\tau - nT_0)$$

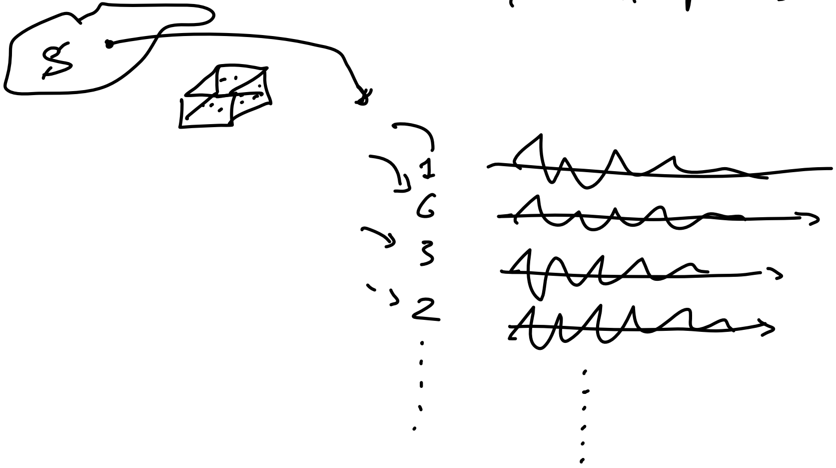
$$= \frac{1}{T_0}\sum_{n=-\infty}^{+\infty} R_i(\tau - nT_0) \Rightarrow$$ autocorr. di $i(t)$ periodicizzata.

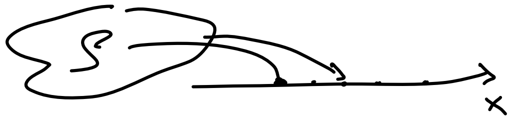
S.E.	S.P.
$\mathcal{E}_s \xleftarrow{\int} \mathcal{E}_s(f)$	$P_s(f) \xrightarrow{\int} P_s$
$\updownarrow \mathcal{F}$	$\updownarrow \mathcal{F}$
$R_s(\tau)$	$R_s(\tau)$

Segnali di energia

" " potenza

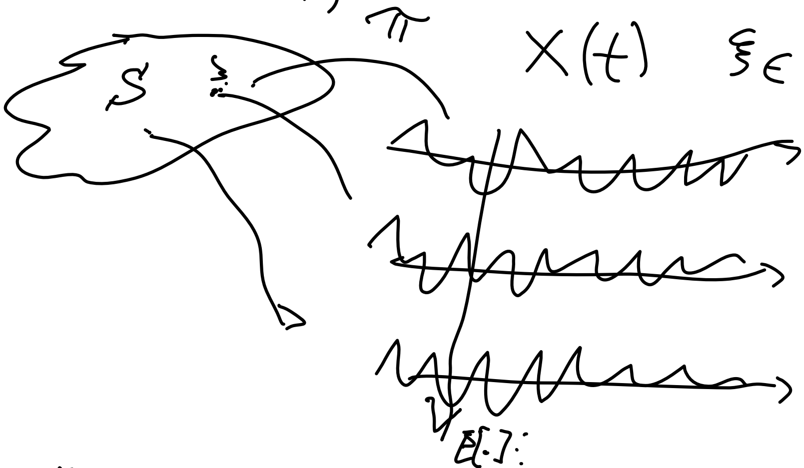






SEGNALI ALEATORI (PROCESSI)

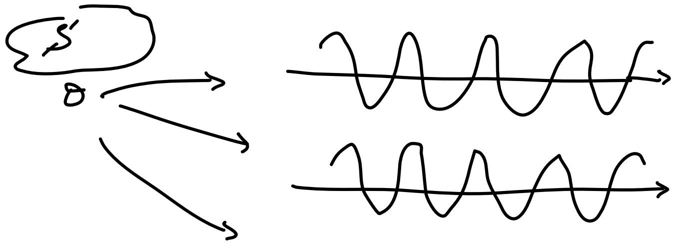
$X(t) \quad \xi \in S \longrightarrow x(t;\xi)$



Realizzazioni o funzioni membro del processo

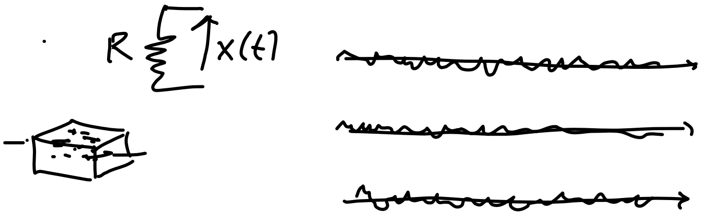
Esempio

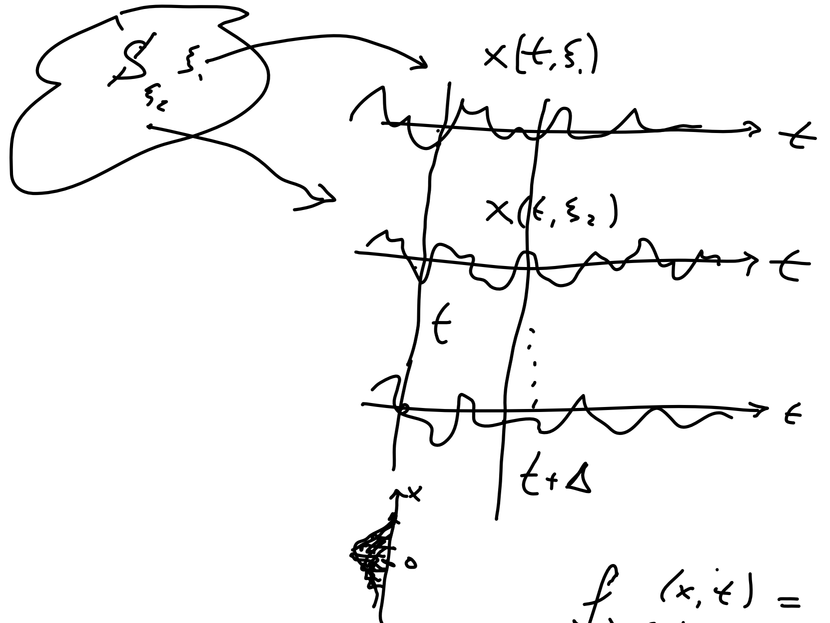
$$X(t) = A\cos(2\pi f_0 t + \Theta) \qquad \Theta \propto f_\Theta(\theta)$$



Processo aleatorio parametrico

non parametrico





$X(t)$ per t fissato è una v.a.

$f_{X(t)}(x,t)$ I ordine

$f_{X(t)}(x,t) = f_{X(t+\Delta)}(x,t+\Delta)$ STAZIONARIO AL I ordine.

X $\quad \xi \in S \to x \in \mathbb{R}$

$f_X(x)$ $\quad \int_{-\infty}^{+\infty} f_X(x)\,dx = 1$

$P_r\{a < X < b\} = \int_a^b f_X(x)\,dx$

$E[X] = \int \xi f_X(\xi)\,d\xi$ $\quad E[X^2] = \int \xi^2 f_X(\xi)\,dx$ $\quad \sigma_X^2 = VAR[X] = E[(X-\mu_X)^2] = E[X^2] - \mu_X^2$

$\mu_X(t) = E[X(t)]$ $\qquad \mu_X(t) = \mu_X$ staz. nella media

$m_X(t) = E[X^2(t)]$ $\qquad m_X(t) = m_X$ staz. nella m.q.

$\Downarrow$

$\sigma_X^2(t) = \sigma_X^2$ staz. nella varianza.

Staz. del I ordine

$f_{X(t)}(x,t) = f_X(x) \ \forall t$ $\Rightarrow$ staz. media

(in senso stretto)

medio quadratico

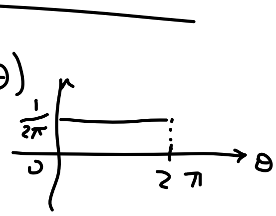
varianza

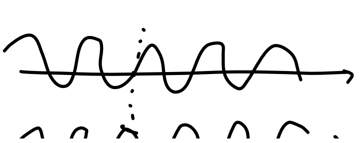
tutti i momenti

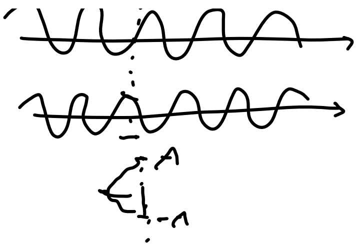
Esempio parametrico

$X(t) = A\cos(2\pi f_0 t + \Theta)$

$\Theta \propto f_\Theta(\theta)$







Teorema della media

$f_X(x)$ X $Y = g(x)$

$E[Y] = \cancel{\int y f_Y(y)\, dy} = \int g(x) f_X(x)\, dx$

$$E[X(t)] = A \int_{(\pi)} \cos(2\pi f_0 t + \theta) f_\Theta(\theta)\, d\theta$$

$$= A \frac{1}{2\pi} \int_0^{2\pi} \cos(2\pi f_0 t + \theta)\, d\theta = \frac{A}{2\pi} \sin(2\pi f_0 t + \theta)\Big]_0^{2\pi}$$

$$= \frac{A}{2\pi} \left(\sin(2\pi f_0 t + 2\pi) - \sin 2\pi f_0 t\right) = 0 \quad \forall t$$

staz. nella media

$$\boxed{\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x}$$

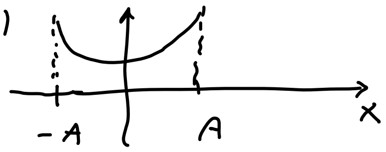
$$E[X^2(t)] = \int_0^{2\pi} A^2 \cos^2(2\pi f_0 t + \theta) \frac{1}{2\pi}\, d\theta =$$

$$= \frac{A^2}{2\pi} \int_0^{2\pi} \left(\frac{1}{2} + \frac{1}{2}\cos(4\pi f_0 t + 2\theta)\right) d\theta$$

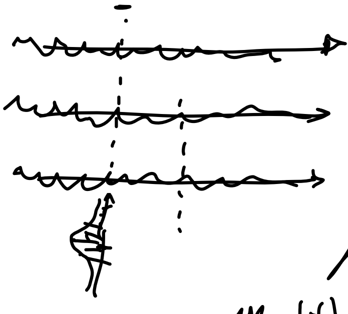
$$= \frac{A^2}{2\pi} \left[\pi + \frac{1}{2} \frac{\sin(4\pi f_0 t + 2\theta)}{2}\Big]_0^{2\pi}\right]$$

$$= \frac{A^2}{2} + \frac{A^2}{8\pi} \left(\sin(4\pi f_0 t + 4\pi) - \sin(4\pi f_0 t)\right) = \frac{A^2}{2} \quad \forall t$$

$$f_{X(t)}(x;t) = f_{X(t)}(x)$$



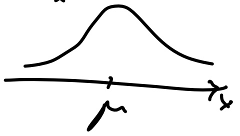
R $X(t)$



Stazionario Gaussiano

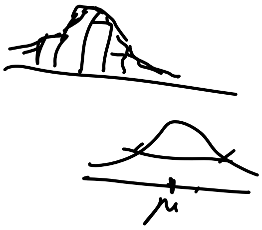
$$f_{X(t)}(x,t) = f_{X(t)}(x) = \mathcal{N}(x; \mu, \sigma_x^2)$$

$$= \frac{1}{\sqrt{2\pi\sigma_x^2}} e^{-\frac{(x-\mu)^2}{2\sigma_x^2}}$$



$$\mu_x(t) = \mu_x = 0$$

$$m_q(t) = m_q = \sigma^2 = \text{cost.} \ \forall t$$



$$\sigma^2 = m_q - \mu_x^2$$

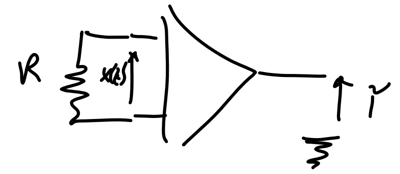
$$\sigma_x^2 = E[(X-\mu_x)^2] = E[X^2 + \mu_x^2 - 2X\mu_x]$$

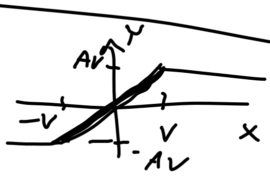
$$= E[X^2] + \mu_x^2 - 2\mu_x \underbrace{E[X]}_{\mu_x} = \underbrace{E[X^2]}_{m_q} - \mu_x^2$$

$$m_q - \mu_x^2$$

$-2E[X] + \mu_x^2 - 2\mu_x E[X] = E[X^2] - \mu_x^2$

$\mu_x \qquad m_2 - \mu_x^2$





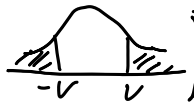
$$y(t) = \begin{cases} A\,x(t) & -V < x(t) < V \\ AV & x(t) > V \\ -AV & x(t) < -V \end{cases}$$

$P_r\{\text{saturazione amplif.}\} =$

$= P_r\{X(t) > V \text{ or } X(t) < -V\}$

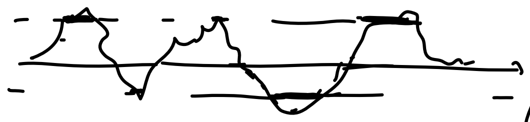
$= 1 - P_r\{\text{non sat.}\}$

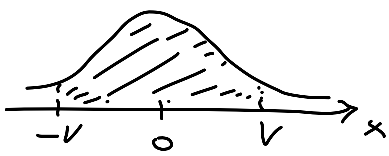
$= 1 - P_r\{-V < X(t) < V\}$

$= 1 - \Big(1 -$  $\Big)$

$= 1 - \left(1 - 2Q\left(\frac{V}{\sigma}\right)\right)$

$= 2Q\left(\frac{V}{\sigma}\right)$ ✓



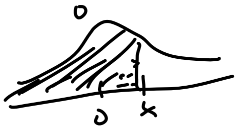


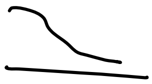
$Q(x)$

$\Phi(x)$

$\sigma^2 = 1$

$Q(x) =$ 

$\Phi(x)$ 



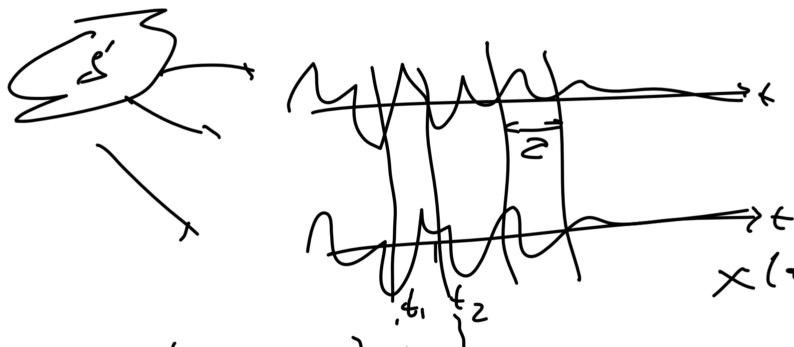
$N(x;\mu,\sigma^2)$

erfc

erfc

$P_r\{a < X < b\} = Q\left(\frac{a-\mu}{\sigma}\right) - Q\left(\frac{b-\mu}{\sigma}\right)$

II ordine



$X(t_1) \quad X(t_2)$

$f_{X(t_1)X(t_2)}(\xi,\eta;t_1,t_2)$

$t\text{-}\tau \quad t$

τ

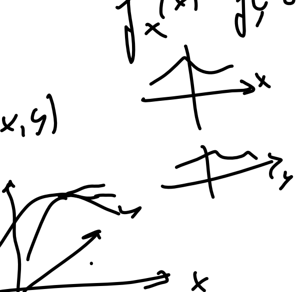
$f_{X(t)X(t-\tau)}(\xi,\eta;t;\tau) = f_{X(t)X(t-\tau)}(\xi,\eta;\tau)$ staz. al II ordine.

$R_X(t;\tau) = E[X(t)X(t-\tau)]$ dipende solo da $\tau = R_X(\tau)$

staz. nella

$X \quad Y \qquad f_X(x) \quad f_Y(y)$

$f_{XY}(x,y)$



$f_X(x) = \int f_{XY}(x,y)\,dy$

$f_Y(y) = \int f_{XY}(x,y)\,dx$

X e Y indipendenti

$f_{XY}(x,y) = f_X(x) f_Y(y)$

$E[XY]$ correlazione

$R_X(t;\tau) = E[X(t)X(t-\tau)]$ dipende solo da τ $= R_X(\tau)$

Funzione di autocorrelazione. Staz. nella autocorrelazione.

$f_{XY}(x,y) = f_X(x) f_Y(y)$

$E[XY]$ correlazione

$E[(X-\mu_x)(Y-\mu_y)] = \mathrm{COV}[XY]$ covarianza

$= E[XY] - \mu_x\mu_y$

X e Y non correlate

$E[XY] = E[X]E[Y]$

$\Rightarrow \mathrm{cov}[XY] = 0$

indipendenza $\Rightarrow$ decorr.

$\cancel{\Leftarrow}$ vero per le var. gaussiane

DEFINIZIONE:

p.a. staz. nella media $E[X(t)] = \mu$

staz. nella autocorr. $E[X(t)X(t-\tau)] = R_X(\tau)$

<u>S</u>TAZIONARIO IN <u>S</u>ENSO <u>L</u>ATO

SSL

(<u>W</u>IDE-<u>S</u>ENSE <u>S</u>TATIONARY

WSS)

STAZ. II ordine $\Longrightarrow$ SSL

$\cancel{\Longleftarrow}$

$\Longleftarrow$ se processi gaussiani (pdf $f_{X(t)X(t-\tau)}(\xi,\eta;\tau)$ gaussiana)

<u>ESEMPIO</u>

processo parametrico

$X(t) = A\cos(2\pi f_0 t + \Theta)$

Θ v.a. $f_\Theta(\theta) = U(0,2\pi)$

$\cos\alpha\cos\beta = \frac{1}{2}\cos(\alpha+\beta) + \frac{1}{2}\cos(\alpha-\beta)$

$E[X(t)] = \mu = 0$

$E[X^2(t)] = \frac{A^2}{2}$

$f_{X(t)}(x,t) = f_{X(t)}(x)$ -> Staz. I ordine.

$R_X(t,\tau) = E[X(t)X(t-\tau)] = E_\Theta[A\cos(2\pi f_0 t + \Theta)A\cos(2\pi f_0(t-\tau)+\Theta)]$

$= A^2 E\left[\frac{1}{2}\cos 2\pi f_0\tau + \frac{1}{2}\cos(2\pi f_0(2t-\tau)+\Theta)\right]$

$= \frac{A^2}{2}\cos 2\pi f_0\tau + \frac{A^2}{2}E[\cos 2\pi f_0(2t-\tau)+\Theta]$

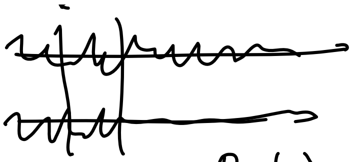
$\hookrightarrow \int_\Theta \cos(2\pi f_0(2t-\tau)+\theta) f_\Theta(\theta)\, d\theta$

$= \int_0^{2\pi} \cos(2\pi f_0(2t-\tau)+\Theta)\frac{1}{2\pi}\, d\theta = 0$

$R_X(t,\tau) = \frac{A^2}{2}\cos 2\pi f_0\tau$ processo è stazionario SSL

$\{ X(t)$

$\{X(t)$



$\mu_x = 0$

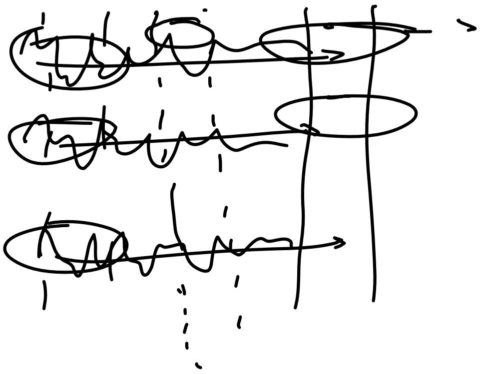
$\sigma_x^2 = \text{cost.}$

$f_x(x) = N(x; 0, \sigma_x^2)$

$R_x(\tau)$ $\Rightarrow$ SSL

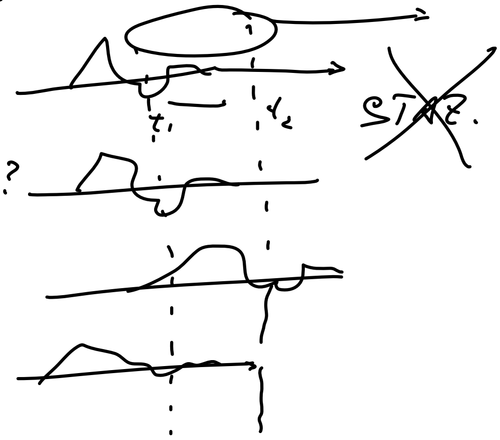
S'

POTENZA



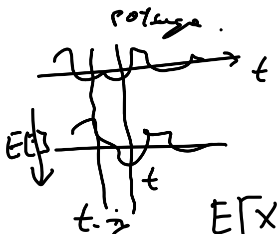
$P_x = ?$

ENERGIA



$\mathcal{E}_x = ?$

PROCESSI (SEGNALI) ALEATORI



$E[X(t)]$

$E[X^2(t)]$

$VAR[X(t)]$

$f_{X(t)}(x;t)$

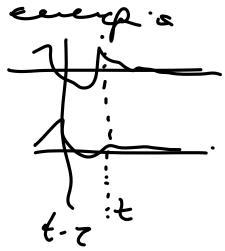
$R_x(t;\tau)=E[X(t)X(t-\tau)]$

$C_x(t;\tau)=E[(X(t)-\mu_x(t))(X(t-\tau)-\mu_x(t-\tau))]$
$=E[X(t)X(t-\tau)]-\mu_x(t)\mu_x(t-\tau)$

$\begin{cases} R_x(t;\tau)=R_x(\tau) \\ E[X(t)]=\mu_x \end{cases}$ SSL

$f_{X(t)X(t-\tau)}(\xi,\eta;t,\tau)=f_x(\xi,\eta;t)$ SSS-II

esempio



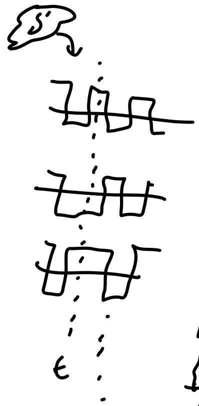
$E[X(t)]=\mu_x(t)$

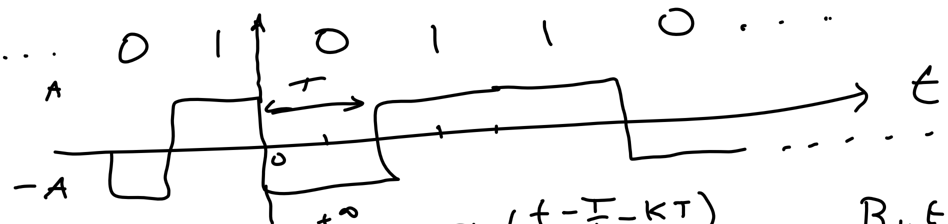
non stazionario

$R_x(t;\tau)=E[X(t)X(t-\tau)]$

$\overline{R_x}(\tau)=\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}R_x(t;\tau)\,dt$

Esempio

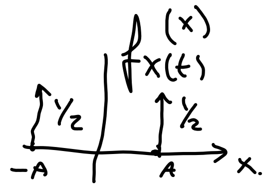




$X(t)=\sum_{k=-\infty}^{+\infty}\beta_k\,\Pi\left(\frac{t-\frac{T}{2}-kT}{T}\right)$

$\beta_k\in\{-A,A\}$

$\Pi=\left(\frac{1}{2},\frac{1}{2}\right)$



$E[X(t)]=\sum_{k=-\infty}^{+\infty}E[\beta_k]\,\Pi\left(\frac{t-\frac{T}{2}-kT}{T}\right)=0$

$-A\frac{1}{2}+A\frac{1}{2}=0$

$R_x(t;\tau)=E[X(t)X(t-\tau)]=E\left[\sum_{k_1}\beta_{k_1}\Pi\left(\frac{t-\frac{T}{2}-k_1T}{T}\right)\sum_{k_2}\beta_{k_2}\Pi\left(\frac{t-\tau-\frac{T}{2}-k_2T}{T}\right)\right]$

$=\sum_{k_1}\sum_{k_2}E[\beta_{k_1}\beta_{k_2}]\,\Pi\left(\frac{t-\frac{T}{2}-k_1T}{T}\right)\Pi\left(\frac{t-\tau-\frac{T}{2}-k_2T}{T}\right)$

$k_1\neq k_2$

-A -A 1/4
-A A 1/4
A -A 1/4
A A 1/4

$=A^2\frac{1}{4}-A^2\frac{1}{4}-A^2\frac{1}{4}+A^2\frac{1}{4}=0$

$\begin{cases}\frac{A^2}{2} & k_1=k_2\\ 0 & k_1\neq k_2\end{cases}$

$k_1=k_2$

$(-A)^2\frac{1}{2}$
$A^2\frac{1}{2}$ $\Big\rangle A^2$

$=\sum_k A^2\,\Pi\left(\frac{t-\frac{T}{2}-kT}{T}\right)\Pi\left(\frac{t-\tau-\frac{T}{2}-kT}{T}\right)$

non è stazionario

non è stazionario

Tesi: Ciclostazionario con periodo T

$t \leftarrow t + nT$

$$A^2 \sum_{k=-\infty}^{+\infty} \Pi\left(\frac{t+nT-\frac{T}{2}-kT}{T}\right)\Pi\left(\frac{t+nT-\tau-\frac{T}{2}-kT}{T}\right)$$

$k' = k-n$

$$A^2 \sum_{k'} \Pi\left(t-\frac{T}{2}-k'T\right)\Pi\left(\frac{t-\tau-\frac{T}{2}-k'T}{T}\right)$$

$R_x(t;\tau)$ periodica in t con periodo T

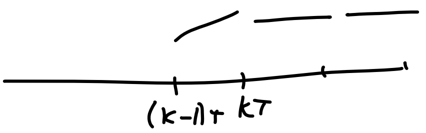
$\Rightarrow$ ciclostazionario con periodo T

$\overline{R_x}(\tau)$ = media nel periodo = $\displaystyle\frac{1}{T}\int_{-T/2}^{T/2} R_x(t;\tau)\,dt$

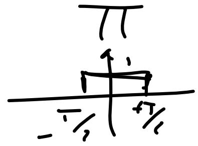
$$= A^2\sum_{k=-\infty}^{+\infty}\frac{1}{T}\int_{-T/2}^{T/2}\Pi\left(t-\frac{T}{2}-kT\right)\Pi\left(t-\tau-\frac{T}{2}-kT\right)dt$$

$\eta = t - kT - \frac{T}{2}$

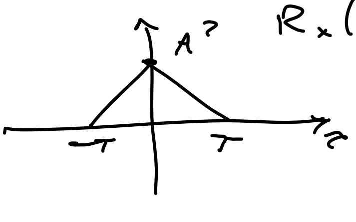
$$= A^2 \sum_{k=-\infty}^{+\infty}\frac{1}{T}\int_{-T-kT}^{-kT}\Pi\left(\frac{\eta}{T}\right)\Pi\left(\frac{\eta-\tau}{T}\right)d\eta$$

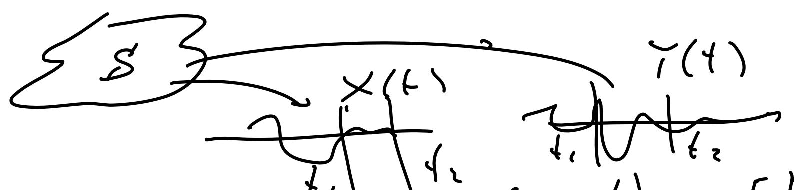


$$= A^2\,\frac{1}{\cancel{T}}\underbrace{\int_{-\infty}^{+\infty}\Pi\left(\frac{\eta}{T}\right)\Pi\left(\frac{\eta-\tau}{T}\right)d\eta}_{\cancel{T}\,\Lambda\left(\frac{\tau}{T}\right)}$$



$$= A^2\,\Lambda\left(\frac{\tau}{T}\right)$$





Mutua correlazione

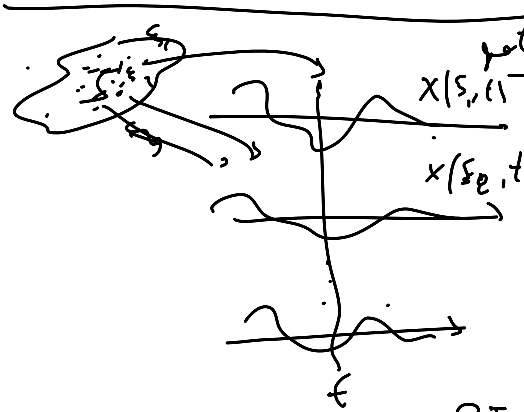
(x,y,t)

$f_{X(t)Y(t)}$

(x,y,t_1,t_2)

$f_{X(t_1)Y(t_2)}$

$E[X(t)Y(t-\tau)]$



$$\Longrightarrow \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}|x(\xi,t)|^2\,dt = P_x(\xi)$$

$$P_x = E[\quad] = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}E[|X(t)|^2]\,dt$$

POTENZA MEDIA

staz.

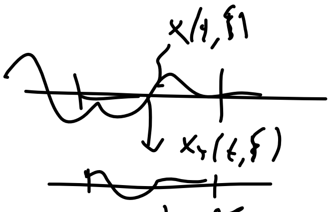
$$= E[|X|^2]$$

STAZ. NELLA MEDIA QUADRATICA

$$P_x = E[|X(t)|^2] = R_x(0)$$

$$R_x(\tau) = E[X(t)X^*(t-\tau)]$$

$\lim_{T\to\infty}\frac{1}{2T}\int \ldots$



$\downarrow \mathcal{F}$

$X_T(f,\xi)$

$\downarrow |\ |^2$

$|X_T(f,\xi)|^2$

$\downarrow E$

$$\lim_{T\to\infty}\frac{E[|X_T(f)|^2]}{2T} = P_x(f)$$

DENSITÀ SPETTRALE DI POTENZA

$$(*)\quad P_x = \int_{-\infty}^{+\infty}P_x(f)\,df$$

$$(**)\quad P_x(f) \overset{\mathcal{F}}{\longleftrightarrow} R_x(\tau)$$

$$\int_{-\infty}^{+\infty}P_x(f)\,df = \lim_{T\to\infty}\frac{1}{2T}\int_{-\infty}^{+\infty}E[|X_T(f)|^2]\,df$$

$$= \lim_{T\to\infty}\frac{1}{2T}E\left[\int_{-\infty}^{+\infty}|X_T(f)|^2\,df\right] = \lim_{T\to\infty}\frac{1}{2T}E\left[\int_{-T}^{T}|x_T(t)|^2\,dt\right]$$

$$= \lim_{T\to\infty}\frac{1}{2T}E\left[\int_{-T}^{T}|X(t)|^2\,dt\right]$$

$$= \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}E[|X(t)|^2]\,dt = E[|X(t)|^2] = P_x$$

$$R_x(\tau) = E[X(t)X^*(t-\tau)] \qquad P_x(f) = \lim_{T\to\infty}\frac{1}{2T}E[|X_T(f)|^2]$$

SSL

$|z|^2 = zz^*$

Tesi: $R_x(\tau) \overset{\mathcal{F}}{\longleftrightarrow} P_x(f)$

T. DI WIENER-KHINCHIN

$$E[|X_T(f)|^2] = E[X_T(f)\,X_T^*(f)]$$

$$= E\left[\int_{-\infty}^{+\infty}x_T(t_1)e^{-j2\pi f t_1}\,dt_1 \int_{-\infty}^{+\infty}x_T^*(t_2)\,e^{j2\pi f t_2}\,dt_2\right]$$

$$= E\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}x(t_1)\,\Pi(t_1\ldots)\,x^*(t_2)\,\Pi(t_2\ldots)\,e^{-j2\pi f(t_1-t_2)}\ldots\right]$$

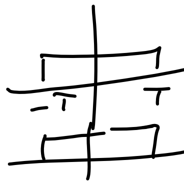
$$= E\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x(t_1)\,\Pi\left(\frac{t_1}{2T}\right)x^*(t_2)\,\Pi\left(\frac{t_2}{2T}\right)e^{-j2\pi f(t_1-t_2)}\,dt_1\,dt_2\right]$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} E[x(t_1)x^*(t_2)]\,\Pi\left(\frac{t_1}{2T}\right)\Pi\left(\frac{t_2}{2T}\right)e^{-j2\pi f(t_1-t_2)}\,dt_1\,dt_2$$

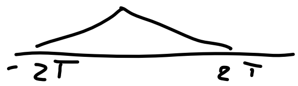
$$\tau = t_1 - t_2 \qquad t = t_1$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \underbrace{E[x(t)x^*(t-\tau)]}_{R_x(\tau)}\,\Pi\left(\frac{t}{2T}\right)\Pi\left(\frac{t-\tau}{2T}\right)e^{-j2\pi f\tau}\,dt\,d\tau$$

$$= \int_{-\infty}^{+\infty} R_x(\tau)\,e^{-j2\pi f\tau}\underbrace{\int_{-\infty}^{+\infty}\Pi\left(\frac{t}{2T}\right)\Pi\left(\frac{t-\tau}{2T}\right)dt}_{2T\,\Lambda\left(\frac{\tau}{2T}\right)}\,d\tau$$

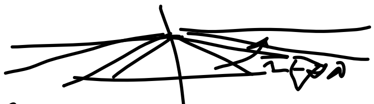


$$= \int_{-\infty}^{+\infty} R_x(\tau)\,e^{-j2\pi f\tau}\,\Lambda\left(\frac{\tau}{2T}\right)2T\,d\tau$$



$$\downarrow \ 1/2T \qquad \lim_{T\to\infty}$$

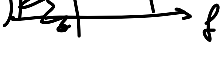
$$P_x(f) = \lim_{T\to\infty}\int_{-\infty}^{+\infty} R_x(\tau)\,e^{-j2\pi f\tau}\,\Lambda\left(\frac{\tau}{2T}\right)d\tau$$



$$= \int R_x(\tau)\,e^{-j2\pi f\tau}\,d\tau = \mathcal{F}[R_x(\tau)] \quad \text{C.V.D.}$$

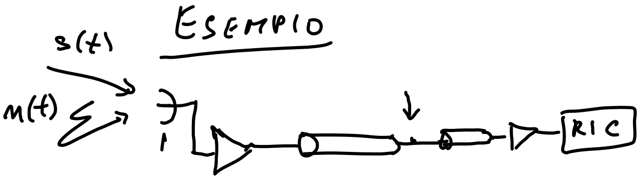
<table>
<tr><th></th><th>Energia</th><th>Potenza</th></tr>
<tr><td>SEGNALI DETERMINISTICI</td><td>$\mathcal{E} = \int_{-\infty}^{+\infty}|x(t)|^2dt$
$\mathcal{E}(f) = |X(f)|^2$
$r(\tau) = \int_{-\infty}^{+\infty}x(t)x^*(t-\tau)dt$
$r(\tau) \overset{\mathcal{F}}{\longleftrightarrow} \mathcal{E}(f)$</td><td>$P = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}|x(t)|^2dt$
$P(f) = \lim_{T\to\infty}\frac{1}{2T}\lim_{U\to\infty}\frac{1}{2U}\int_{-U}^{U}|X_T(f;t_0)|^2dt_0$
$r(\tau) = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}x(t)x^*(t-\tau)dt$
$r(\tau) \longleftrightarrow P(f)$</td></tr>
<tr><td>SEGNALI ALEATORI</td><td>$\mathcal{E}(t) = \int_{-\infty}^{+\infty}E[|X(t)|^2]dt$
⋮</td><td>STAZ. $E[|X(t)|^2] = P_x$
$P_x(f) = \lim_{T\to\infty}\frac{1}{2T}E[|X_T(f)|^2]$
SSL
$R_x(\tau) = E[X(t)X^*(t-\tau)]$
$R_x(\tau) \longleftrightarrow P_x(f)$</td></tr>
</table>

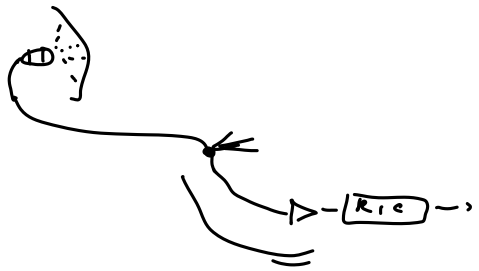
$$x(t) \longrightarrow P_x(f)$$

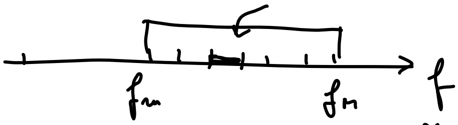


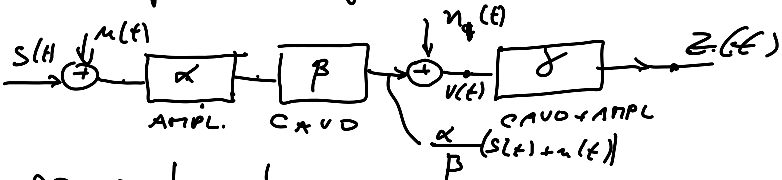
$$R_x(\tau) \xleftarrow{\mathcal{F}^{-1}}$$

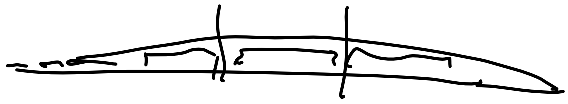
ESEMPIO



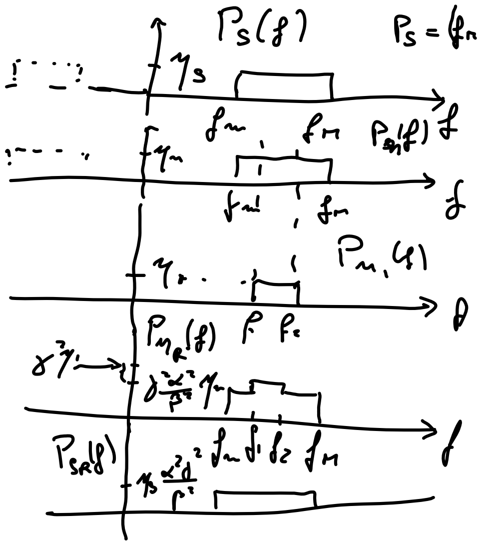








$s(t)$ passa-banda



$P_s = (f_M - f_m)\eta_s 2$

$P_n = (f_M - f_m)\eta_n 2$

$P_{n_1} = (f_2 - f_1)\eta_1 2$

$$v(t) = \frac{\alpha}{\beta}(s(t)+n(t)) + n_q(t)$$

$$z(t) = \gamma\left(\frac{\alpha}{\beta}(s(t)+n(t)) + n_q(t)\right) = s_R(t) + n_R(t)$$

$$r(t) = s(t) + n(t)$$

$$\Downarrow$$

$$R_r(\tau),\ P_r(f)$$

incoerenti

$$z(t) = \underbrace{\frac{\gamma\alpha}{\beta}s(t)}_{s_R(t)} + \underbrace{\frac{\gamma\alpha}{\beta}n(t) + \gamma n_1(t)}_{n_R(t)}$$

$$P_{s_R}(f) = \frac{\gamma^2\alpha^2}{\beta^2}P_s(f)$$

$$P_{n_R}(f) = \frac{\alpha^2\gamma^2}{\beta^2}P_n(f) + \gamma^2 P_{n_1}(f)$$

$$\left(\frac{S}{N}\right)_R = \frac{P_{s_R}}{P_{n_R}} = \frac{\frac{\alpha^2\cancel{\gamma^2}}{\beta^2}P_s}{\frac{\alpha^2\cancel{\gamma^2}}{\beta^2}P_n + \cancel{\gamma^2}P_{n_1}} = \frac{P_s}{P_n + \frac{\beta^2}{\alpha^2}P_{n_1}}$$

$$\hat{h} \to y(t) = (h * s)(t)$$

$s(t)$ SStaz. in senso lato $\to$ $\boxed{h(t)}$ $\to$ $Y(f) = \underbrace{H(f)}_{\text{Risposta armonica}} S(f)$

$P_s(f)$

$\updownarrow \mathcal{F}$

$R_s(\tau) \triangleq E[s(t)s^*(t-\tau)]$

$R_y(\tau)$?

$P_y(f)$?

Tesi: $P_y(f) = |H(f)|^2 P_s(f)$

$\updownarrow$

$R_y(\tau) = (\tau_h * R_s)(\tau)$ $\qquad \tau_h(\tau) \triangleq \int_{-\infty}^{+\infty} h(t)\, h(t-\tau)\, dt$

$s(t) \to \boxed{h(t)} \to y(t)$

$$R_y(t,\tau) = E[y(t)y^*(t-\tau)]$$

$$= E\left[\int_{-\infty}^{+\infty} h(\eta)s(t-\eta)\,d\eta \int_{-\infty}^{+\infty} h^*(\alpha)s^*(t-\tau-\alpha)\,d\alpha\right]$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h(\eta)h^*(\alpha)\underbrace{E[s(t-\eta)s^*(t-\tau-\alpha)]}_{t-\eta-t+\tau+\alpha}\,d\alpha\, d\eta$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h(\eta)h^*(\alpha)R_s(\tau-\eta+\alpha)\,d\alpha\, d\eta$$

$$\gamma = \eta - \alpha$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h(\eta)h^*(\eta-\gamma)R_s(\tau-\gamma)\,d\eta\, d\gamma$$

segnale di energia

$$= \int_{-\infty}^{+\infty} R_s(\tau-\gamma)\underbrace{\int_{-\infty}^{+\infty} h(\eta)h^*(\eta-\gamma)\,d\eta}\, d\gamma$$

$$\int_{-\infty}^{+\infty} R_s(\tau-\gamma)\tau_h(\gamma)\,d\gamma \; \boxed{= (\tau_h * R_s)(\tau) \quad R_y(\tau)}$$

$$\boxed{P_y(f) = |H(f)|^2 P_s(f)}$$

f di trasf dell'energia

Esempio

$n(t)$ SSL

$s(t)$ SSL $\to (+) \to$ $z(t) = s(t)+n(t)$

$Z(f) = S(f)+N(f)$

	E	P.
det.	•	•
rand.	x	•

$$R_z(t;\tau) = E[z(t)z^*(t-\tau)]$$

$$= E[(s(t)+n(t))(s^*(t-\tau)+n^*(t-\tau))]$$

$$= E[s(t)s^*(t-\tau)] + E[n(t)n^*(t-\tau)]$$

$n(t)$ e $s(t)$ congiuntamente

$$= E[s(t)\,s^*(t-\tau)] + E[n(t)\,n^*(t-\tau)] + E[n(t)\,s^*(t-\tau)] + E[s(t)\,n^*(t-\tau)]$$

$n(t)$ e $s(t)$ congiuntamente SSL

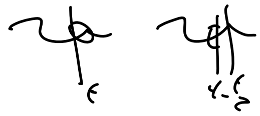
$$R_z(\tau) = R_s(\tau) + R_n(\tau) + R_{ns}(\tau) + R_{sn}(\tau)$$

$$P_z(f) = P_s(f) + P_n(f) + P_{ns}(f) + P_{sn}(f)$$

$$E[n(t)\,s^*(t-\tau)] = \underbrace{E[n(t)]}_{0}\,\underbrace{E[s^*(t-\tau)]}_{0} = 0 \quad \forall \tau$$

DEF. $n(t)$ e $s(t)$ sono <u>scorrelati</u>

$$E[n(t)\,s^*(t)] = E[n(t)]\,E[s^*(t)]$$



$n(t)$ e $s(t)$ sono <u>ortogonali</u>

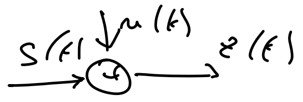
$$E[n(t)\,s^*(t)] = 0$$

$n(t)$ e $s(t)$ sono <u>incoerenti</u>

$$E[n(t)\,s^*(t-\tau)] = E[n(t)]\,E[s^*(t-\tau)] = 0 \quad \forall \tau$$

$n(t)$ e $s(t)$ sono ortogonali

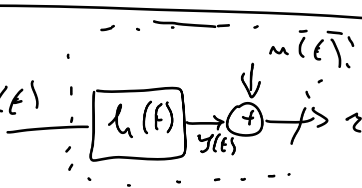
$$R_z(0) = R_s(0) + R_n(0)$$
$$P_z = P_s + P_n$$



$n(t)$ e $s(t)$ sono incoerenti

$$R_z(\tau) = R_s(\tau) + R_n(\tau)$$
$$P_z(f) = P_s(f) + P_n(f)$$

<u>Esempio</u>



$$z(t) = \underbrace{(h * s)(t)}_{s_R(t)} + \underbrace{n(t)}_{n_R(t)}$$

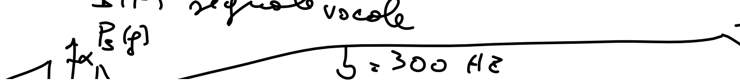
$s(t)$ p.a. SSL $P_s(f)$
$n(t)$ p.a. SSL $P_n(f)$
$s(t)$ e $n(t)$ incoerenti.

$$P_y(f) = |H(f)|^2 P_s(f) \longleftrightarrow R_y(\tau) = (r_h * R_s)(\tau)$$

$$P_z(f) = |H(f)|^2 P_s(f) + P_n(f) \longleftrightarrow R_z(\tau) = (r_h * R_s)(\tau) + R_n(\tau)$$

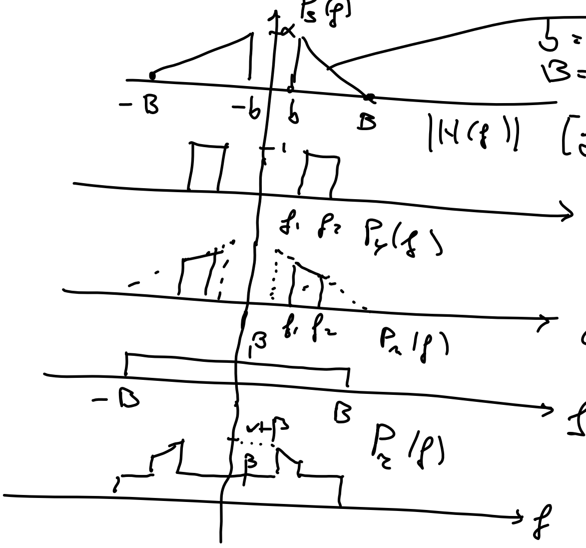
$$\left(\frac{S}{N}\right)_R = \frac{P_{s_R}}{P_{n_R}} = \frac{P_y}{P_n} = \frac{\int_{-\infty}^{+\infty} |H(f)|^2 P_s(f)\,df}{\int_{-\infty}^{+\infty} P_n(f)\,df}$$

$s(t)$ segnale vocale



$$-\frac{\alpha}{B-b} f + \alpha$$

$s(f)$ segnale vocale



$b = 300\ Hz$
$B = 4000\ Hz$

$|H(f)|$ $[f_1, f_2] = [500, 3000]\ Hz$

$$\left(\frac{S}{N}\right)_R = \frac{P_y}{P_n} =$$

$$-\frac{\alpha}{B-b} f + x$$

$$-\frac{\alpha}{B-b} B + x = 0$$

$$x = +\frac{\alpha B}{B-b}$$

$$-\frac{\alpha}{B-b} f + \frac{\alpha B}{B-b}$$

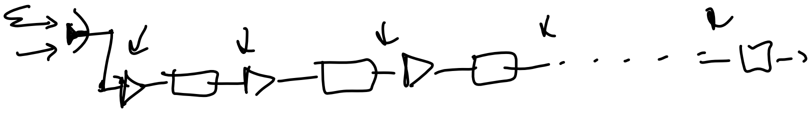
$$\ell(f) = -\frac{\alpha}{B-b}(f - B)$$

$$P_y = (\ell(f_2) + \ell(f_1))(f_2 - f_1)$$

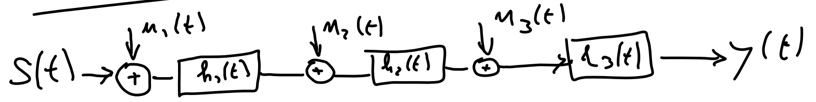
$$P_n = 2B\beta$$

$$\left(\frac{S}{N}\right)_{R\,dB} = 10\log_{10}\left(\frac{S}{N}\right)_R$$

MV



SISTEMI A CASCATA



$$y(t) = \underbrace{(h_3 * h_2 * h_1 * s)(t)}_{s_R(t)} + \underbrace{(h_3 * h_2 * h_1 * n_1)(t) + (h_3 * h_2 * n_2)(t) + (h_3 * n_3)(t)}_{n_R(t)}$$

$$Y(f) = H_3(f) H_2(f) H_1(f) S(f) + H_3(f) H_2(f) H_1(f) N_1(f) + H_3(f) H_2(f) N_2(f) + H_3(f) N_3(f)$$

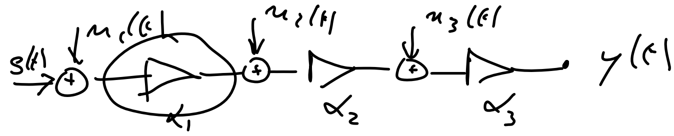
$$P_y(f) = \underbrace{|H_3(f)|^2 |H_2(f)|^2 |H_1(f)|^2 P_s(f)}_{P_{S_R}(f)} + \underbrace{|H_3(f)|^2 |H_2(f)|^2 |H_1(f)|^2 P_{n_1}(f) + |H_3(f)|^2 |H_2(f)|^2 P_{n_2}(f) + |H_3(f)|^2 P_{n_3}(f)}_{P_{n_R}(f)}$$

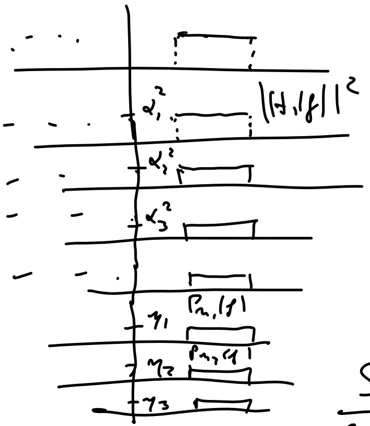
$$\left(\frac{S}{N}\right)_R = \frac{P_{SR}}{P_{nR}} = \frac{\int |H_3(f)|^2 |H_2(f)|^2 |H_1(f)|^2 P_s(f)\, df}{\int |H_3(f)|^2 |H_2(f)|^2 |H_1(f)|^2 P_{n_1}(f)\, df + \int |H_3(f)|^2 |H_2(f)|^2 P_{n_2}(f)\, df + \int |H_3(f)|^2 P_{n_3}(f)\, df}$$

Cascata di amplificatori e attenuatori:

$$\int |H_3(f)|^2 |H_2(f)|^2 |H_1(f)|^2 P_{n_1}(f)\,df + \int |H_3(f)|^2 |H_2(f)|^2 P_{n_2}(f)\,df + \int |H_3(f)|^2 P_{n_3}(f)\,df$$

Cascata di amplificatori o attenuatori:

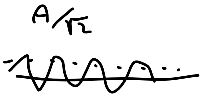




$$y(t) = \underbrace{\alpha_1\alpha_2\alpha_3\, s(t)}_{s_R(t)} + \underbrace{\alpha_1\alpha_2\alpha_3\, n_1(t) + \alpha_2\alpha_3\, n_2(t) + \alpha_3\, n_3(t)}_{n_R(t)}$$

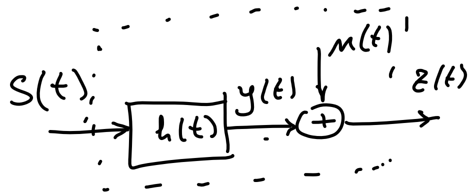
$$P_y(f) = \underbrace{\alpha_1^2\alpha_2^2\alpha_3^2\, P_s(f)}_{P_{s_R}(f)} + \underbrace{\alpha_1^2\alpha_2^2\alpha_3^2 P_{n_1}(f) + \alpha_2^2\alpha_3^2 P_{n_2}(f) + \alpha_3^2 P_{n_3}(f)}_{P_{n_R}(f)}$$

$$\frac{S}{N} = \frac{P_{s_R}}{P_{n_R}} = \frac{\alpha_1^2\alpha_2^2\alpha_3^2\, P_s}{\alpha_1^2\alpha_2^2\alpha_3^2 P_{n_1} + \alpha_2^2\alpha_3^2 P_{n_2} + \alpha_3^2 P_{n_3}}$$

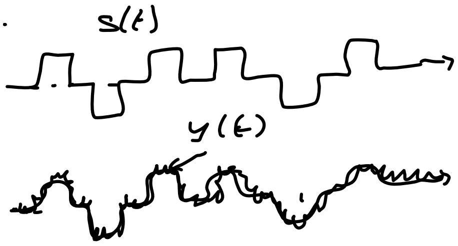


$$= \frac{P_s}{P_{n_1} + \frac{1}{\alpha_1^2} P_{n_2} + \frac{1}{\alpha_1^2\alpha_2^2} P_{n_3}}$$

$\sqrt{P_s} = 1\,\mu V$



Es.



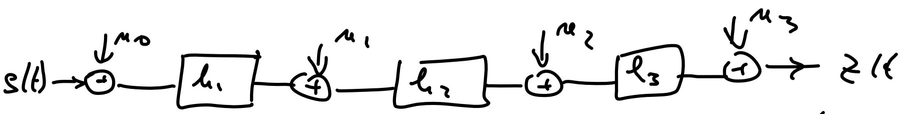
NOTAZIONE

Processi — Deterministici

$R_s(t;\tau)=E[s(t)s^*(t-\tau)]$

$\mathcal{R}_s(\tau)=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}s(t)s^*(t-\tau)\,dt$

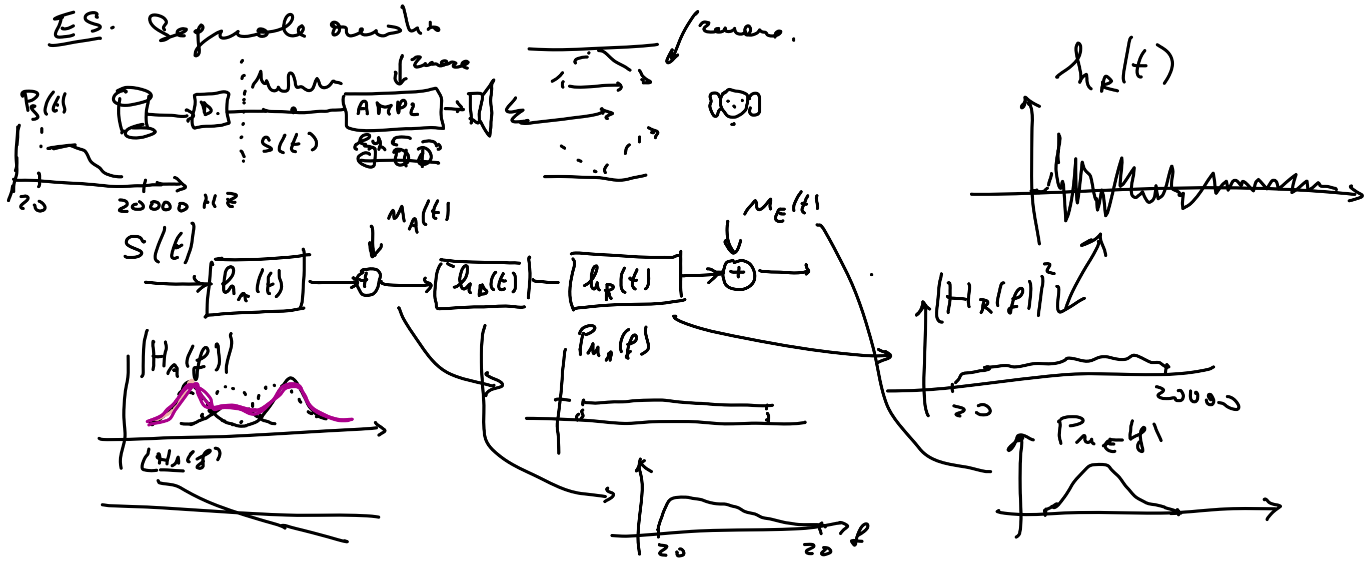
$\langle s(t)s^*(t-\tau)\rangle$

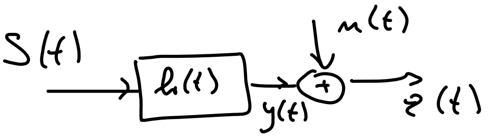


$$z(t)=\underbrace{(h_3*h_2*h_1*s)(t)}_{y(t)}+\underbrace{(h_3*h_2*h_1*n_0)(t)+(h_3*h_2*n_1)(t)+(h_3*n_2)(t)+n_3(t)}_{n(t)}$$

$$h(t)=(h_3*h_2*h_1)(t).$$

ES. Segnale musicale





s(t) p.a. SSL
$E[s(t)]=0$
$P_s(f)$

n(t) p.a. SSL
$E[n(t)]=0$
$P_n(f)$

s(t) e n(t) sono incoerenti:

$E[s(t)n(t-\tau)]=0\quad\forall\tau$

$$z(t)=\overbrace{(h*s)(t)}^{s_R(t)}+\overbrace{n(t)}^{n_R(t)}$$

$$Z(f)=H(f)S(f)+N(f)$$

$$P_z(f)=\underline{|H(f)|^2P_s(f)}+P_n(f)$$

$$\left(\frac{S}{N}\right)=\frac{P_{s_R}}{P_{n_R}}=\frac{\int_{-\infty}^{+\infty}|H(f)|^2P_s(f)\,df}{\int_{-\infty}^{+\infty}P_n(f)\,df}$$

ESEMPIO

s(t)

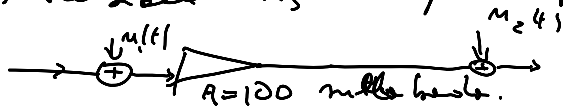
~~[illegible]~~ s(t) tensione $\sqrt{P_s}=10\,\mu V$ piatto in frequenza nella banda

n(t) . n(t) $n_2(t)$ [88 - 100] MHz

s(t) ~~...~~

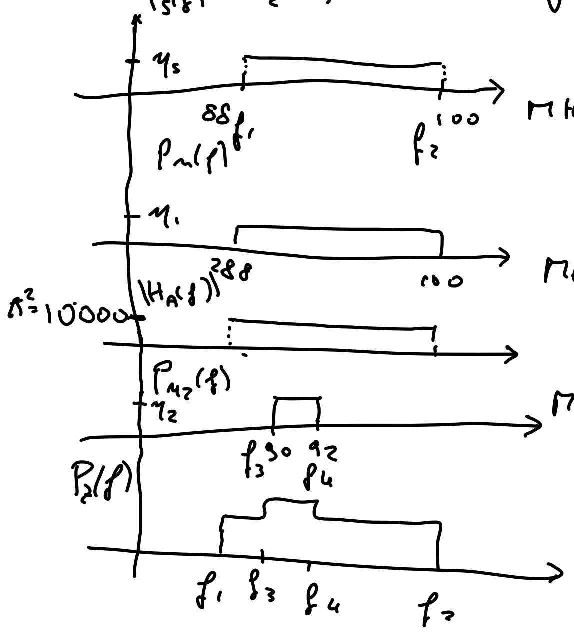
n(t)

$s(t)$ tensione $\sqrt{P_s} = 10\,\mu V$ piatto in frequenza nella banda $[88-100]$ MHz



$n(t)$ tensione $\sqrt{P_{n_1}} = 3\,\mu V$ piatto nella stessa banda

$n_2(t)$ tensione $\sqrt{P_{n_2}} = 1\,\mu V$ piatto nella banda $[90-92]$ MHz.



$$P_s = 100(\mu V)^2 = (f_2 - f_1)\eta_s 2$$

$$\eta_s = \frac{P_s}{2(f_2-f_1)} = \frac{100}{2\cdot 12}\,\frac{(\mu V)^2}{MHz} = 4.17\,\frac{(\mu V)^2}{MHz}$$

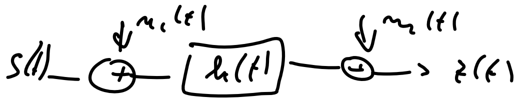
$$P_{n_1} = 9(\mu V)^2$$

$$\eta_1 = \frac{P_{n_1}}{2(f_2-f_1)} = \frac{9}{24}\,\frac{(\mu V)^2}{MHz} = 0.38$$

$$P_{n_2} = 1(\mu V)^2$$

$$\eta_2 = \frac{P_{n_2}}{2(f_4-f_3)} = \frac{1}{4} = 0.25\,\frac{(\mu V)^2}{MHz}$$

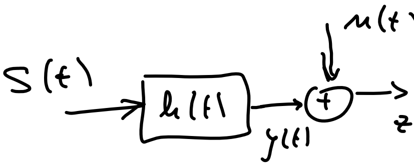
$P_z =$



$$z(t) = \underbrace{(h*s)(t)}_{A s(t)} + \underbrace{(h*n_1)(t)}_{A n_1(t)} + \underbrace{n_2(t)}_{n_2(t)}$$

$$\left(\frac{S}{N}\right) = \frac{A^2 P_s}{A^2 P_{n_1} + P_{n_2}} = \frac{10^4\cdot 100}{10^4\cdot 9 + 1} = \frac{10^6}{9\cdot 10^4 + 1}$$

$$(SNR)_{dB} = 10\log_{10}\left(\ \right) = \_\_$$



$$z(t) = \underbrace{(h*s)(t)}_{S_R(t)} + \underbrace{n(t)}_{n_R(t)}$$

$$\left(\frac{S}{N}\right) = \frac{P_{SR}}{P_{nR}} = \frac{\int_{-\infty}^{+\infty} |H(f)|^2 P_s(f)\,df}{\int_{-\infty}^{+\infty} P_n(f)\,df}$$

$$e(t) = z(t) - s(t)$$

$$e(t) = z(t) - K s(t-t_0)$$

$$z(t) = K s(t-t_0) + \underline{e(t)}$$ e contiene dist. e rumore

$z(t) = K s(t - t_0) + e(t)$ contiene (dist. e rumore)

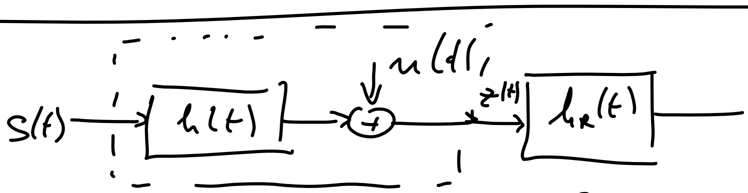
$$\left(\frac{S}{N}\right) = \frac{\text{Potenza di segnale indistorto}}{\text{Potenza della distorsione} + \text{Potenza rumore}}$$

$$= \frac{K^2 P_s}{\int_{-\infty}^{+\infty} \rho_c(f) P_s(f)\, df + P_n}$$

|Hc(f)| 2 + K2 − 2K |Hc(f)|cos(2πf t0 + φc(f)).

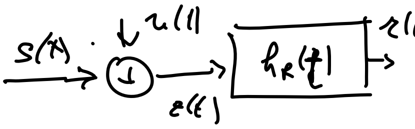
$$\rho_c(f) = |H(f)|^2 + K^2 - 2K |H(f)| \cos(2\pi f t_0 + \phi(f))$$

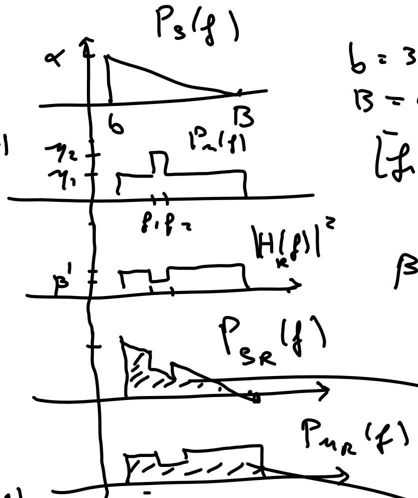
$$h(t) \overset{\mathcal{F}}{\longleftrightarrow} H(f) = |H(f)|\, e^{j \angle H(f)}$$



Esempio

$s(t)$ Segnale vocale





$b = 300$ Hz
$B = 4000$ Hz.
$[f_1 \; f_2] = [1000 \;\; 1100]$ Hz.

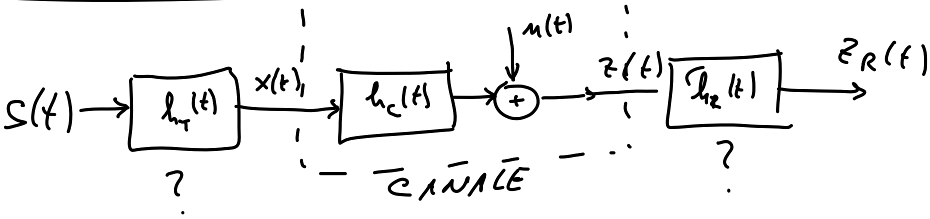
$\beta < 1$

$$z(t) = \underbrace{(h_R * s)(t)}_{s_R(t)} + \underbrace{(h_R * n)(t)}_{n_R(t)}$$

$$P_z(f) = \underbrace{|H_R(f)|^2 P_s(f)}_{P_{sR}(f)} + \underbrace{|H_R(f)|^2 P_n(f)}_{P_{nR}(f)}$$

$$\left(\frac{S}{N}\right) = \frac{P_{sR}}{P_{nR}}$$

$$\left(\frac{S}{N}\right) = \frac{K^2 P_s}{P_{dist.} + P_{nR}} = \ldots\ldots$$



$$z_R(t) = \underbrace{(h_R * h_c * h_T * s)(t)}_{s_R(t)} + \underbrace{(h_R * n)(t)}_{n_R(t)}$$

$Z_R(f) = H_R(f) H_c(f) H_T(f) S(f) + H_R(f) N(f)$

$$\underbrace{\qquad\qquad}_{s_R(t)} \qquad \underbrace{\qquad\qquad}_{n_R(t)}$$

$$Z_R(f) = H_R(f)\, H_C(f)\, H_T(f)\, S(f) + H_R(f) N(f)$$

$$P_{Z_R}(f) = |H_R(f)|^2 |H_C(f)|^2 |H_T(f)|^2 P_S(f) + |H_R(f)|^2 P_n(f)$$

$$\left(\frac{S}{N}\right)_R = \frac{\int |H_R(f)|^2 |H_C(f)|^2 |H_T(f)|^2 P_S(f)\, df}{\int |H_R(f)|^2 P_n(f)\, df}$$

$H_R(f)$?
$H_T(f)$?

$$s_R(t) = K\, s(t - t_0)$$

$$H_R(f)\, H_C(f)\, H_T(f) = K e^{-j2\pi f t_0} \qquad f \in I_S$$

BANDA DI INTERESSE

$$\begin{cases} |H_R(f)|\,|H_C(f)|\,|H_T(f)| = K \Leftarrow \\ \angle H_R(f) + \angle H_C(f) + \angle H_T(f) = -2\pi f t_0 \Leftarrow \end{cases}$$

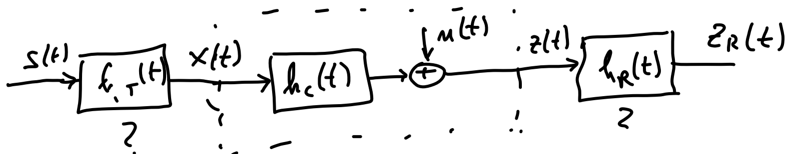
$$\frac{P_{S_R}}{P_{n_R}} = \frac{K^2 P_S}{\int |H_R(f)|^2 P_n(f)\, df} \Leftarrow \text{massimizzare} ???$$

$$\begin{cases} H_T(f) \\ H_R(f) \end{cases} ??$$

$$\boxed{\min \frac{P_{n_R}}{P_{S_R}} \quad P_x}$$

vincolo

$$|H_T(f)|\,|H_C(f)|\,|H_R(f)| = K$$

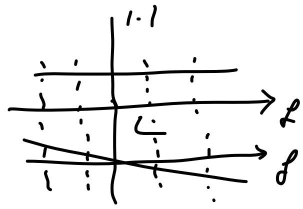


$$z_R(t) = \underbrace{(h_R * h_C * h_T * s)(t)}_{s_R(t)} + \underbrace{(h_R * n)(t)}_{n_R(t)}$$

$$Z_R(f) = H_R(f)\, H_c(f)\, H_T(f)\, S(f) + H_R(f)\, N(f)$$

$$P_R(f) = \underbrace{|H_R(f)|^2 |H_c(f)|^2 |H_T(f)|^2 P_s(f)}_{P_{SR}(f)} + \underbrace{|H_R(f)|^2 P_n(f)}_{P_{nR}(f)}$$

$$\left(\frac{S}{N}\right) = \frac{P_{SR}}{P_{nR}} = \frac{\int |H_R(f)|^2 |H_c(f)|^2 |H_T(f)|^2 P_s(f)\, df}{\int |H_R(f)|^2 P_n(f)}$$



$$H_R(f)\, H_c(f)\, H_T(f) = K e^{-j 2\pi f t_0} \qquad f \in I_f$$

$$|H_R(f)|^2 |H_c(f)|^2 |H_T(f)|^2 = K^2$$

$$\left(\frac{S}{N}\right) = \frac{K^2 P_s}{\int |H_R(f)|^2 P_n(f)} = \frac{P_{SR}}{P_{nR}}$$

Nuova funzione di costo

$$\min \frac{P_{nR}}{P_{SR}} P_x = \frac{\int |H_R(f)|^2 P_n(f)\, df \int |H_T(f)|^2 P_s(f)\, df}{\int |H_T(f)|^2 |H_c(f)|^2 |H_R(f)|^2 P_s(f)\, df} = \frac{\int |H_R(f)|^2 P_n(f)\, df \cdot \int |H_T(f)|^2 P_s(f)\, df}{K^2 P_s}$$

$$\begin{cases} H_{T0}(f) \text{ e } H_{R0}(f) = \underset{H_T(f)\, H_R(f)}{\arg\min} \dfrac{P_{nR} P_x}{P_{SR}} \\ H_T(f)\, H_c(f)\, H_R(f) = K e^{-j 2\pi f t_0} \quad f \in I_f \end{cases}$$

contano solo i moduli.

Dis. di Swartz: $V, W \in L_2$

$$\left| \int V \cdot W^* \right|^2 \leq \int |V|^2 \cdot \int |W|^2$$

$=$ se $V = \alpha W$ $\Rightarrow$ il minimo è ottenuto quando

$$\begin{cases} |H_{R0}(f)|\, P_n(f)^{1/2} = \alpha\, |H_{T0}(f)|\, P_s(f)^{1/2} \\ |H_{T0}(f)|\, |H_c(f)|\, |H_{R0}(f)| = K \end{cases}$$

$$|H_{R0}(f)| = \alpha\, |H_{T0}(f)| \left(\frac{P_s(f)}{P_n(f)}\right)^{1/2} = \alpha \frac{K}{|H_c(f)|\,|H_{R0}(f)|} \left(\frac{P_s(f)}{P_n(f)}\right)^{1/2}$$

$$\boxed{|H_{R0}(f)|^2 = \frac{\alpha K}{|H_c(f)|} \left(\frac{P_s(f)}{P_n(f)}\right)^{1/2}}$$

$$\boxed{\ldots \; |H_c(f)| \left(P_n(f)\right) \ldots}$$

$$|H_{T_0}(f)| = \frac{1}{\alpha}|H_{R_0}(f)|\left(\frac{P_n(f)}{P_s(f)}\right)^{1/2} = \frac{1}{\alpha}\frac{K}{|H_{T_0}(f)||H_c(f)|}\left(\frac{P_n(f)}{P_s(f)}\right)^{1/2}$$

$$\boxed{|H_{T_0}(f)|^2 = \frac{K}{\alpha}\frac{1}{|H_c(f)|}\left(\frac{P_n(f)}{P_s(f)}\right)^{1/2}}$$

Esempio

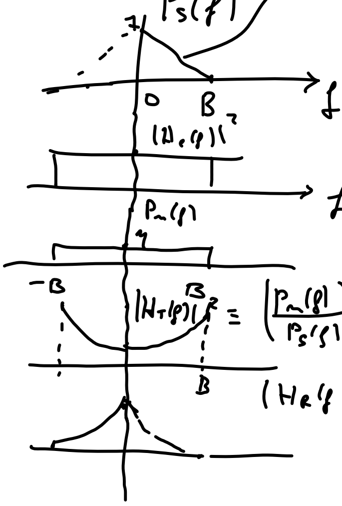
$$\to \boxed{H_T(f)} \to \oplus \xrightarrow{} \boxed{H_R(f)} \to \qquad n(t)\downarrow$$

$$|H_T(f)|^2 = \left(\frac{P_n(f)}{P_s(f)}\right)^2 \qquad |H_R(f)|^2 = \left(\frac{P_s(f)}{P_n(f)}\right)^2$$

PRE-ENFASI

ESEMPIO

$-\frac{1}{B}f+1$



$|H_c(f)| =$ ~~cost~~.

$H_T($

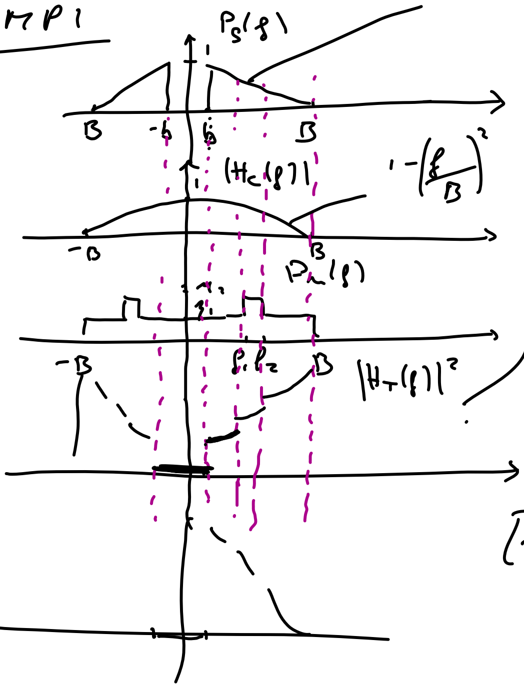
$$|H_T(f)|^2 = \left(\frac{P_n(f)}{P_s(f)}\right)^{1/2} \equiv \frac{1}{\sqrt{-\frac{1}{B}f+1}}$$

$$|H_R(f)|^2 = \left(\frac{P_s(f)}{P_n(f)}\right)^{1/2} \equiv \sqrt{-\frac{1}{B}f+1}$$

ESEMPI

$-\frac{1}{B-b}(f-B)$

$1-\left(\frac{f}{B}\right)^2$



$$|H_{T_0}(f)|^2 = \frac{K}{\alpha}\frac{1}{|H_c(f)|}\left(\frac{P_n(f)}{P_s(f)}\right)^{1/2}$$

$$|H_{R_0}(f)| = \frac{\alpha K}{|H_c(f)|}\left(\frac{P_s(f)}{P_n(f)}\right)^{1/2}$$

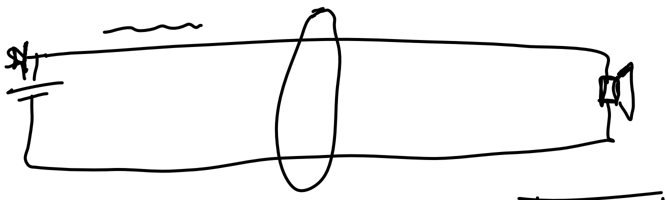
$[b, f_1]$ $\quad \frac{K}{\alpha}\frac{1}{1-\left(\frac{f}{B}\right)^2}\frac{\sqrt{\eta_1}}{\sqrt{-\frac{1}{B-b}(f-B)}}$

$[f_1, f_2]$ $\quad \frac{K}{\alpha}\frac{1}{1-\left(\frac{f}{B}\right)^2}\frac{\sqrt{\eta_2}}{\sqrt{-\frac{1}{B-b}(f-B)}}$

$[f_2\ B]$ . . . . . .

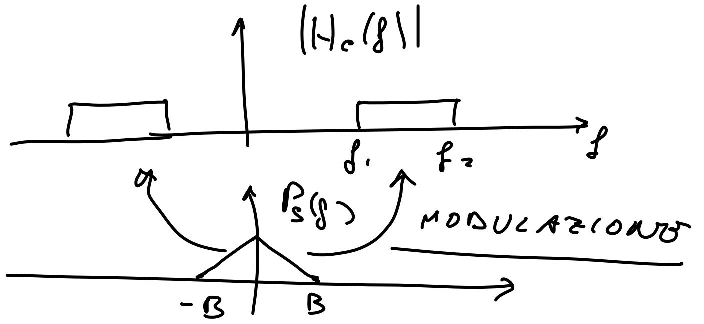
MODULAZIONE

$s(t) \longrightarrow$ CAVO $\longrightarrow$ Destinazione.

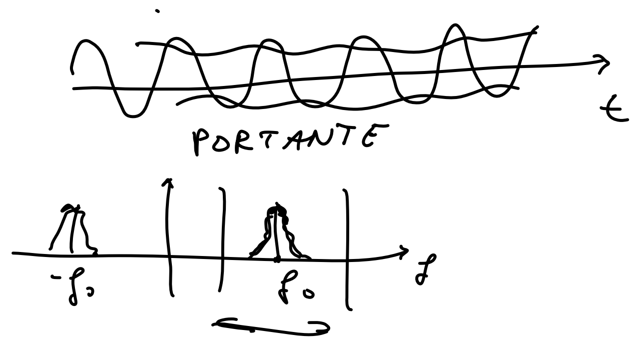


$s(t) \to$ Trasf. $\to$ canale $\to$ Trasf.$^{-1}$ $\to \hat{s}(t)$

ESEMPIO: CANALE PASSA-BANDA



MOD. LINEARE SU PORTANTE SINUSOIDALE



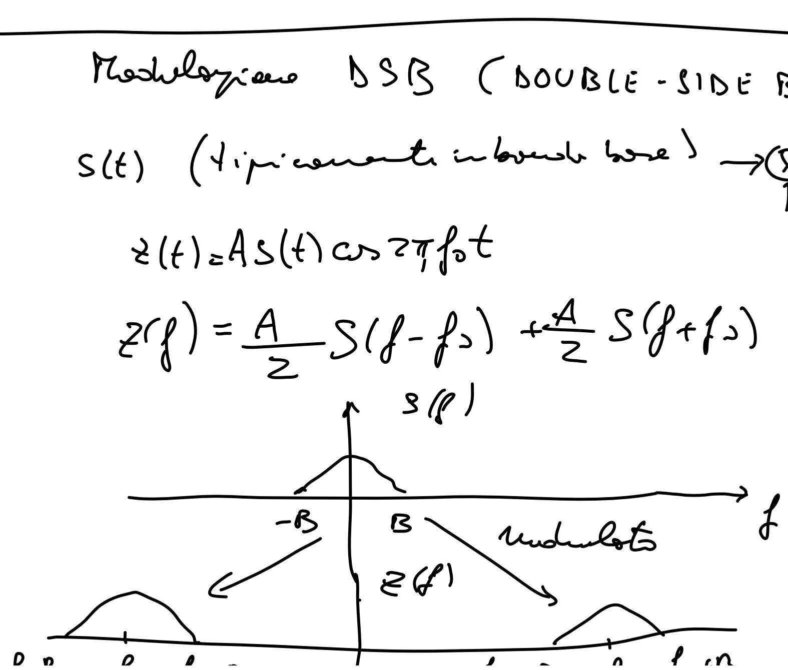
Modulazione DSB (DOUBLE-SIDE BAND)

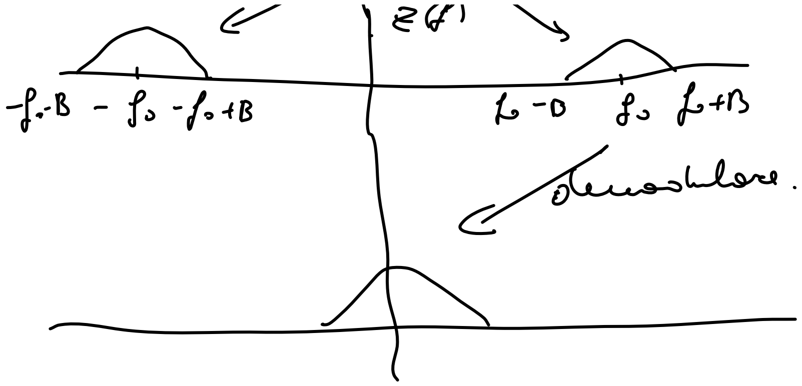
$s(t)$ (tipicamente in banda base) $\to \otimes \to z(t)$

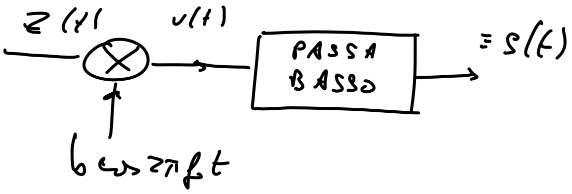
$\uparrow A\cos 2\pi f_0 t$

$$z(t) = A\,s(t)\cos 2\pi f_0 t$$

$$Z(f) = \frac{A}{2} S(f-f_0) + \frac{A}{2} S(f+f_0)$$





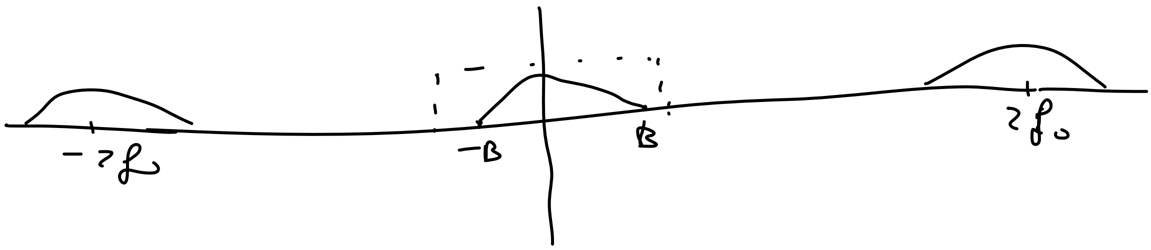


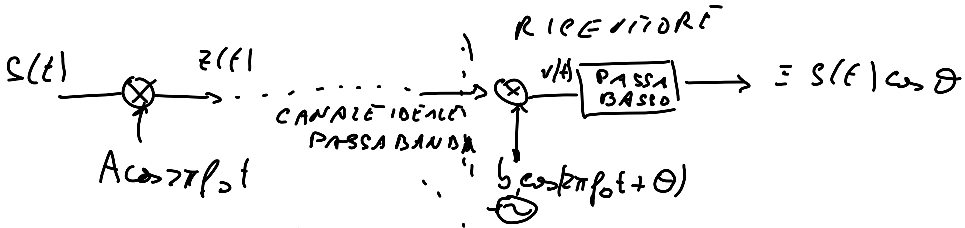
$\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$

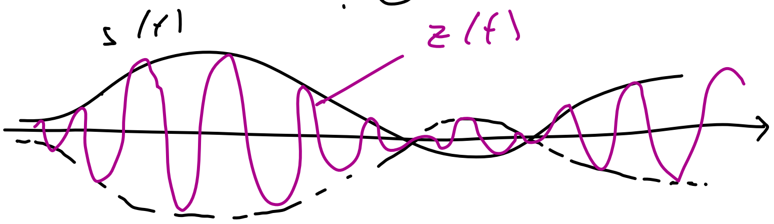
$$v(t) = A\,s(t)\cos 2\pi f_0 t \cdot b\cos 2\pi f_0 t$$

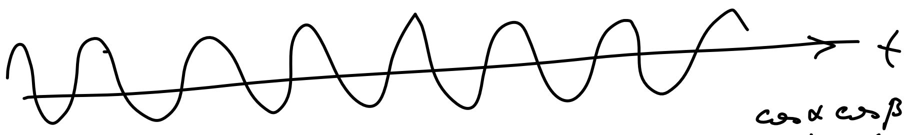
$$= \frac{A b\, s(t)}{2} + \frac{A b\, s(t)}{2}\cos 2\pi 2 f_0 t$$

$$V(f) = \underbrace{\frac{Ab}{2} S(f)} + \frac{A b}{4} S(f - 2f_0) + \frac{Ab}{4} S(f + 2f_0)$$









$\cos\alpha \cos\beta = \frac{1}{2}\cos(\alpha+\beta) + \frac{1}{2}\cos(\alpha-\beta)$

Errore di fase al ricevitore:

$$v(t) = z(t)\, b\cos(2\pi f_0 t + \theta) = A\, s(t)\cos 2\pi f_0 t \; b\cos(2\pi f_0 t + \theta)$$

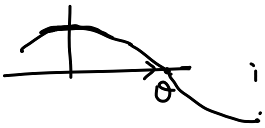
$$= \frac{b A s(t)}{2}\cos(2\pi 2 f_0 t + \theta) + \underbrace{\frac{b A}{2} s(t)\cos\theta}$$

$$= \frac{bAs(t)}{2}\cos(2\pi 2 f_0 t + \theta) + \frac{bA}{2} s(t)\cos\theta$$

eliminato dal filtro passa-basso

$\theta = 0$ no problem.

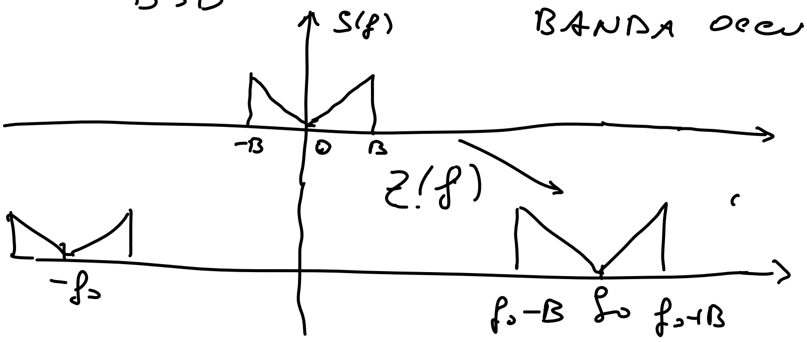
$\theta \simeq 0$



$\theta = \pi/2$ segnale cancellato.

$\theta = \pi$ segnale invertito

DSB



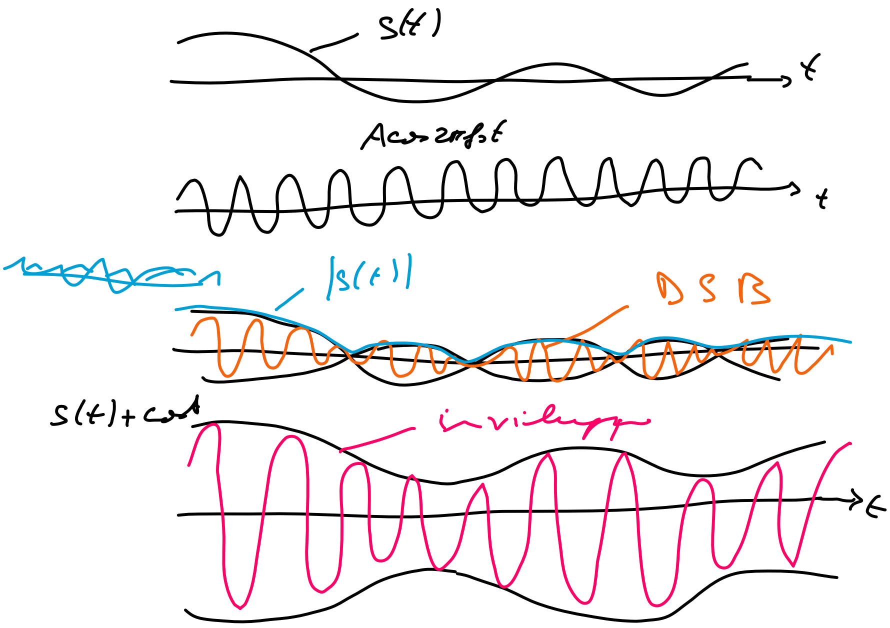
BANDA OCCUPATA B Hz

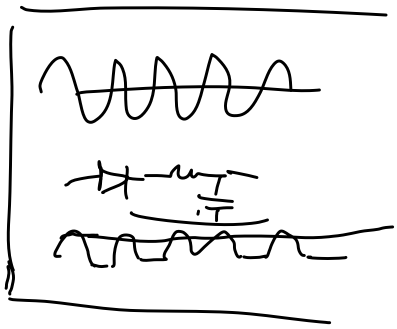
" " 2B Hz

$$\eta = \frac{\text{Banda prima della mod.}}{\text{Banda dopo la mod.}} = \frac{1}{2}$$

efficienza spettrale 1/2

MODULAZIONE AM o DI INVILUPPO



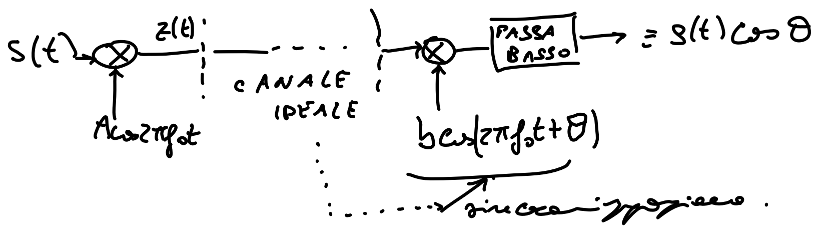


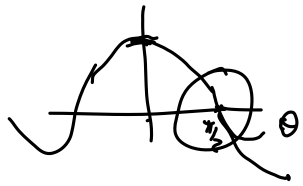
$$z(t) = A(1 + ks(t))\cos 2\pi f_0 t$$

COND. DELL'INVILUPPO

$1 + ks(t) > 0 \quad \forall t$

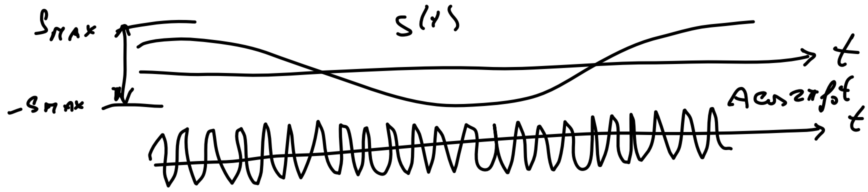
DSB

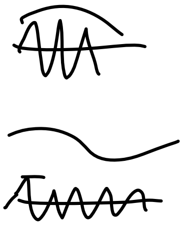


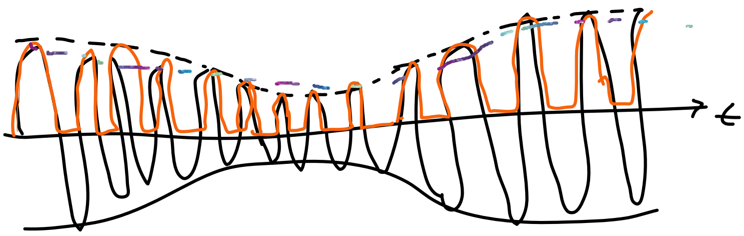


AM AMPLITUDE MODULATION
o MODI DI INVILUPPO

$$z(t) = A(1+ks(t))\cos 2\pi f_0 t = A\cos 2\pi f_0 t + Ak\, s(t)\cos 2\pi f_0 t$$





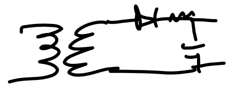


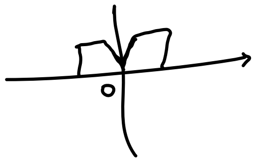
$1+ks(t) > 0 \quad \forall t$

$1 - kS_{MAX} > 0$

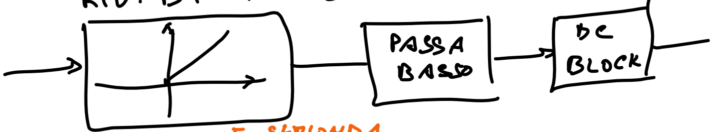
$$k < \frac{1}{S_{MAX}}$$

CONDIZIONE DELL'INVILUPPO

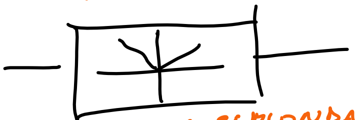




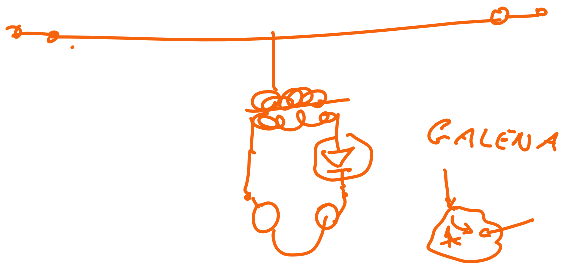
RIV. DI INVILUPPO

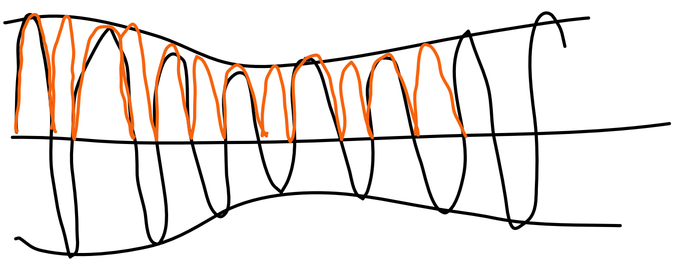


A SEMPLICE SEMIONDA



A DOPPIA SEMIONDA

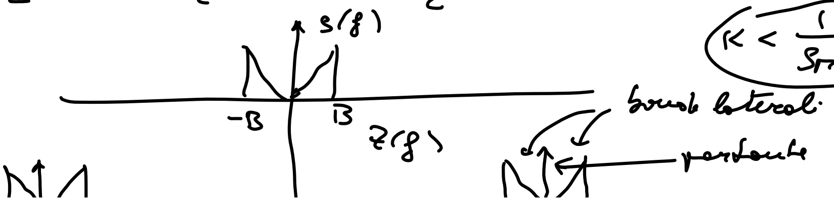


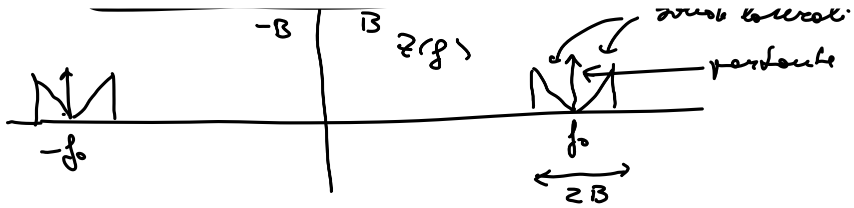


$$z(t) = A(1+ks(t))\cos 2\pi f_0 t = A\cos 2\pi f_0 t + Ak\, s(t)\cos 2\pi f_0 t$$

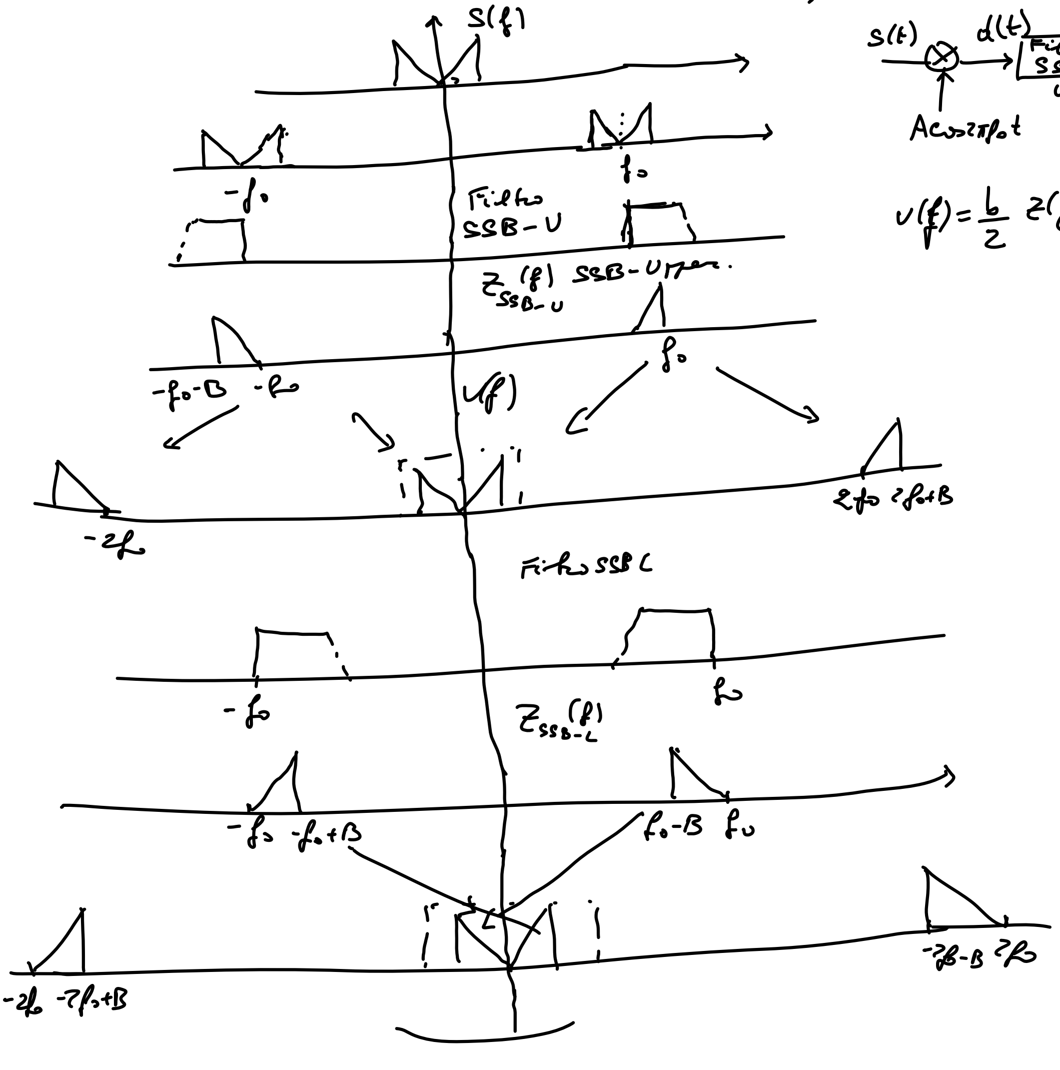
$$Z(f) = \frac{A}{2}\delta(f-f_0) + \frac{A}{2}\delta(f+f_0) + \frac{Ak}{2}S(f-f_0) + \frac{Ak}{2}S(f+f_0)$$

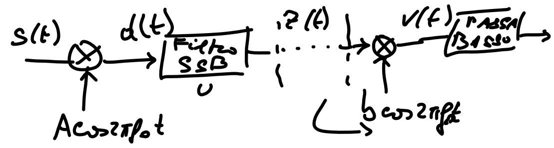
$$k < \frac{1}{S_{MAX}}$$





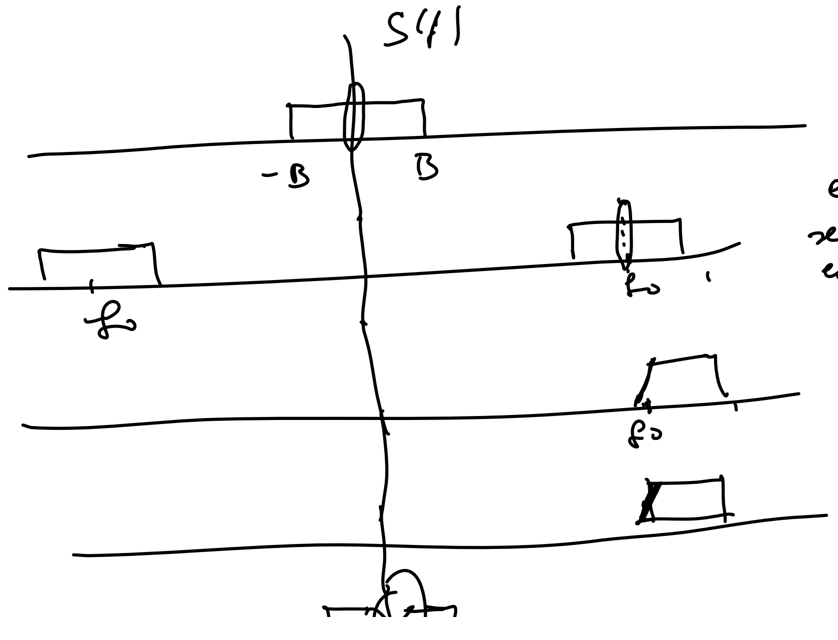
MODULAZIONE SSB (SINGLE SIDEBAND)
(A SINGOLA BANDA LATERALE)



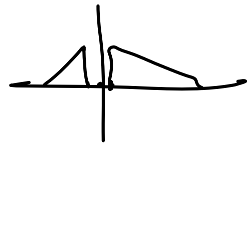


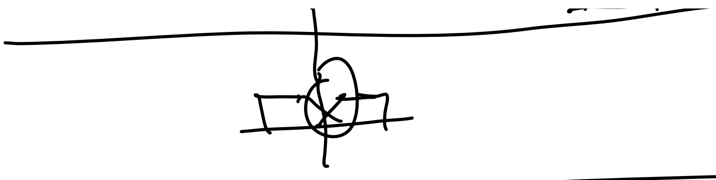
$$v(f) = \frac{b}{2} z(f-f_0) + \frac{b}{2} z(f+f_0)$$

$$\eta_{spettrale} = \frac{\cancel{B}}{\cancel{B}} = 1 = \frac{\text{Banda segnale modulante}}{\text{Banda del segnale modulato}}$$



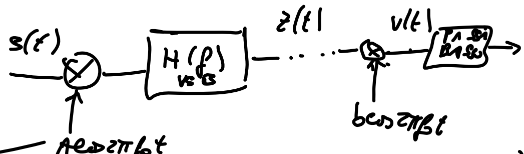
è critica per segnali che hanno contenuti alle basse frequenze.



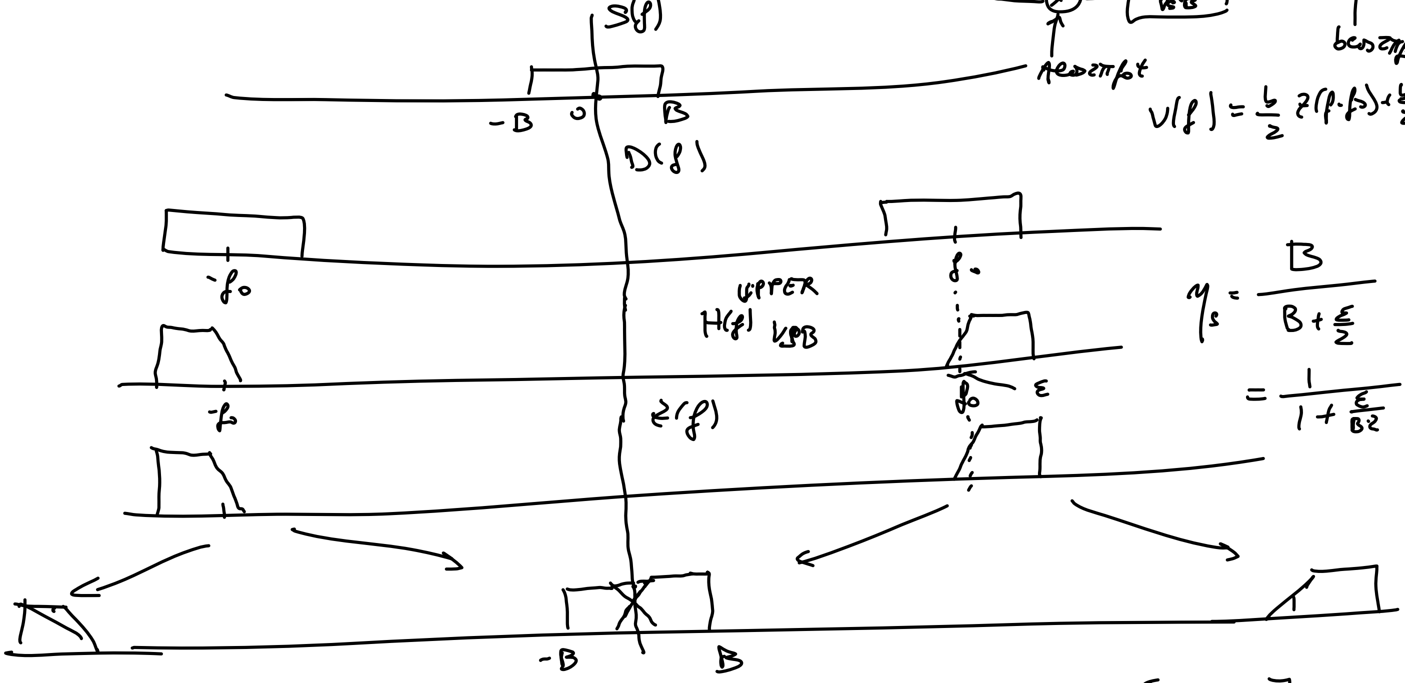


VSB VESTIGIAL SIDE-BAND

(MODULAZIONE A BANDA VESTIGIALE)



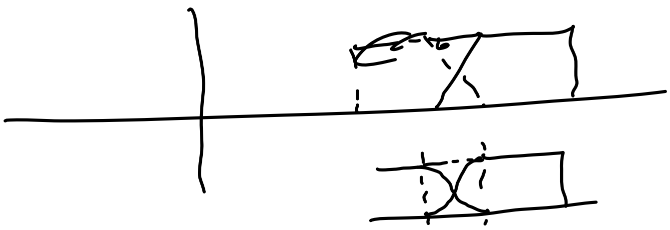
$$V(f) = \frac{b}{2} Z(f-f_0) + \frac{b}{2} Z(f+f_0)$$

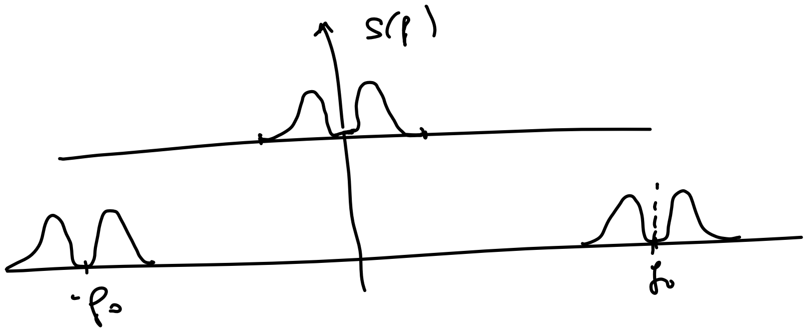


$$\eta_s = \frac{B}{B+\frac{\varepsilon}{2}} = \frac{1}{1+\frac{\varepsilon}{B2}}$$

$$H(f+f_0) + H(f-f_0) = \text{cost.} \quad f \in [-B, B]$$

$\Rightarrow$ eliminata la distorsione!!





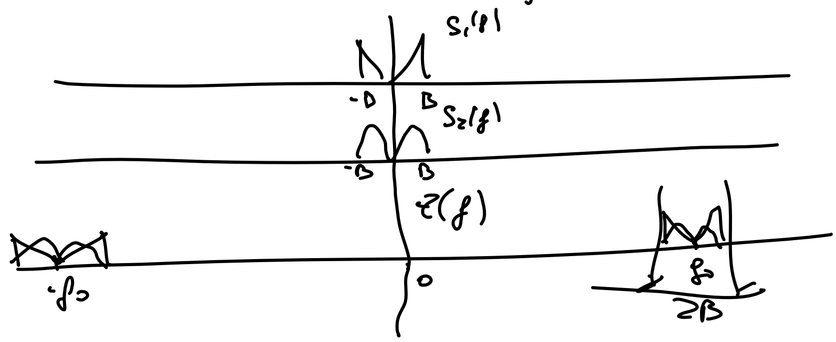
MODULAZIONE QAM (QUADRATURE AMPLITUDE MODULATION)

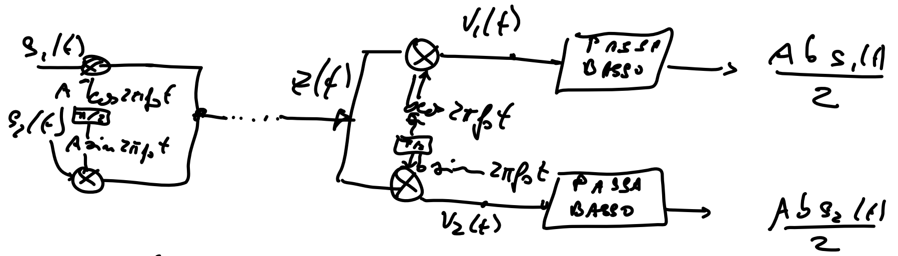
$$z(t) = A\, s_1(t) \cos 2\pi f_0 t + A s_2(t) \sin 2\pi f_0 t$$

$$\mathcal{F}[\sin 2\pi f_0 t] = \frac{1}{2j}\left(\delta(f-f_0) - \delta(f+f_0)\right)$$

$$Z(f) = \frac{A}{2} S_1(f-f_0) + \frac{A}{2} S_1(f+f_0) + \frac{A}{2j} S_2(f-f_0) - \frac{A}{2j} S_2(f+f_0)$$

$$Z(f) = \frac{A}{2}S_1(f-f_0) + \frac{A}{2}S_1(f+f_0) + \frac{A}{2j}S_2(f-f_0) - \frac{A}{2j}S_2(f+f_0)$$





$\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$

$\cos\alpha \sin\beta =$

$= \sin(\alpha+\beta) - \sin(\beta-\alpha)$

$\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$

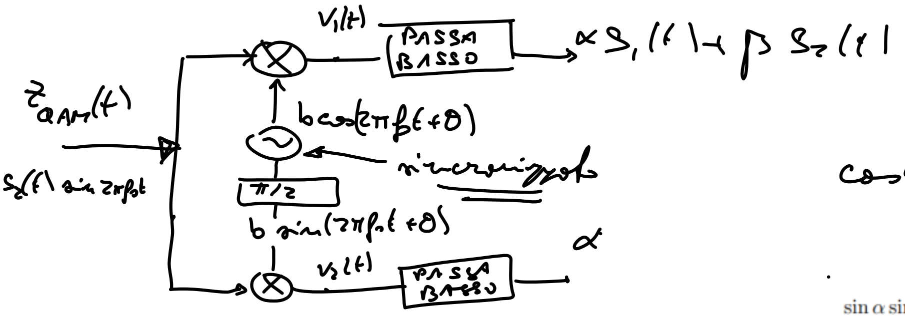
$$V_1(t) = b\cos 2\pi f_0 t\left[A s_1(t)\cos 2\pi f_0 t + A s_2(t)\sin 2\pi f_0 t\right]$$

$$= Ab\, s_1(t)\cos^2 2\pi f_0 t + Ab\, s_2(t)\cos 2\pi f_0 t \sin 2\pi f_0 t$$

$$= \frac{Ab\, s_1(t)}{2} + \underbrace{\frac{Ab\, s_1(t)}{2}\cos 2\pi 2 f_0 t}_{} + \underbrace{\frac{Ab\, s_2(t)}{2}\sin 2\pi 2 f_0 t}_{}$$

$$V_2(t) = b\sin 2\pi f_0 t\left[A s_1(t)\cos 2\pi f_0 t + A s_2(t)\sin 2\pi f_0 t\right]$$

$$= \underbrace{\frac{bA\, s_1(t)}{2}\sin 2\pi 2 f_0 t}_{} + \frac{bA\, s_2(t)}{2} - \underbrace{\frac{bA\, s_2(t)}{2}\cos 2\pi 2 f_0 t}_{}$$



$= A s_1(t)\cos 2\pi f_0 t + A s_2(t)\sin 2\pi f_0 t$

$\cos\alpha \cos\beta = \frac{1}{2}\left[\cos(\alpha+\beta) + \cos(\alpha-\beta)\right]$

$\sin\alpha\sin\beta = \frac{1}{2}\left(\cos(\alpha-\beta) - \cos(\alpha+\beta)\right)$

$$\begin{cases} v_1(t) = b\cos(2\pi f_0 t + \theta)\left[A s_1(t)\cos 2\pi f_0 t + A s_2(t)\sin 2\pi f_0 t\right] \\ v_2(t) = b\sin(2\pi f_0 t + \theta)\left[A s_1(t)\cos 2\pi f_0 t + A s_2(t)\sin 2\pi f_0 t\right] \end{cases}$$

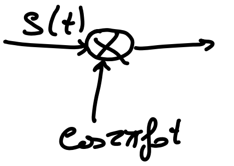
$$\begin{cases} v_1(t) = \frac{bA}{2}s_1(t)\cancel{\cos(2\pi 2 f_0 + \theta)} + \frac{Ab\, s_1(t)}{2}\cos\theta + \frac{Ab}{2}s_2(t)\cancel{\sin(2\pi 2 f_0 t + \theta)} - \frac{Ab}{2}s_2(t)\sin\theta \\ v_2(t) = \frac{bA}{2}s_1(t)\cancel{\sin(2\pi 2 f_0 + \theta)} + \frac{Ab\, s_1(t)}{2}\sin\theta - \frac{Ab}{2}s_2(t)\cancel{\cos(2\pi 2 f_0 t + \theta)} + \frac{Ab}{2}s_2(t)\cos\theta \end{cases}$$

$$\begin{cases} v_1(t) = \frac{Ab}{2}s_1(t)\cos\theta + \frac{Ab}{2}s_2(t)\sin\theta \\ v_2(t) = \frac{Ab}{2}s_1(t)\sin\theta + \frac{Ab}{2}s_2(t)\cos\theta \end{cases}$$

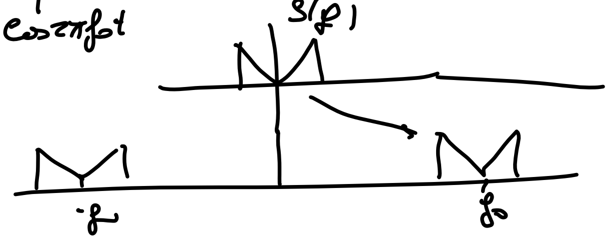
MOD: DSB, AM, SSB, VSB, QAM

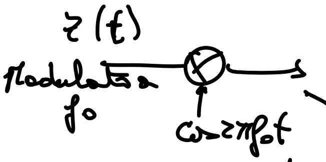
OMODINA ⟷ ETERODINA

DSB, AM, SSB, VSB, QAM



$s(t)\cos 2\pi f_0 t \leftrightarrow \frac{1}{2}S(f-f_0)+\frac{1}{2}S(f+f_0)$

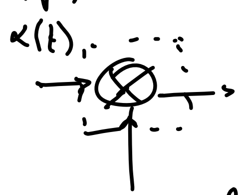




$\frac{1}{2}Z(f-f_0)+Z(f+f_0)$

$\alpha(f)$

segnale passa banda a freq. f_1

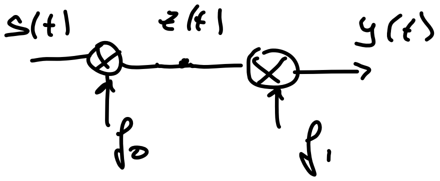


$\frac{1}{2}\alpha(f-f_2)+\frac{1}{2}\alpha(f+f_2)$

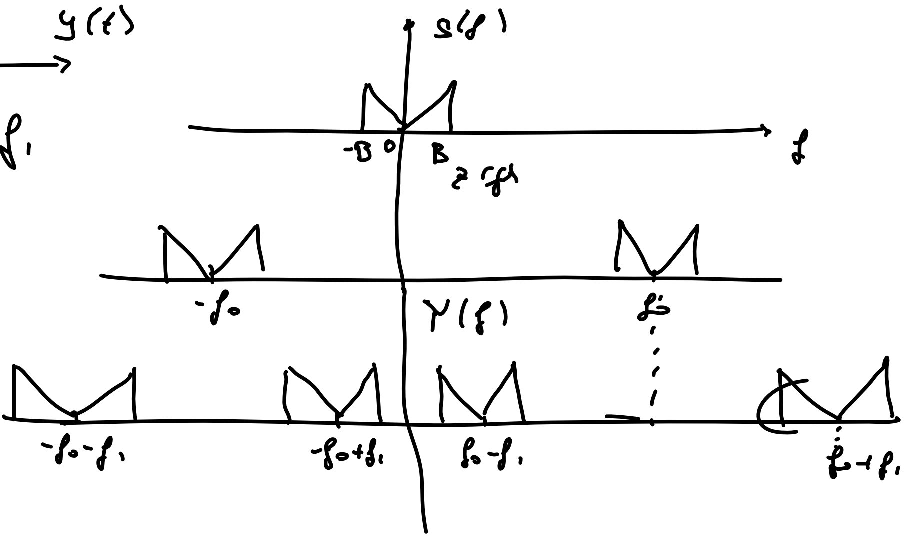
sin. a freq. f_2

eterodina

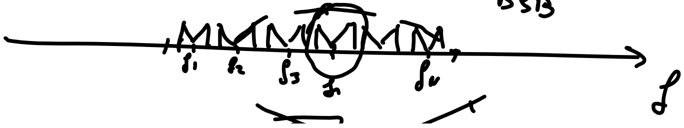
$f_1 = f_2$ omodina

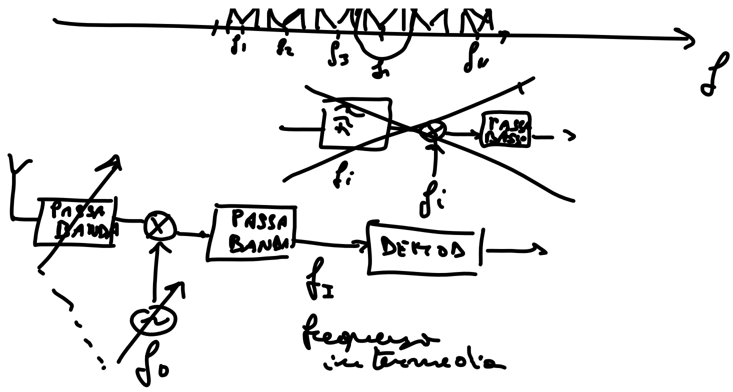


$Y(f) = \frac{1}{2}Z(f-f_1)+\frac{1}{2}Z(f+f_1)$



RICEVITORE A SUPERETERODINA





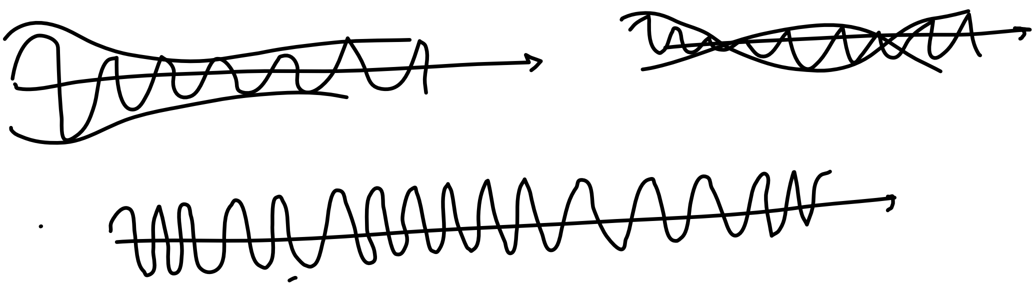
$f_i - f_I \quad f_I$

$$f_1(s(t))\cos 2\pi f_0 t + f_2(s(t))\sin 2\pi f_0 t$$

MODULAZIONE ANGOLARE ?

$$z(t) = A\cos(2\pi f_0 t + \psi(t))$$

FO. MOD AM.



MODULAZIONE DI FASE (PM)

$\psi(t) = S_\varphi s(t)$ $\qquad z(t) = A\cos(2\pi f_0 t + S_\varphi s(t))$

MODULAZIONE DI FREQUENZA (FM)

$\psi(t) = S_f \int^t s(\xi)\,d\xi$ $\qquad z(t) = A\cos(2\pi f_0 t + S_f \int^t s(\xi)\,d\xi)$

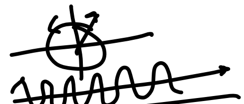
$z(t) = A\cos(2\pi f_0 t + \psi(t))$ MOD. ANGOLARE

FASE ISTANTANEA $\varphi(t) = 2\pi f_0 t + \psi(t)$

FREQUENZA ISTANTANEA $f(t) = \dfrac{\varphi'(t)}{2\pi} = f_0 + \dfrac{1}{2\pi}\dot\psi(t)$

$$\begin{cases} PM: & f_{PM}(t) = f_0 + \dfrac{S_\varphi}{2\pi}\dot s(t) \\ FM: & f_{FM}(t) = f_0 + S_f s(t) \end{cases}$$

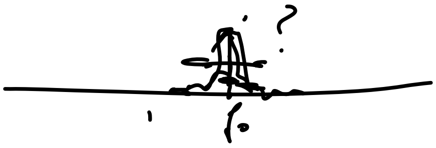
$\cos 2\pi f_0 t$



$\omega(t) = \varphi'(t)$

$|\psi(t)| \ll 1$

$|\psi(t)| \gg 1$



$$z(t) = A\cos(2\pi f_0 t + \psi(t)) = A\,\mathrm{Re}\left[e^{j2\pi f_0 t} e^{j\psi(t)}\right]$$

$$= A\,\mathrm{Re}\left[e^{j2\pi f_0 t}\left(1 + j\psi(t) + j^2\psi^2(t) + \dots\right)\right]$$

→ segnale ci a banda infinita!!

$|\psi(t)| \ll 1$

$$|\varphi(t)| \ll 1$$

$$z(t) \simeq z_{bs}(t) = A\,\mathrm{Re}\left[e^{j2\pi f_0 t}\,(1+j\varphi(t))\right]$$

$\cos 2\pi f_0 t + j\sin 2\pi f_0 t$

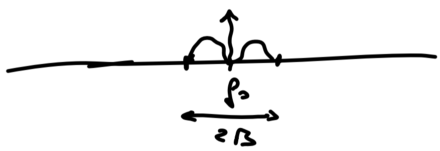
segnale a banda infinita!!

$$z_{bs}(t) = A\cos 2\pi f_0 t - A\varphi(t)\sin 2\pi f_0 t$$] segnale a banda stretta.

$$Z_{bs}(f) = \frac{A}{2}\delta(f-f_0) + \frac{A}{2}\delta(f+f_0) - \frac{A}{2j}\Phi(f-f_0) + \frac{A}{2j}\Phi(f+f_0)$$

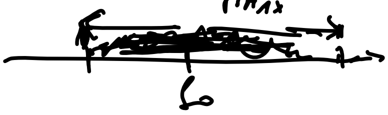
PM $\varphi(t) = S_\varphi s(t)$

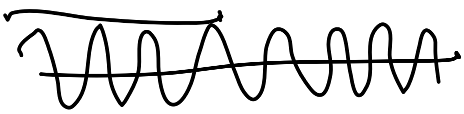
FM $\varphi(t) = S_f \int^t s(\xi)\,d\xi$



$$|\varphi(t)| \gg 1$$

- Molto complicato
- in generale a banda infinita





$$z(t) = A\cos(2\pi f_0 t + \varphi(t))$$

$$f(t) = f_0 + \frac{1}{2\pi}\dot\varphi(t)$$

PM $f(t) = f_0 + \frac{1}{2\pi} S_\varphi \dot s(t)$

FM $f(t) = f_0 + \underbrace{\frac{1}{2\pi} S_f s(t)}_{\text{Deviazione di frequenza } \Delta f(t)}$ ⟸ FM

$$|\Delta f|_{max} = \frac{1}{2\pi} S_f |s(t)|_{max}$$

BANDA DI CARSON

$$B_c = 2B\left(1 + \frac{|\Delta f|_{max}}{B}\right)$$

m RAPPORTO DI DEV. DI FREQUENZA
O
INDICE DI MODULAZIONE

$m \ll 1 \implies B_c \simeq 2B$ (Mod. a banda stretta)

$m \gg 1 \implies B_c \simeq 2|\Delta f|_{max}$

FM $\quad z(t) = A\cos\left(2\pi f_0 t + S_f \int^t s(\xi)\,d\xi\right)$

$$\frac{dz(t)}{dt} = -A(2\pi f_0 + S_f s(t))\sin\left(2\pi f_0 t + S_f\int^t s(\xi)\,d\xi\right)$$

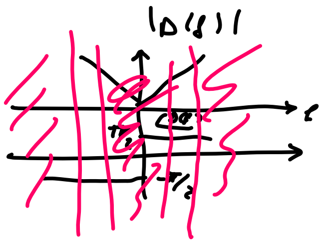
⟹ MOD DI INVILUPPO

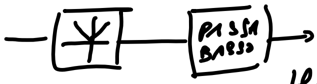
DISCRIMINATORE DI FREQUENZA

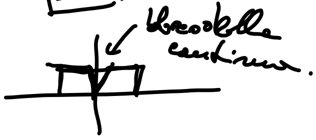
$z(t)$ → $\frac{d}{dt}$ → RIV. → DC

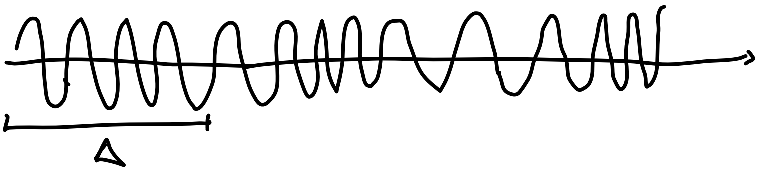
DISCRIMINATORE DI FREQUENZA

$\boxed{\frac{d}{dt}} \longrightarrow D(f) = j2\pi f$





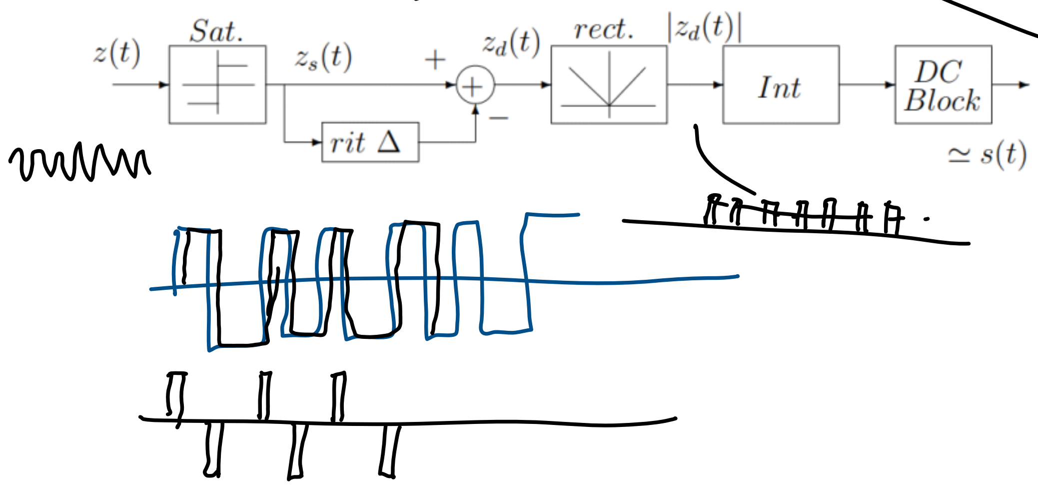




$$f_i = \frac{\#\text{ZEROCROSSING}}{2\Delta}$$

approx. alla derivata

≃ DISCR. DI FREQUENZA

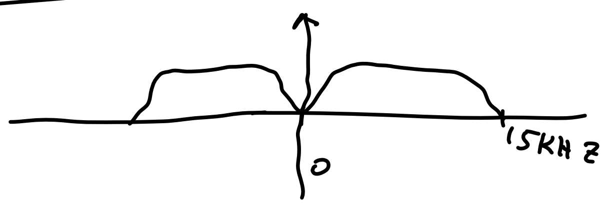


STEREO COMPATIBILE

s_L

s_R

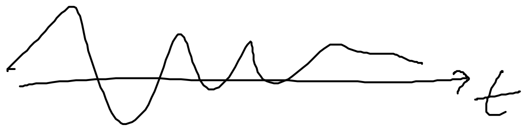
MONO: $\frac{s_L(t)+s_R(t)}{2}$



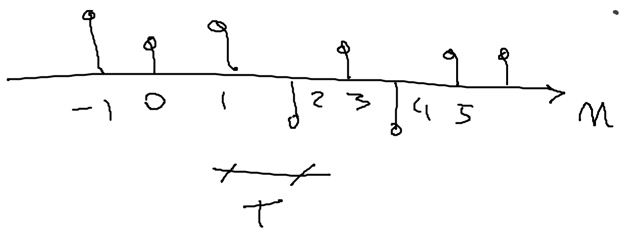
$$\frac{s_L(t)-s_R(t)}{2}$$

Lezione 16

giovedì 26 novembre 2020 08.54



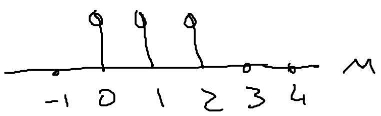
x(t)



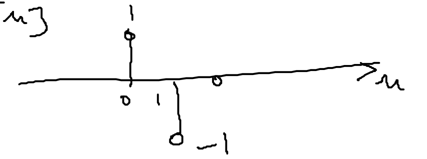
x[n]

Esercizio di Convoluzione lineare T.D.

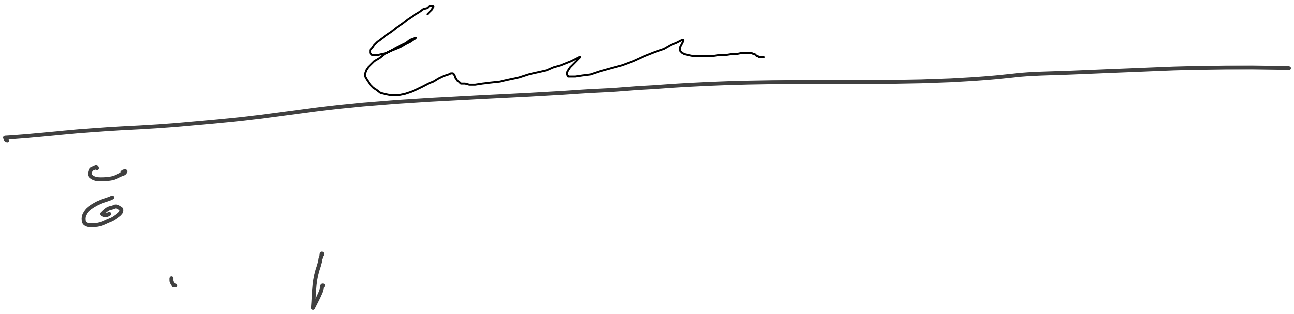
x[n]



y[n]



z[n]



x[n]

X(f)

TRAST. DI FOURIER TEMPO DISCRETO

$$\boxed{X(\nu) = \sum_{n=-\infty}^{+\infty} x[n]\, e^{-j2\pi\nu n}}$$

$$X(f) = \int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi f t}\, dt$$

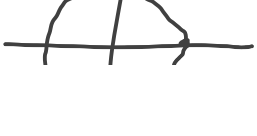
DTFT (DISCRETE-TIME FOURIER TRANSFORM)

ν FREQUENZA (NORMALIZZATA) (ADIMENSIONALE)

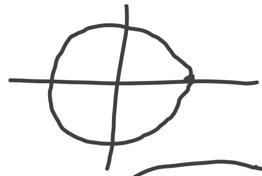
$X(\nu)$ periodica con periodo 1

$$e^{-j2\pi(\nu+1)n} = e^{-j2\pi\nu n} \cdot e^{-j2\pi n}$$

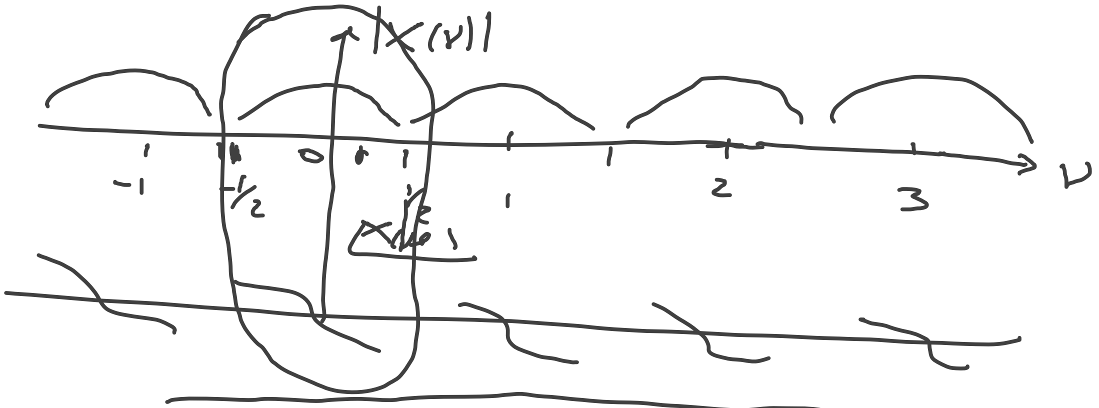
1



$$e^{-j2\pi(\nu+1)n} = e^{-j2\pi\nu n}\underbrace{e^{-j2\pi n}}_{1}$$



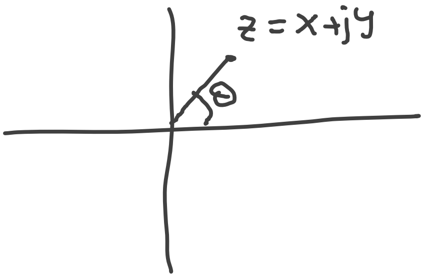
$$\nu = \frac{f}{f_c}$$



$$\boxed{x[n] = \int_{-1/2}^{1/2} X(\nu)\, e^{j2\pi\nu n}\, d\nu}$$

$DTFT^{-1}$ ANTITRASFORMATA

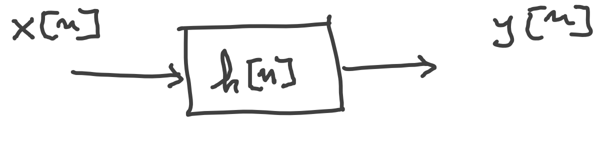
LA FASE DI UN NUMERO COMPLESSO

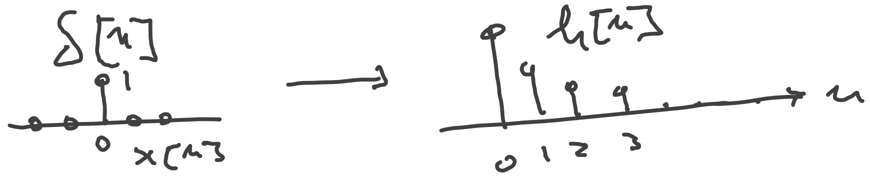


$$\theta = tg^{-1}\frac{y}{x}$$

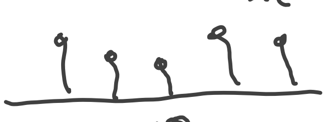
$$\theta = \underline{atan2}(x,y)$$

SISTEMI LINEARI TEMPO-DISCRETO





x[n]



$$x[n] = \sum_{m=-\infty}^{+\infty} \delta[n-m]\, x[m]$$

$$\delta[n-m]\,x[m] \longrightarrow h[n-m]\,x[m]$$

$$\longrightarrow \sum_m$$

$\delta[n-m]\,x[m]$

$\sum_m \longrightarrow \sum_m$

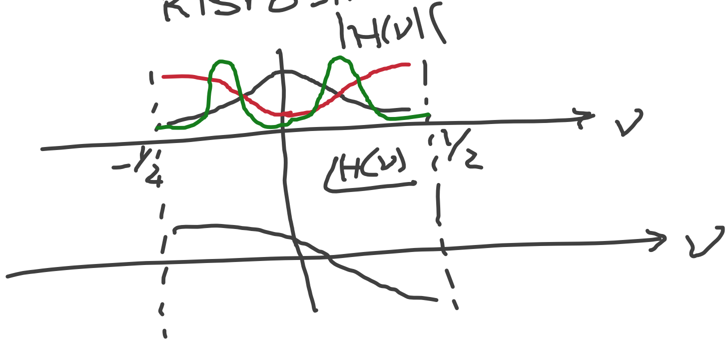
$$\boxed{y[n] = \sum_{m=-\infty}^{+\infty} x[m]\,h[n-m]}$$

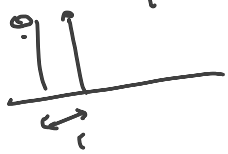
CONV. LINEARE

$$= \sum_{m=-\infty}^{+\infty} h[m]\,x[n-m]$$

$$Y(\nu) = \underbrace{H(\nu)}_{\text{FUNZIONE DI RISPOSTA ARMONICA}} X(\nu)$$

$Y(f) = H(f)\,X(f)$

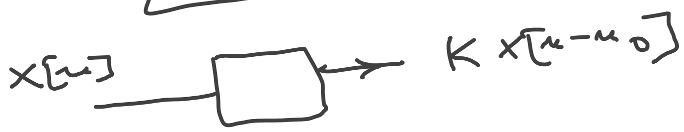


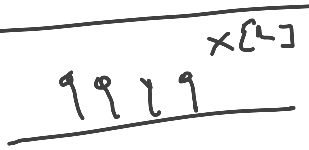


NON DISTORCENTE

$$\left.\begin{array}{l} |H(\nu)| = K \\ \angle H(\nu) = -2\pi\nu n_0 \end{array}\right\} \forall \nu \in I_x$$

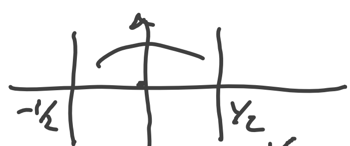
per le frequenze di interesse.

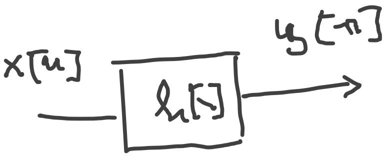




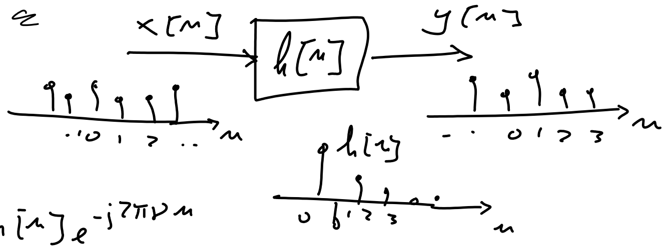
$\longleftrightarrow$ DF

$$X(\nu) = \sum_{n=-\infty}^{+\infty} x[n]\,e^{-j2\pi\nu n}$$





$$x[n] = \int_{-1/2}^{1/2} X(\nu)\,e^{j2\pi\nu n}\,d\nu$$



$$y[n] = (h * x)[n] = \sum_{k=-\infty}^{+\infty} h[k]\,x[n-k]$$

DTFT

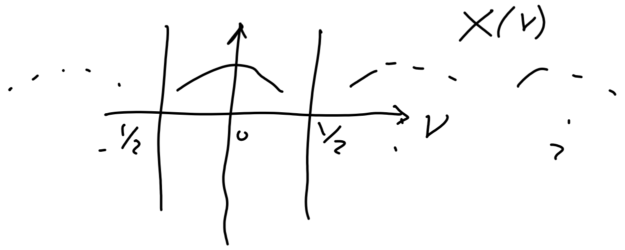
$$H(\nu) = \sum_{n=-\infty}^{+\infty} h[n]\,e^{-j2\pi\nu n}$$

$$X(\nu) = \sum_{n=-\infty}^{+\infty} x[n]\,e^{-j2\pi\nu n}$$

$$Y(\nu) = \sum_{n=-\infty}^{+\infty} y[n]\,e^{-j2\pi\nu n}$$

$$Y(\nu) = H(\nu)\,X(\nu)$$

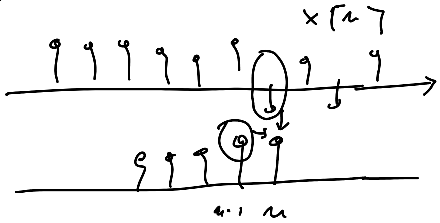
$$H(\nu) = \frac{Y(\nu)}{X(\nu)}$$ Risposta armonica



Esempio:

y[n] soldi in tasca ~~al~~ alla fine del giorno n

x[n] soldi spesi o guadagnati



$$y[n] = y[n-1] + x[n]$$

$$y[-1] = C$$

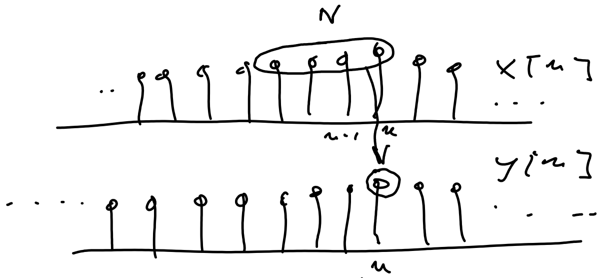
$$y[1] = C + x[1]$$

$$y[2] = y[1] + x[2] = (C + x[1]) + x[2]$$

$$y[3] = y[2] + x[3] = C + x[1] + x[2] + x[3]$$

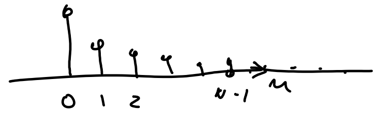
$$\vdots$$

$$y[n] = C + \sum_{i=1}^{n} x[i]$$

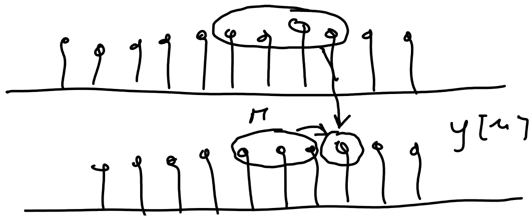


$\boxed{\text{FIR}}$

Finite Impulse Response

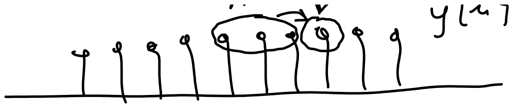


$$y[n] = \sum_{k=0}^{N-1} h[k]\,x[n-k]$$



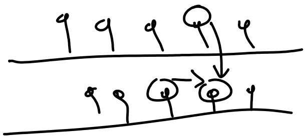
IIR

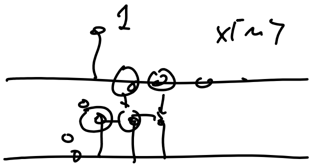
Infinite Impulse Response



$$\boxed{y[n] = \sum_{k=1}^{M} a_k\, y[n-k] + \sum_{k=0}^{N-1} b_k\, x[n-k]}$$

Esempio IIR





$$y[n] = a\, y[n-1] + b\, x[n]$$

Risposta impulsiva $y[-1] = 0$, $x[n] = \delta[n]$

$$y[0] = a\, y[-1] + b = b$$

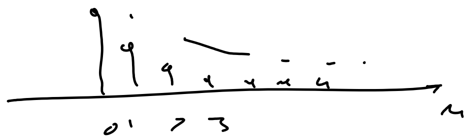
$$y[1] = a\, y[0] + = a b$$

$$y[2] = a\, y[1] = a^2 b$$

$$y[3] = a\, y[2] = a^3 b$$

$$\vdots$$

$$y[n] = a^n b$$



$$h[n] = a^n b\, u[n]$$

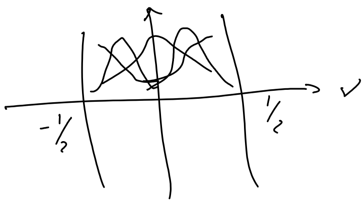
$$\boxed{y[n] = \sum_{k=1}^{M} a_k\, y[n-k] + \sum_{k=0}^{N-1} b_k\, x[n-k]}$$ FORMA CANONICA

DTFT

$$Y(\nu) = \sum_{k=1}^{M} a_k e^{-j2\pi\nu k}\, Y(\nu) + \sum_{k=0}^{N-1} b_k e^{-j2\pi\nu k}\, X(\nu)$$

$$Y(\nu)\left(1 - \sum_{k=1}^{M} a_k e^{-j2\pi\nu k}\right) = X(\nu) \sum_{k=0}^{N-1} b_k e^{-j2\pi\nu k}$$

$$\boxed{H(\nu) = \frac{Y(\nu)}{X(\nu)} = \frac{\sum_{k=0}^{N-1} b_k e^{-j2\pi\nu k}}{1 - \sum_{k=1}^{M} a_k e^{-j2\pi\nu k}}}$$



Z-TRASFORMATA (T. DI LAPLACE T.D.)

$$\ldots + x[-2]z^{2} + x[-1]z + x[0] + x[1]z^{-1} + x[2]z^{-2} + \ldots$$

sequenza $\longleftrightarrow$ polinomio(z)

DT

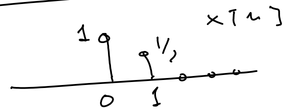
$$\boxed{X(z) = \sum_{n=-\infty}^{+\infty} x[n]\, z^{-n}} \qquad z \in \mathbb{C}$$

D.Z

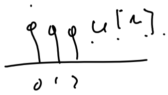
Esempio

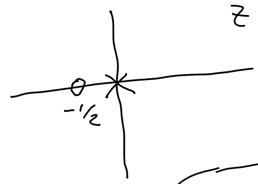
x[n]

Esempio

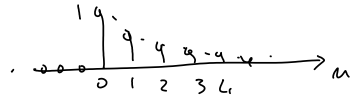


$$X(z) = 1 + \frac{1}{2} z^{-1} = z^{-1}\left(z + \frac{1}{2}\right) = \frac{z + 1/2}{z}$$





Esempio $x[n] = a^n u[n]$ $\quad 0 < a < 1$



$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \qquad |x| < 1$$

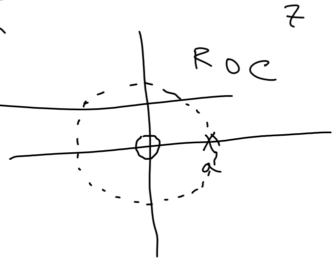
$$X(z) = \sum_{n=0}^{\infty} a^n z^{-n} = \sum_{n=0}^{\infty} (a z^{-1})^n = \frac{1}{1 - a z^{-1}}$$

ROC: $|a z^{-1}| < 1$

Region of Convergence

$$= \frac{z}{z - a}$$

$\left|\frac{a}{z}\right| < 1 \qquad |z| > a$



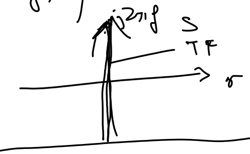
T.C.

$x(t)$

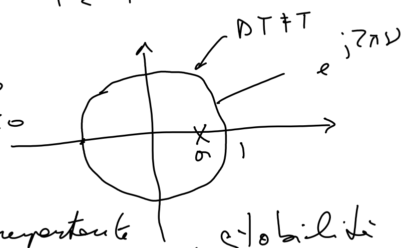
$$X(s) = \int_{-\infty}^{+\infty} x(t)\, e^{-st}\, dt$$

$s = j2\pi f$

$$X(f) = \int x(t)\, e^{-j2\pi f t}\, dt$$



cerchio unitario



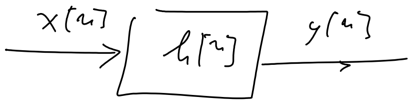
Molto importante perchè? stabilità

$$X(z) = \sum_{k=-\infty}^{+\infty} x[n]\, z^{-n} \xrightarrow{z = e^{j2\pi\nu}} X(\nu) = X(z)\Big|_{z = e^{j2\pi\nu}}$$

T. della convoluzione:

$$y[n] = (h * x)[n]$$

$$Y(z) = H(z) X(z)$$



$H(z)$

FUNZIONE DI TRASFERIMENTO

Prova:

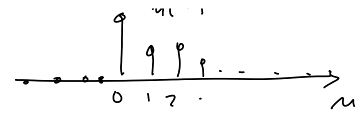
$$y[n] = \sum_{k=-\infty}^{+\infty} h[k]\, x[n-k]$$

$$Y(z) = \sum_{n=-\infty}^{+\infty} y[n] z^{-n} = \sum_{n=-\infty}^{+\infty} \sum_{k=-\infty}^{+\infty} h[k]\, x[n-k]\, z^{-n}$$

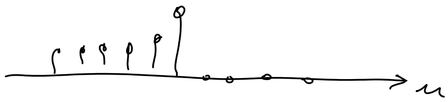
$$= \sum_{k=-\infty}^{+\infty} h[k] \underbrace{\sum_{n=-\infty}^{+\infty} x[n-k]\, z^{-n}}_{z^{-k} X(z)} = \underbrace{\sum_{k=-\infty}^{+\infty} h[k]\, z^{-k}}_{H(z)} X(z)$$

SIST. CAUSALE

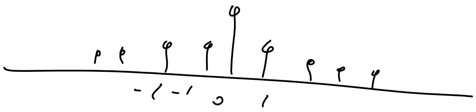
$h[n]$



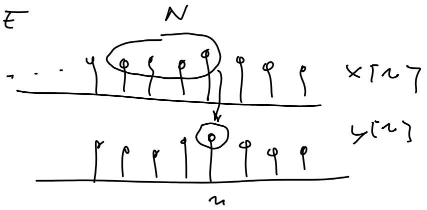
SIST. ANTICAUSALE



SIST. NON CAUSALE



FIR CAUSALE



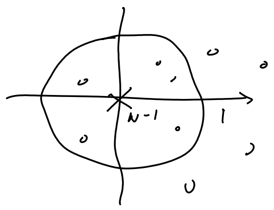
$z^2+1=0$

$z^2=-1$

$$y[n]=\sum_{k=0}^{N-1} h[k]\,x[n-k]$$

$$Y(z)=H(z)X(z) \qquad H(z)=\sum_{n=0}^{N-1} h[n]\,z^{-n}$$

$$=h[0]+h[1]z^{-1}+h[2]z^{-2}+\dots+h[N-1]z^{-N+1}$$



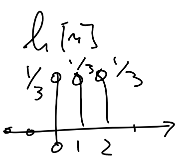
$$=z^{-N+1}\underbrace{\left(h[0]z^{N-1}+h[1]z^{N-2}+h[2]z^{N-3}+\dots+h[N-1]\right)}_{\text{polinomio di ordine } N-1}$$

$\frac{1}{z^{N-1}}$

$N-1$ poli origine

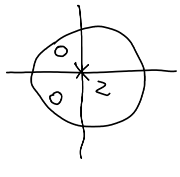
$N-1$ zeri (se complessi compaiono in coppie complesse coniugate)

Esempio



$$H(z)=\frac{1}{3}+\frac{1}{3}z^{-1}+\frac{1}{3}z^{-2}$$

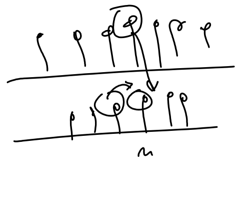
$$=\frac{1}{3}\left[1+z^{-1}+z^{-2}\right]=\frac{z^{-2}}{3}\left[z^2+z+1\right]$$



$$=\frac{1}{3}\frac{1}{z^2}\left[z^2+z+1\right]$$

$$z_{1,2}=\frac{-1\pm\sqrt{1-4}}{2}=\frac{-1\pm\sqrt{-3}}{2}=\frac{-1\pm j\sqrt{3}}{2}$$

Esempio IIR

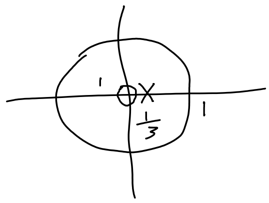


$$y[n]=\frac{1}{3}y[n-1]+2x[n]$$

$$Y(z)=\frac{1}{3}z^{-1}Y(z)+2X(z)$$

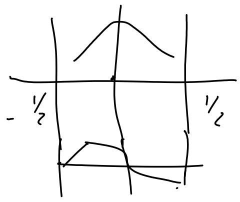
$$Y(z)\left(1-\frac{1}{3}z^{-1}\right) = 2X(z)$$

$$H(z)=\frac{Y(z)}{X(z)} = \frac{2}{1-\frac{1}{3}z^{-1}} = \frac{2z}{z-\frac{1}{3}}$$



$$H(\nu) = H(z)\Big|_{z=e^{j2\pi\nu}} = \frac{2}{1-\frac{1}{3}e^{-j2\pi\nu}}$$

$$= \frac{2}{1-\frac{1}{3}\cos 2\pi\nu+\frac{1}{3}j\sin 2\pi\nu}$$



$$|H(\nu)| = \frac{2}{\sqrt{\left(1-\frac{1}{3}\cos 2\pi\nu\right)^2+\frac{1}{9}\sin^2 2\pi\nu}} = \frac{2}{\sqrt{1+\frac{1}{9}\cos^2 2\pi\nu-\frac{2}{3}\cos 2\pi\nu+\frac{1}{9}\sin^2 2\pi\nu}}$$

$$= \frac{2}{\sqrt{\frac{10}{9}-\frac{2}{3}\cos 2\pi\nu}}$$

$$\angle H(\nu) = -\text{atan2}\left(1-\frac{1}{3}\cos 2\pi\nu;\ \frac{1}{3}\sin 2\pi\nu\right)$$

$$y[n] = \sum_{k=1}^{M} a_k\, y[n-k] + \sum_{k=0}^{N-1} b_k\, x[n-k]$$ Forma canonica

$$Y(z) = \sum_{k=1}^{M} a_k z^{-k} Y(z) + \sum_{k=0}^{N-1} b_k z^{-k} X(z)$$

$$Y(z)\left(1-\sum_{k=1}^{M} a_k z^{-k}\right) = \sum_{k=0}^{N-1} b_k z^{-k}\, X(z)$$

$$\boxed{H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{N-1} b_k z^{-k}}{1-\sum_{k=1}^{M} a_k z^{-k}}}$$ IIR

$$= \frac{z^{-N+1}\left(\sum_{k=0}^{N-1} b_k z^{N-1-k}\right)}{z^{-M}\left(z^{M}-\sum_{k=1}^{M} a_k z^{M-k}\right)}$$

$N-1$ zeri

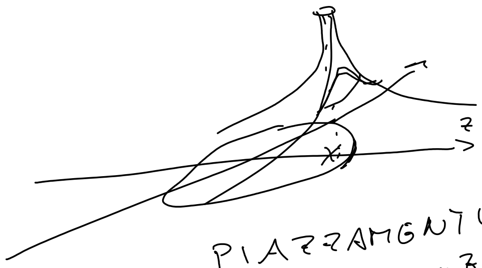
M poli

z^{M-N+1} $\begin{cases} M-N+1>0 & \text{zeri nell'origine} \\ M-N+1<0 & \text{poli nell'origine} \end{cases}$

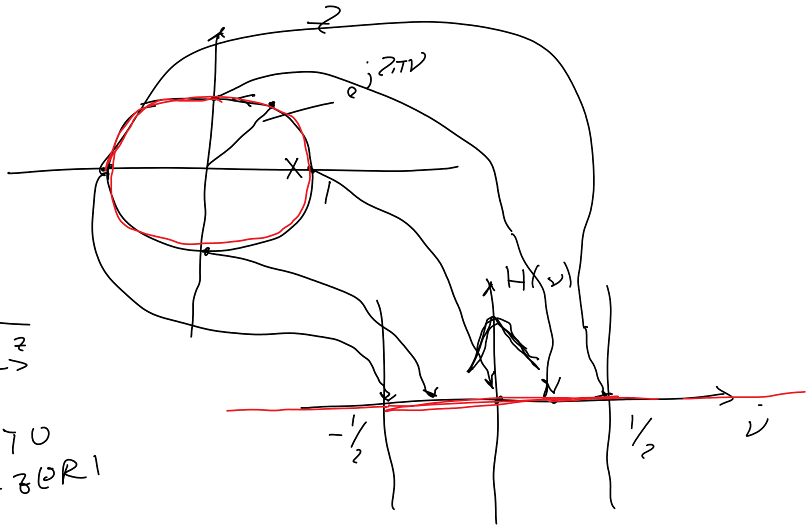
$$z^{M-N+1} \begin{cases} M-N+1 > 0 & \text{zeri nell'origine} \\ M-N+1 < 0 & \text{poli nell'origine} \end{cases}$$

COME INFERIRE O STIMARE APPROSSIMATIVAMENTE LA RISPOSTA ARMONICA?

$$H(\nu) = \mathcal{H}(z)\Big|_{z=e^{j2\pi\nu}}$$

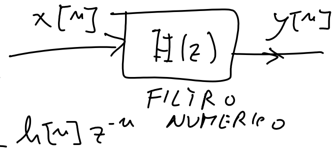


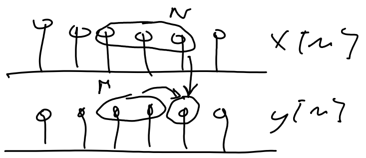
PIAZZAMENTO POLI-ZERI



Lezione 18

giovedì 3 dicembre 2020 08.58





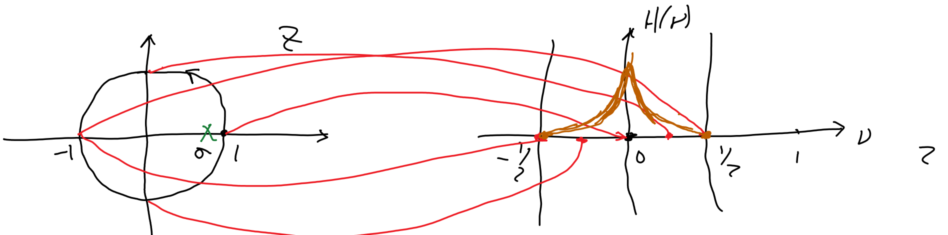
$$H(z) = \sum_{n=-\infty}^{+\infty} h[n]\, z^{-n}$$

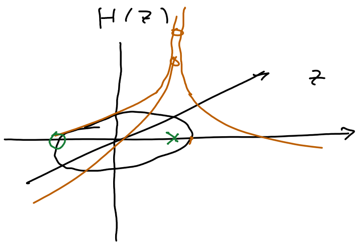
$$y[n] = \sum_{k=1}^{M} a_k\, y[n-k] + \sum_{k=0}^{N-1} b_k\, x[n-k]$$

$$H(\nu) = \sum_{n=-\infty}^{+\infty} h[n]\, e^{-j2\pi\nu n} = H(z)\Big|_{z=e^{j2\pi\nu n}}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{N-1} b_k z^{-k}}{1 - \sum_{k=1}^{M} a_k z^{-k}}$$

ν freq. numeriche (adimensionali)





$$H(z) = \frac{z+1}{z-a} = \frac{1+z^{-1}}{1-a z^{-1}}$$

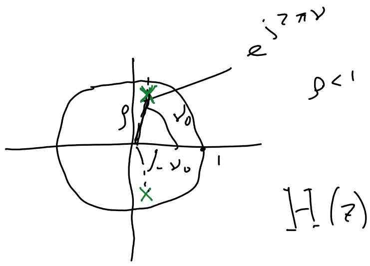
$$\frac{Y(z)}{X(z)} = \frac{1+z^{-1}}{1-a z^{-1}}$$

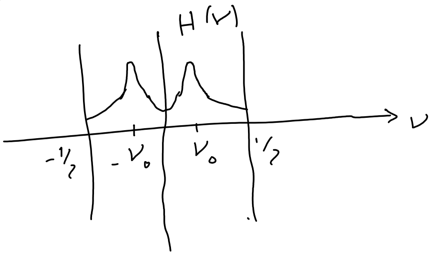
$$Y(z)(1-a z^{-1}) = (1+z^{-1})X(z)$$

$$y[n] - a\, y[n-1] = x[n] + x[n-1]$$

$$\boxed{y[n] = a\, y[n-1] + x[n] + x[n-1]}$$

Esempio (PASSA BANDA)





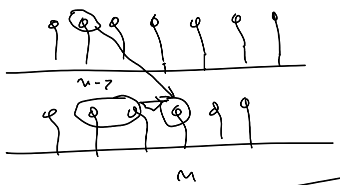
$$H(z) = \frac{1}{(z - \rho e^{j2\pi\nu_0})(z - \rho e^{-j2\pi\nu_0})}$$

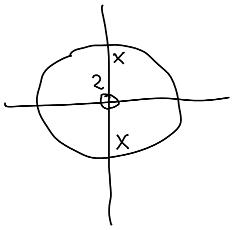
$$\cos x = \frac{e^{jx} + e^{-jx}}{2}$$

$$= \frac{1}{z^2 - z\rho e^{-j2\pi\nu_0} - z\rho e^{j2\pi\nu_0} + \rho^2}$$

$$= \frac{1}{z^2 - z\rho\, 2\cos 2\pi\nu_0 + \rho^2} = \frac{z^{-2}}{1 - 2\rho\cos 2\pi\nu_0\, z^{-1} + \rho^2 z^{-2}}$$

$$\frac{Y(z)}{X(z)} \qquad \boxed{y[n] = 2\rho\cos 2\pi\nu_0\, y[n-1] - \rho^2 y[n-2] + x[n-2]}$$

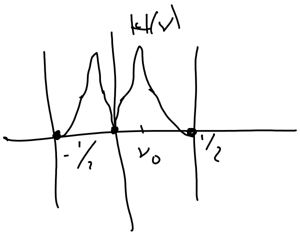


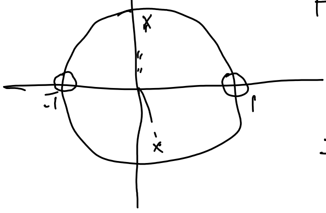


$$H(z) = \frac{z^2}{(z-\rho e^{j2\pi\nu_0})(z-\rho e^{-j2\pi\nu_0})}$$

$$= \frac{z^2}{z^2 - 2\rho\cos 2\pi\nu_0 z + \rho^2} = \frac{1}{1 - 2\rho\cos 2\pi\nu_0 z^{-1} + \rho^2 z^{-2}}$$

$$\boxed{y[n] = 2\rho\cos 2\pi\nu_0\, y[n-1] - \rho^2 y[n-2] + x[n]}$$



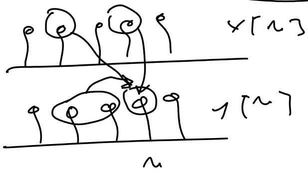


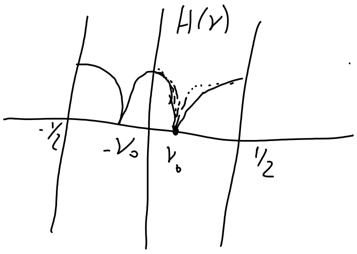
$$H(z) = \frac{(z-1)(z+1)}{(z-\rho e^{j2\pi\nu_0})(z-\rho e^{-j2\pi\nu_0})}$$

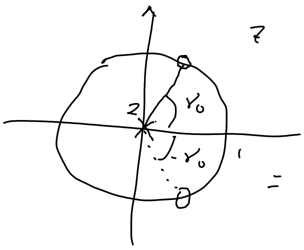
$$= \frac{z^2-1}{z^2 - 2\rho\cos 2\pi\nu_0 z + \rho^2}$$

$$= \frac{1 - z^{-2}}{1 - 2\rho\cos 2\pi\nu_0 z^{-1} + \rho^2 z^{-2}}$$

$$\boxed{y[n] = 2\rho\cos 2\pi\nu_0\, y[n-1] - \rho^2 y[n-2] + x[n] - x[n-2]}$$

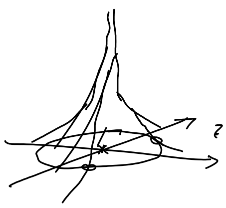






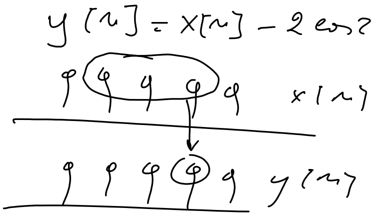
z $\quad H(z) = \frac{(z-e^{j2\pi\nu_0})(z-e^{-j2\pi\nu_0})}{z^2}$

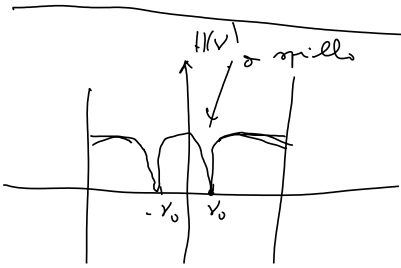
$$= \frac{z^2 - 2\cos 2\pi\nu_0 z + 1}{z^2}$$

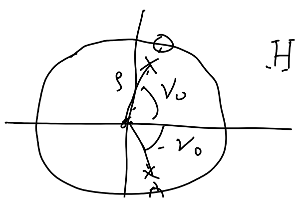


$$= 1 - 2\cos 2\pi\nu_0 z^{-1} + z^{-2} \qquad \text{FIR}$$

$\frac{Y(z)}{X(z)}$ $\quad y[n] = x[n] - 2\cos 2\pi\nu_0\, x[n-1] + x[n-2]$

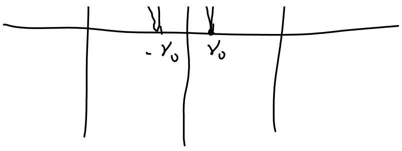


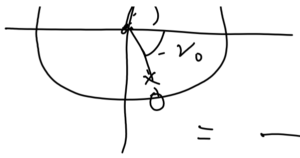




$$H(z) = \frac{(z-e^{j2\pi\nu_0})(z-e^{-j2\pi\nu_0})}{(z-\rho e^{j2\pi\nu_0})(z-\rho e^{-j2\pi\nu_0})}$$

z^2 ...





$$(z-\rho e^{\ldots})(z-\rho e^{\ldots}) = \frac{z^2 - 2\cos 2\pi\nu_0 z + 1}{z^2 - 2\rho\cos 2\pi\nu_0 z + \rho^2}$$

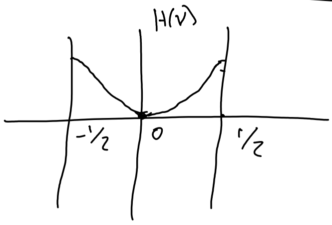
$$\frac{Y(z)}{X(z)} = \frac{1 - 2\cos 2\pi\nu_0 z^{-1} + z^{-2}}{1 - 2\rho\cos 2\pi\nu_0 z^{-1} + \rho^2 z^{-2}}$$

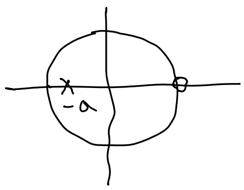
$$y[n] = 2\rho\cos 2\pi\nu_0\, y[n-1] - \rho^2 y[n-2] + x[n] - 2\cos 2\pi\nu_0\, x[n-1] + x[n-2]$$

$$Y(z)\left(1 - 2\rho\cos 2\pi\nu_0 z^{-1} + \rho^2 z^{-2}\right) = X(z)\left(1 - 2\cos 2\pi\nu_0 z^{-1} + z^{-2}\right)$$

$$\underbrace{Y(z)}_{y[n]} - 2\rho\cos 2\pi\nu_0 \underbrace{z^{-1}Y(z)}_{y[n-1]} + \rho^2 \underbrace{z^{-2}Y(z)}_{y[n-2]} = \underbrace{X(z)}_{x[n]} - 2\cos 2\pi\nu_0 \underbrace{z^{-1}X(z)}_{x[n-1]} + \underbrace{z^{-2}X(z)}_{x[n-2]}$$

$0 < a < 1$

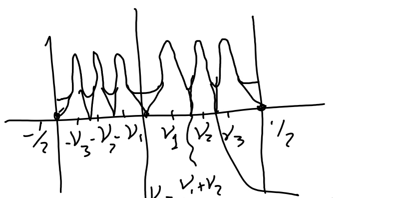




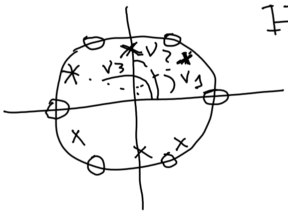
$$H(z) = \frac{z-1}{z+a} = \frac{1 - z^{-1}}{1 + a z^{-1}}$$

$$y[n] = -a\,y[n-1] + x[n] - x[n-1]$$

COMB



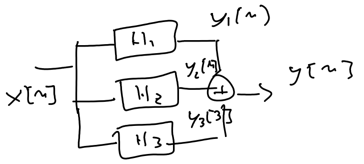
$\nu_a = \frac{\nu_1+\nu_2}{2}$ $\nu_b = \frac{\nu_2+\nu_3}{2}$



$$H(z) = \frac{(z-1)(z+1)(z-e^{j2\pi\nu_a})(z-e^{-j2\pi\nu_a})}{(z-\rho e^{j2\pi\nu_1})(z-\rho e^{-j2\pi\nu_1})(z-\rho e^{j2\pi\nu_2})} \cdot \frac{(z-e^{j2\pi\nu_b})(z-e^{-j2\pi\nu_b})}{(z-\rho e^{-j2\pi\nu_2})(z-\rho e^{j2\pi\nu_3})(z-\rho e^{-j2\pi\nu_3})}$$

$x[n] \longrightarrow \boxed{H(z)} \longrightarrow y[n]$

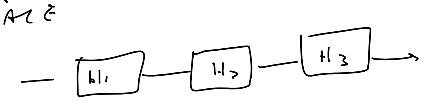
STRUTTURA PARALLELA



$$Y_1(z) = H_1(z)X(z)$$
$$Y_2(z) = H_2(z)X(z)$$
$$Y_3(z) = H_3(z)X(z)$$

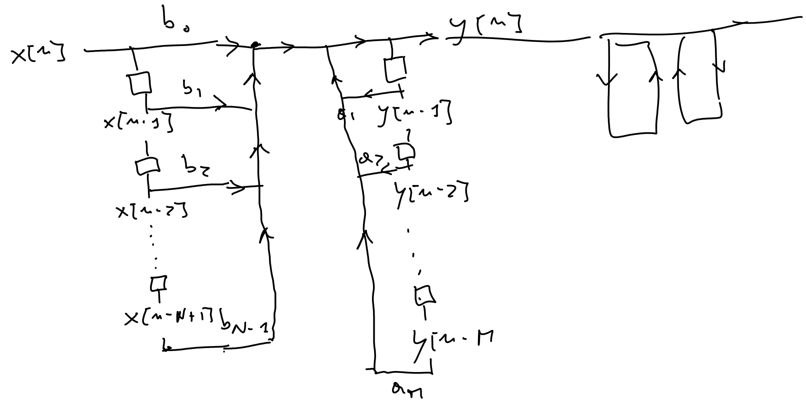
$$Y(z) = \underbrace{\left(H_1(z) + H_2(z) + H_3(z)\right)}_{H(z)} X(z)$$

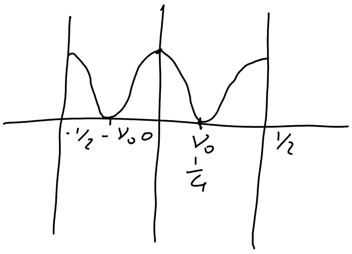
STRUTTURA SERIALE



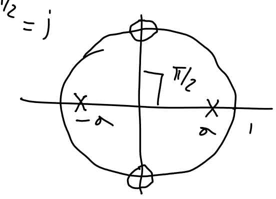
$$Y(z) = \underbrace{H_1(z)\,H_2(z)\,H_3(z)}_{H(z)} X(z)$$

$$y[n] = \sum_{k=1}^{M} a_k\, y[n-k] + \sum_{k=0}^{N-1} b_k\, x[n-k]$$



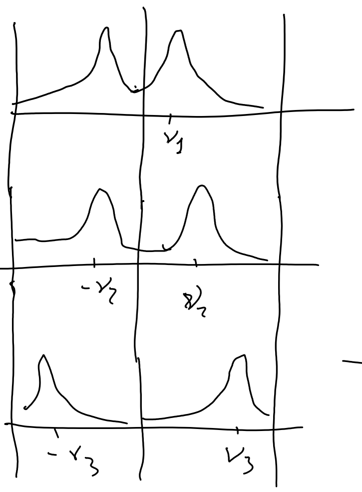


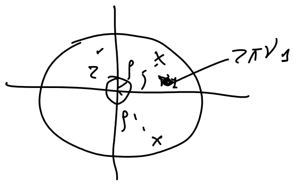
$e^{j\pi/2} = j$



$$H(z) = \frac{(z-j)(z+j)}{(z-a)(z+a)} = \frac{z^2+1}{z^2-a^2} = \frac{1+z^{-2}}{1-a^2 z^{-2}}$$

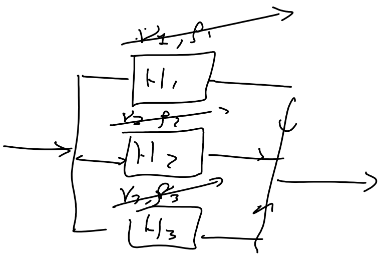
$$y[n] = a^2 y[n-2] + x[n] + x[n-2]$$



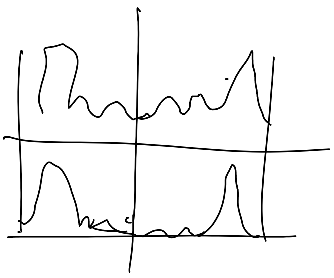


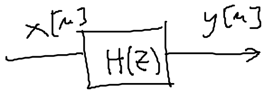
$$H_1(z) = \frac{z^2}{z^2 - 2\rho_1 \cos 2\pi\nu_1 z + \rho_1^2} = \frac{1}{1 - 2\rho_1 \cos 2\pi\nu_1 z^{-1} + \rho_1^2 z^{-2}}$$

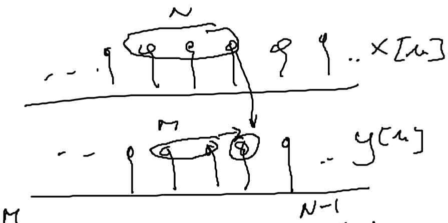
$$y_1[n] = 2\rho_1 \cos 2\pi\nu_1\, y_1[n-1] - \rho_1^2\, y_1[n-2] + x[n]$$



EQUALIZZATORE
GRAFICO



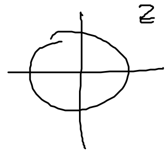




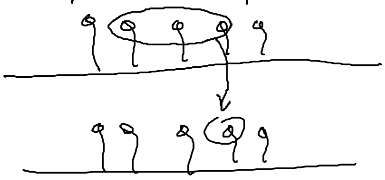
IIR

$$y[n] = \sum_{K=1}^{M} a_K\, y[n-K] + \sum_{K=0}^{N-1} b_K\, x[n-K]$$

$H(\nu)$



FIR Finite Impulse Response

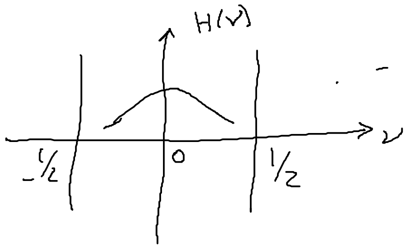


$$y[n] = \sum_{K=0}^{N-1} b_K\, x[n-K]$$

$\Downarrow$

$h[K]$

$\{b_K\}$? per ottenere una data risposta armonica $H(\nu)$?

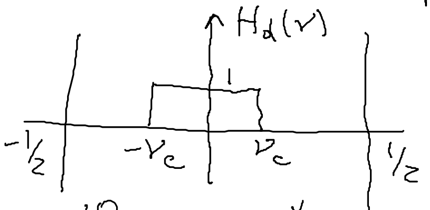


$$H(\nu) = \sum_{k=-\infty}^{+\infty} h[k]\, e^{-j2\pi\nu k} \qquad \text{DTFT}$$

$\updownarrow$

$$h[k] = \int_{-1/2}^{1/2} H(\nu)\, e^{j2\pi\nu k}\, d\nu \qquad \text{IDTFT}$$

ESEMPIO: Progettare un filtro FIR passa-basso con freq. di taglio ν_c



$$h_d[n] = \int_{-1/2}^{1/2} H_d(\nu)\, e^{j2\pi\nu n}\, d\nu = \int_{-\nu_c}^{\nu_c} e^{j2\pi\nu n}\, d\nu$$

$n \neq 0$

$$= \left.\frac{e^{j2\pi\nu n}}{j2\pi n}\right|_{-\nu_c}^{\nu_c} = \frac{e^{j2\pi\nu_c n} - e^{-j2\pi\nu_c n}}{j2\pi n} = \frac{\sin 2\pi\nu_c n}{\pi n}$$

$\downarrow$ e l'Hopital

$$\left.\frac{2\cancel{\pi}\nu_c \cos 2\pi\nu_c n}{\cancel{\pi}}\right|_{n=0} = 2\nu_c$$

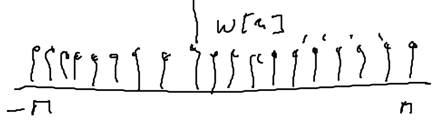
$n=0$

$2\nu_c$

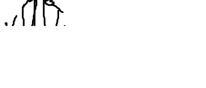
$$h_d[n] = 2\nu_c \frac{\sin 2\pi\nu_c n}{2\nu_c \pi n} = 2\nu_c \operatorname{sinc} 2\nu_c n$$

(1) Estensione infinita
(2) Non causale

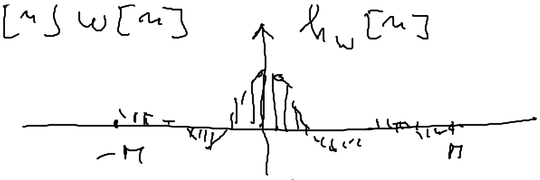
NON FISICA REALIZZABILITA'



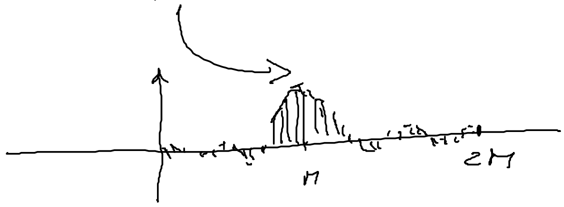
$$h_w[n] = h_d[n]\, w[n]$$



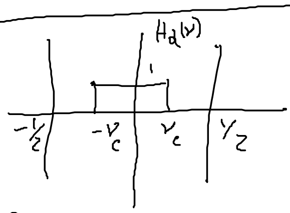
$$h_w[n] = h_d[n]\, w[n]$$



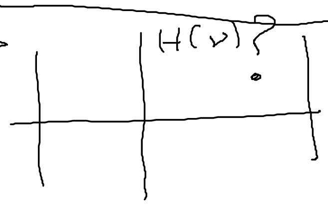
$$h[n] = h_w[n-M] = h_d[n-M]\, w[n-M]$$

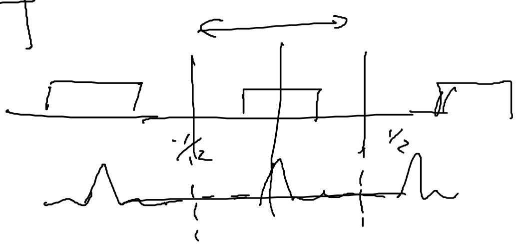


$$\boxed{h[n] = 2\nu_c \,\mathrm{sinc}\, 2\nu_c(n-M) \qquad n=0,\dots,2M}$$



Troncamento + Traslazione





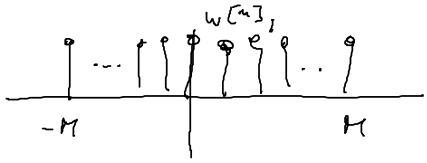
TRONCAMENTO

$$h_w[n] = h_d[n]\, w[n]$$

$$H_w(\nu) = H_d(\nu) \otimes W(\nu) \triangleq \int_{-1/2}^{1/2} H_d(\eta)\, W(\nu-\eta)\, d\eta$$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \qquad |x|<1$$

$$\sum_{n=0}^{N-1} x^n = \frac{1-x^N}{1-x}$$



$$W(\nu) = \sum_{n=-M}^{M} e^{-j2\pi\nu n} = \sum_{m=0}^{2M} e^{-j2\pi\nu(m-M)} = e^{+j2\pi\nu M} \sum_{m=0}^{2M} \left(e^{-j2\pi\nu}\right)^m$$

$$m = n+M$$

$$= e^{+j2\pi\nu M}\, \frac{1-e^{-j2\pi\nu(2M+1)}}{1-e^{-j2\pi\nu}} = e^{+j\frac{2\pi\nu(N-1)}{2}}\, \frac{1-e^{-j2\pi\nu N}}{1-e^{-j2\pi\nu}}$$

$$N = 2M+1 \qquad \frac{N-1}{2} = M$$

$$= \frac{e^{+j\pi\nu N}}{e^{+j\pi\nu}}\; \frac{1-e^{-j2\pi\nu N}}{1-e^{-j2\pi\nu}} = \frac{\dfrac{e^{j\pi\nu N} - e^{-j\pi\nu N}}{2j}}{\dfrac{e^{+j\pi\nu} - e^{-j\pi\nu}}{2j}}$$

Funzione di DIRICHELET

$$\boxed{W(\nu) = \frac{\sin \pi N\nu}{\sin \pi\nu}}$$

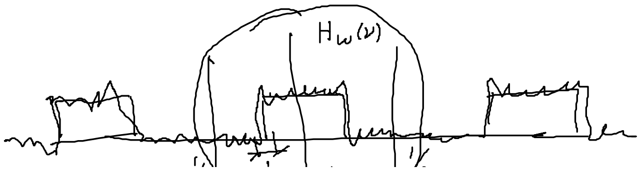
≃ sinc periodica

$$\sin \pi N\nu = 0 \quad \pi N\nu = k\pi \quad \nu = \frac{k}{N}$$

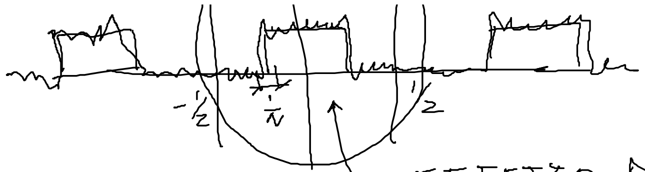
$$\sin \pi\nu = 0 \quad \pi\nu = k\pi$$

$$\nu = 0 \to \text{Hopital} \to \left.\frac{\cos \pi N\nu \cdot \pi N}{\cos \pi\nu \cdot \pi}\right|_{\nu=0} = N$$

$$H_w(\nu) = H_d(\nu) \otimes \frac{\sin \pi N\nu}{\sin \pi\nu}$$



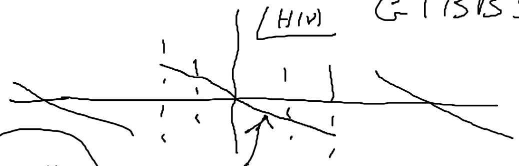
$$H_w(\nu) = H_d(\nu) \circledast \frac{\sin \pi N \nu}{\sin \pi \nu}$$



$N \to \infty$

EFFETTO DI GIBBS

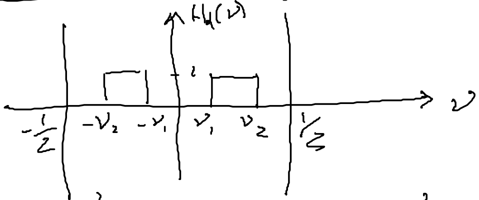
EFFETTO DELLA TRASLAZIONE



$$h[n] = h_w[n-M]$$

$$H(\nu) = e^{-j2\pi\nu M} H_w(\nu) = \underbrace{e^{-j2\pi\nu M}\left(H_d(\nu) \circledast \frac{\sin \pi N \nu}{\sin \pi \nu}\right)}$$

ESEMPIO: Progettare un filtro passa-banda in $[\nu_1, \nu_2]$



$$h_d[n] = \int_{-1/2}^{1/2} H_d(\nu) e^{j2\pi\nu n} d\nu =$$

$$= \int_{-\nu_2}^{-\nu_1} e^{j2\pi\nu n} d\nu + \int_{\nu_1}^{\nu_2} e^{j2\pi\nu n} d\nu = \left.\frac{e^{j2\pi\nu n}}{j2\pi n}\right]_{-\nu_2}^{-\nu_1} + \left.\frac{e^{j2\pi\nu n}}{j2\pi n}\right]_{\nu_1}^{\nu_2}$$

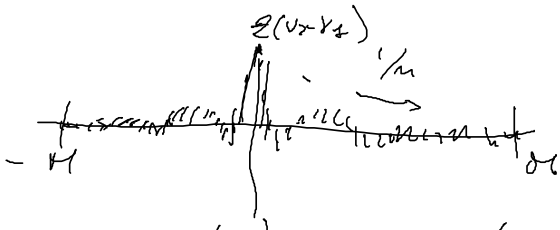
$$= \frac{e^{-j2\pi\nu_1 n} - e^{-j2\pi\nu_2 n} + e^{j2\pi\nu_2 n} - e^{j2\pi\nu_1 n}}{j2\pi n}$$

$$= \frac{-\sin 2\pi\nu_1 n + \sin 2\pi\nu_2 n}{\pi n}$$

$n = 0$

$$\left.\frac{-2\pi\nu_1 \cos 2\pi\nu_1 n + 2\pi\nu_2 \cos 2\pi\nu_2 n}{\pi}\right|_{n=0}$$

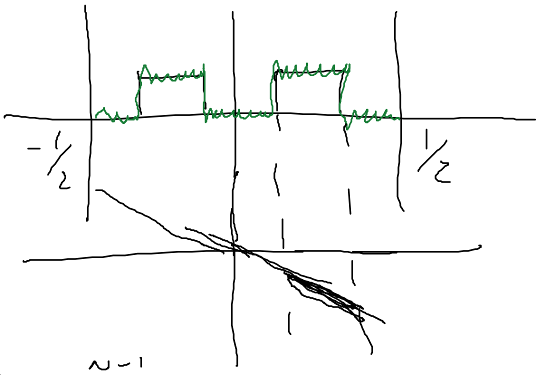
$$\frac{-2\pi\nu_1 + 2\pi\nu_2}{\pi} = 2(\nu_2 - \nu_1)$$

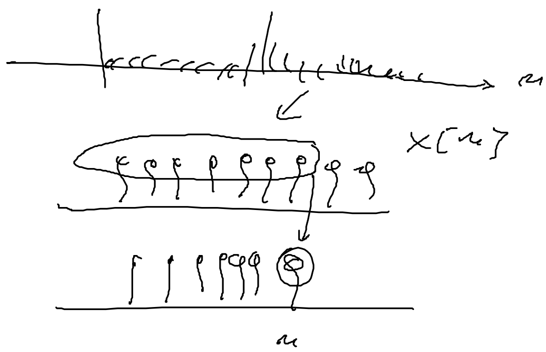


$N = 2M+1$

$$h[n] = \frac{\sin 2\pi\nu_2(n-M) - \sin 2\pi\nu_1(n-M)}{\pi(n-M)}$$

$n = 0, \dots, 2M+1$





$$y[n] = \sum_{k=0}^{N-1} h[k]\, x[n-k]$$

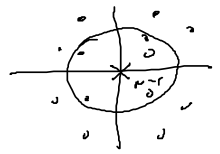
$$H(z) = \sum^{N-1} h[k] z^{-k}$$

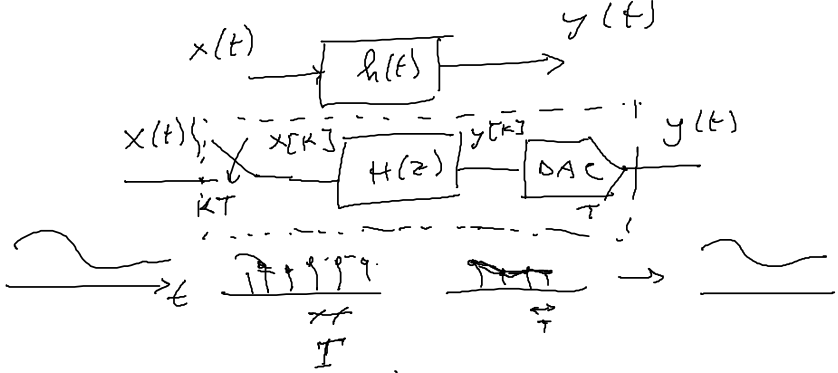
$$H(z) = \sum_{k=0}^{N-1} h[k]\, z^{-k} \qquad y[n] = \sum_{k=0} h[k]\, x[n-k] \qquad Y(z) = H(z)\, X(z)$$

$$= h[0] + h[1] z^{-1} + h[2] z^{-2} + \dots + h[N-1] z^{-N+1}$$

$$= z^{-N+1}\left(h[0] z^{N-1} + h[1] z^{N-2} + \dots + h[N-1]\right)$$

$$= \frac{1}{z^{N-1}} \qquad N-1 \text{ zeri}$$

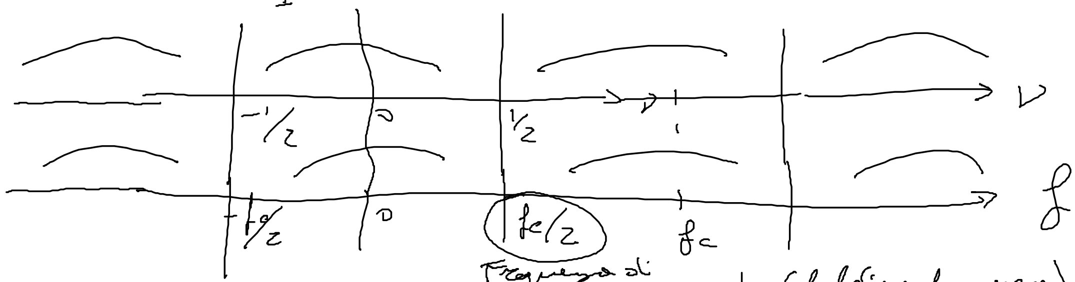




$\nu = \dfrac{f}{f_c}$ =

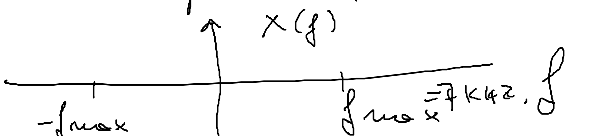
T intervallo di campionamento [sec]

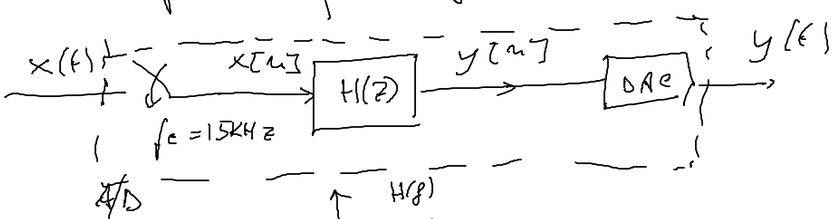
$\frac{1}{T}$ frequenza di campionamento $[Hz] = [\frac{1}{sec}]$



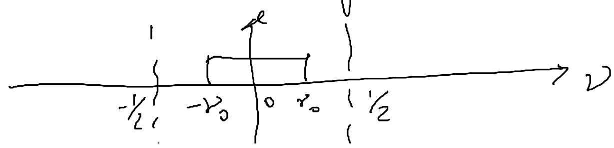
Frequenza di ripiegamento (folding frequency)

ES. Progettare un filtro passa basso con freq. di taglio $f_0 = 5KHz$ mediante un sistema numerico che campiona il segnale d'ingresso a $15 KHz$ (f_C). Si assuma che il segnale d'ingresso ha freq. massima pari a $7 KHz$.



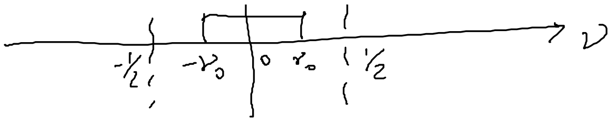


⇓ filtro numerico



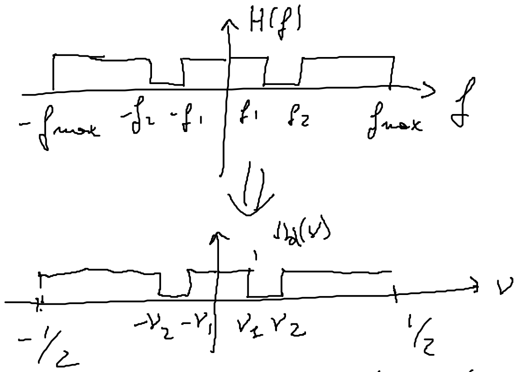
$$\nu_0 = \frac{f_0}{f_c} = \frac{5000}{15000} = \frac{1}{3}$$

$$\nu_0 < \frac{1}{2}$$



$(\nu_0 < 1/2)$

Esempio Progettare un filtro a banda passante $[f_1, f_2] = [2KHz, 4KHz]$ mediante un sistema numerico a freq. di campionamento di $20 KHz$.
Si assuma che il segnale all'ingresso abbia un contenuto spettrale che non supera i 10 KHz. (f_{max})



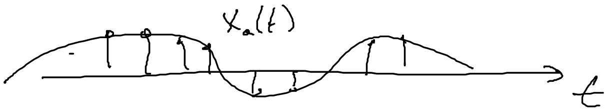
$$\nu_1 = \frac{f_1}{f_c} = \frac{2000}{20000} = \frac{1}{10}$$

$$\nu_2 = \frac{f_2}{f_c} = \frac{4000}{20000} = \frac{2}{10}$$

FIR (con il metodo della finestra) (o metodo della SERIE DI FOURIER)

$$h_d[n] = \int_{-1/2}^{1/2} H_d(\nu) e^{+j2\pi\nu n} d\nu = \ldots\ldots$$

$$h[n] = h_d[n-M] \qquad n = 0, \ldots, 2M+1$$

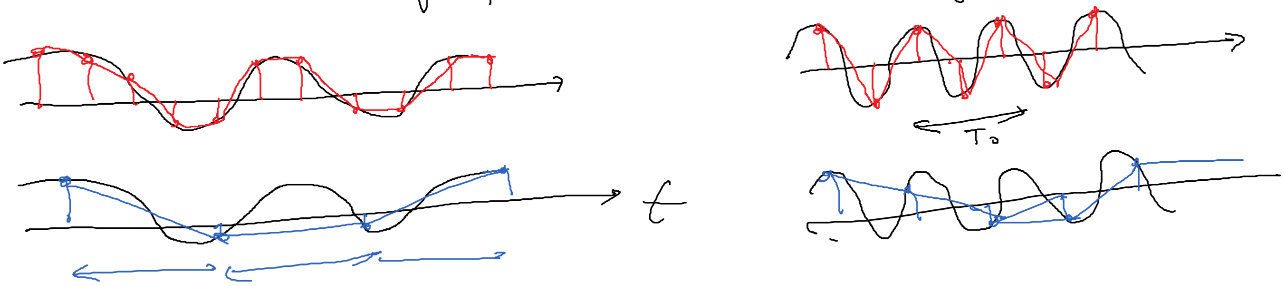


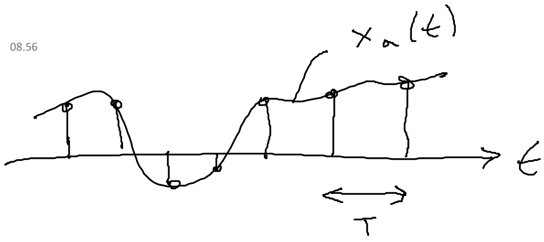
T ? $\quad f_c = \frac{1}{T}$?

$$\boxed{f_c > 2 f_{max}}$$

$$f_c > 2 f_0$$

$$f_0 = \frac{1}{T_0}$$

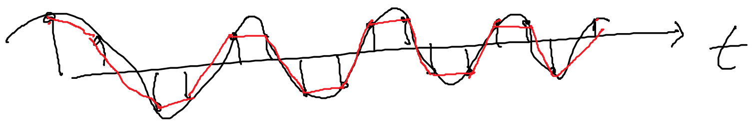


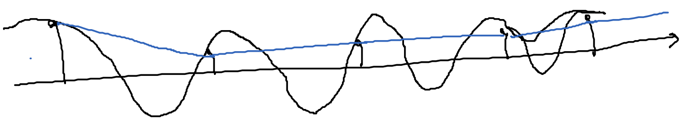


T INTERVALLO DI CAMPIONAMENTO [sec]

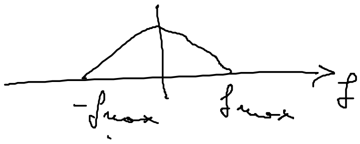
$f_c = \frac{1}{T}$ FREQUENZA DI CAMPIONAMENTO $\left[\frac{1}{sec}\right] = [Hz]$
$= \left[\frac{campioni}{sec}\right]$

CRITERIO INTUITIVO



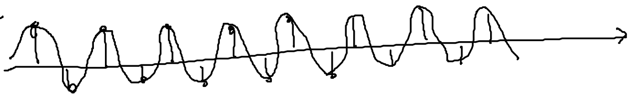


$\rightarrow$ in frequenza $X_a(f)$



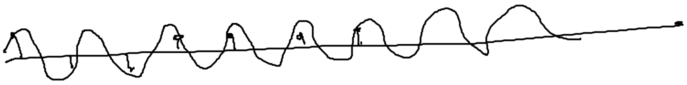
f_{max}

campionato "bene"

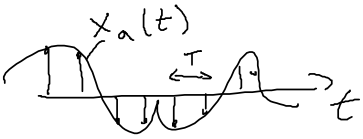


$T < \frac{1}{2 f_{max}} \Rightarrow \boxed{\frac{1}{T} > 2 f_{max}}$

Mal campionato



$f_c > 2 f_{max}$
Criterio di Nyquist.



$\underline{x_a(t)} \xrightarrow{t = nT} \underline{x[n]}$

$x_a(t) \longleftrightarrow X_a(f)$

$x[n] = x_a(nT)$

$\downarrow$ DTFT

$$X(\nu) = \sum_{n=-\infty}^{+\infty} x[n]\, e^{-j2\pi\nu n}$$

$x[n] \xleftrightarrow{DTFT} X(\nu)$

$$x[n] = \int_{-1/2}^{1/2} X(\nu)\, e^{j2\pi\nu n}\, d\nu$$

$$= \sum_{n=-\infty}^{+\infty} x_a(nT)\, e^{-j2\pi\nu n} = \sum_{n=-\infty}^{+\infty} \int_{-\infty}^{+\infty} X_a(f)\, e^{j2\pi f n T}\, df \;\; e^{-j2\pi\nu n}$$

$$= \int_{-\infty}^{+\infty} X_a(f) \underbrace{\sum_{n=-\infty}^{+\infty} e^{j2\pi n (fT-\nu)}}_{P=1 \;\; \sum_{n=-\infty}^{+\infty} \delta(fT-\nu-n)}\, df$$

FORMULA DI POISSON

$$\boxed{\frac{1}{P}\sum_{n=-\infty}^{+\infty} e^{-j\frac{2\pi}{P}nx} = \sum_{n=-\infty}^{+\infty} \delta(x - nP)}$$

$$= \int_{-\infty}^{+\infty} X_a(f) \sum_{n=-\infty}^{+\infty} \delta(fT-\nu-n)\, df$$

$$= \sum_{n=-\infty}^{+\infty} \int_{-\infty}^{+\infty} X_a(f)\, \delta(fT-\nu-n)\, df$$

$fT - \nu - n = 0$

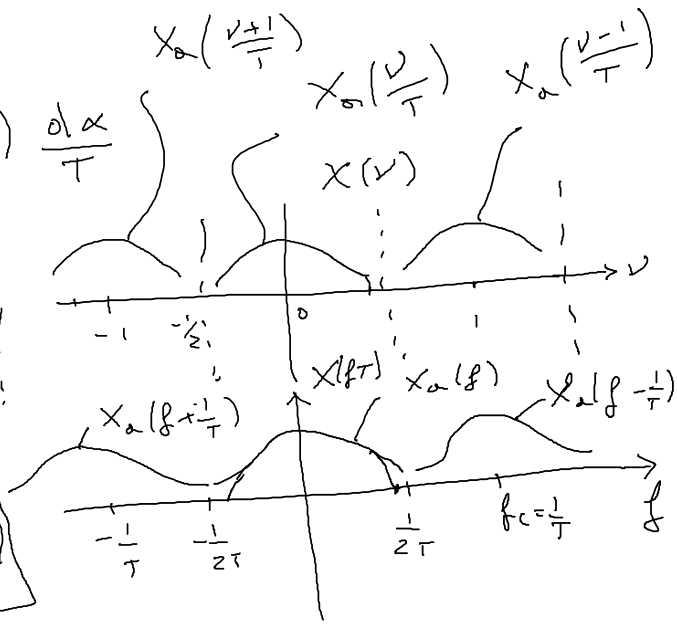
$fT = \nu + n$

$f = \frac{\nu + n}{T}$

$\alpha = fT \qquad d\alpha = df\,T$

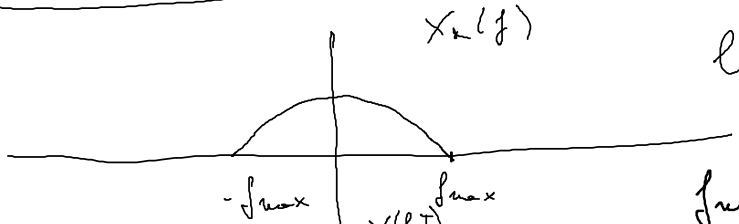
$$= \sum_{n=-\infty}^{+\infty} \int_{-\infty}^{+\infty} X_a\left(\frac{\alpha}{T}\right) \delta(\alpha - \nu - n)\, \frac{d\alpha}{T}$$

$$\boxed{X(\nu) = \frac{1}{T} \sum_{n=-\infty}^{+\infty} X_a\left(\frac{\nu + n}{T}\right)}$$



$\nu \triangleq \frac{f}{f_c} = fT$

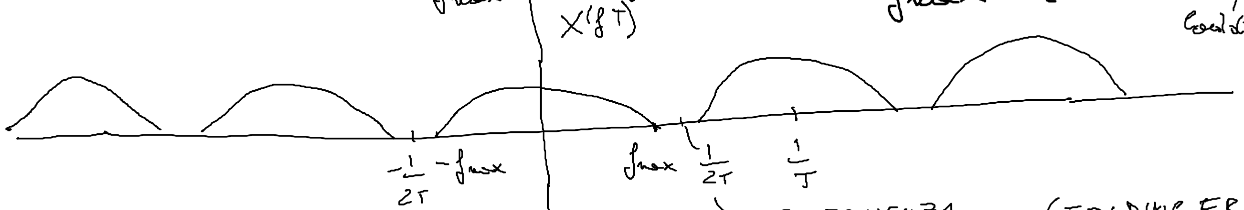
$$\boxed{X(fT) = \frac{1}{T} \sum_{n=-\infty}^{+\infty} X_a\left(f + \frac{n}{T}\right)}$$



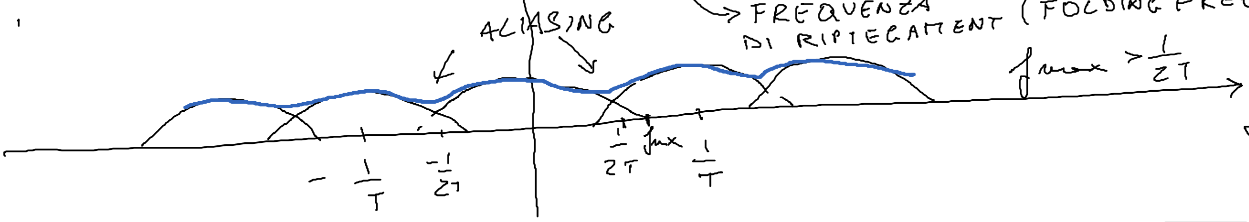
limitato in banda

$f_{max} < \frac{1}{2T} \implies \frac{1}{T} > 2 f_{max}$

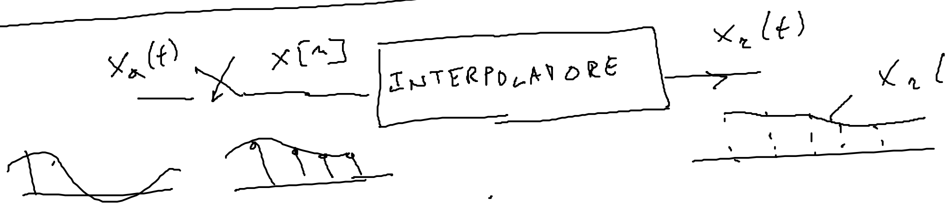
condiz. Nyquist rispettata



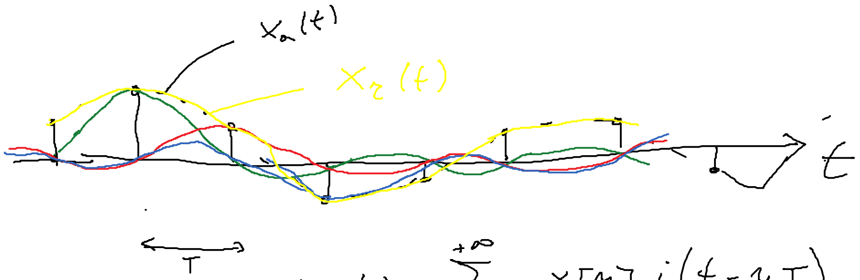
→ FREQUENZA DI RIPIEGAMENTO (FOLDING FREQUENCY)



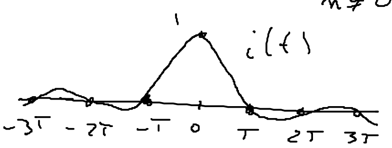
$f_{max} > \frac{1}{2T} \qquad \frac{1}{T} < 2 f_{max}$ Nyquist no!



$x_r(t) \overset{?}{=} x_a(t)$

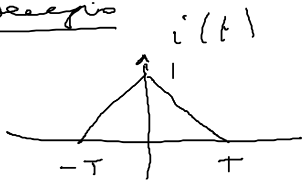


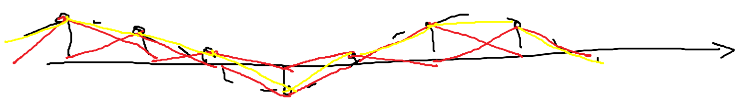
$$i(t) = \begin{cases} 1 & t = 0 \\ 0 & t = nT \\ & n \neq 0 \end{cases}$$

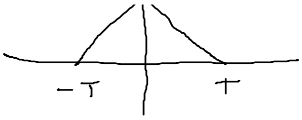


$$x_r(t) = \sum_{n=-\infty}^{+\infty} x[n]\, i(t - nT) \overset{?}{\Longleftrightarrow} x_a(t)$$

<u>Esempio</u>

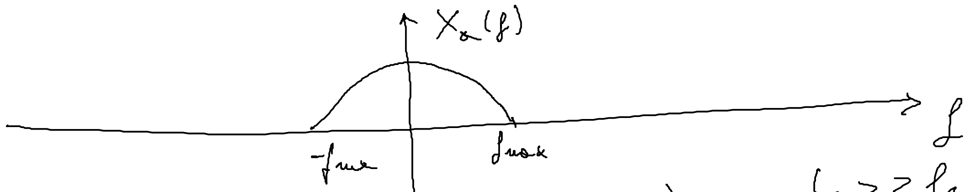




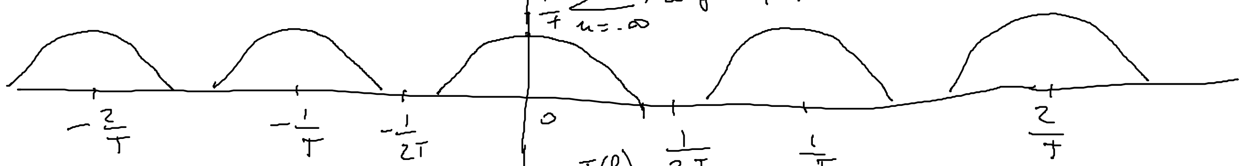


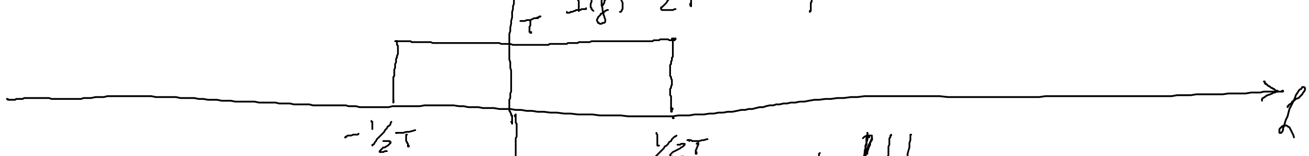
$$X_r(f) = \sum_{n=-\infty}^{+\infty} x[n]\, I(f)\, e^{-j2\pi f n T} = I(f) \sum_{n=-\infty}^{+\infty} x[n]\, e^{-j2\pi f n T}$$

$$= I(f) \underset{DTFT}{X}(fT) = I(f)\, \frac{1}{T} \sum_{n=-\infty}^{+\infty} X_a\left(f - \frac{n}{T}\right)$$

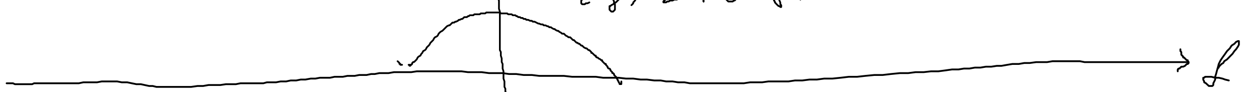


$\frac{1}{T} > 2 f_{max}$



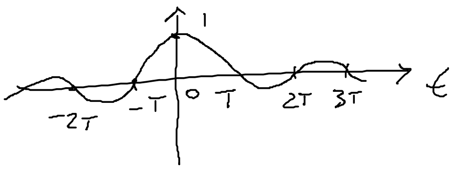


$X_r(f) = X_a(f)$!!!



$$I(f) = T\, \Pi\left(\frac{f}{1/T}\right) \longleftrightarrow i(t) = \cancel{T}\frac{1}{\cancel{T}}\, \mathrm{sinc}\, \frac{1}{T} t$$

$$\boxed{i(t) = \mathrm{sinc}\, \frac{1}{T} t}$$



1. $x_a(t)$ limitato in banda a f_{max}

2. $\frac{1}{T} > 2 f_{max}$ (cond. di Nyquist)

!!!!

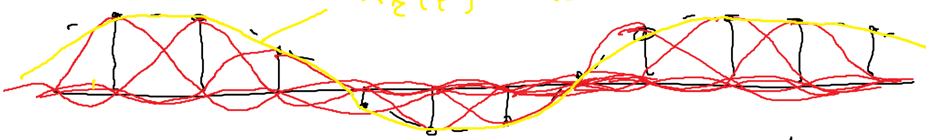
$\Downarrow$

$$\boxed{x_r(t) = \sum_{n=-\infty}^{+\infty} x[n]\, \mathrm{sinc}\left(\frac{t - nT}{T}\right)} = x_a(t)$$

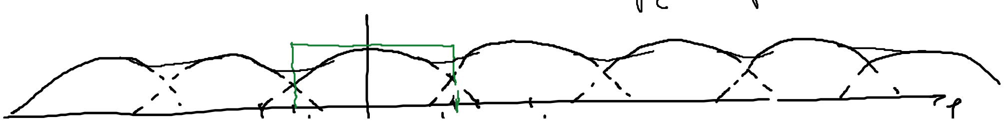
FORMULA DI INTERPOLAZIONE CARDINALE

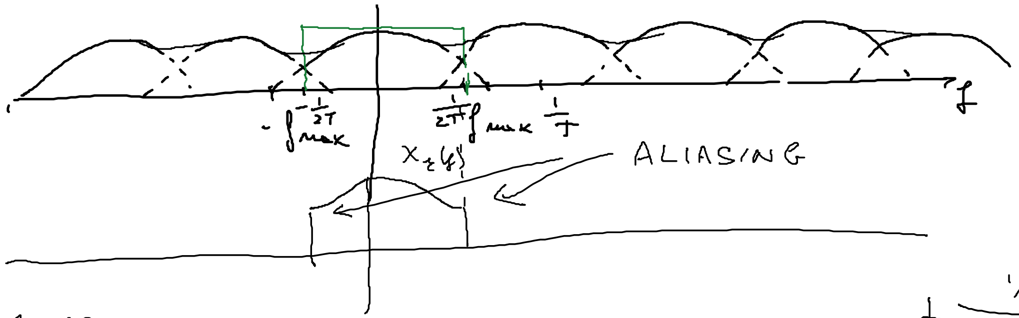
Shannon
Witaker
Witaker
Kotelnikov

$x_r(t) = x_a(t)$



$f_c < 2 f_{max}$



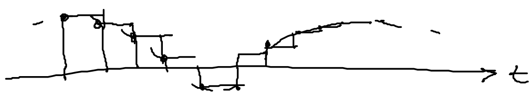


PRATICAMENTE:

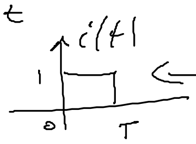
$i(t) = \text{sinc}\,\frac{t}{T}$ ha estensione infinita

SINC TRONCATO $\quad i_N(t) = \begin{cases} i(t) & -NT<t<NT \\ 0 & \text{altrove} \end{cases}$

ALTRE INTERPOLAZIONI: $\quad$ S/H $\quad$ SAMPLE-AND-HOLD

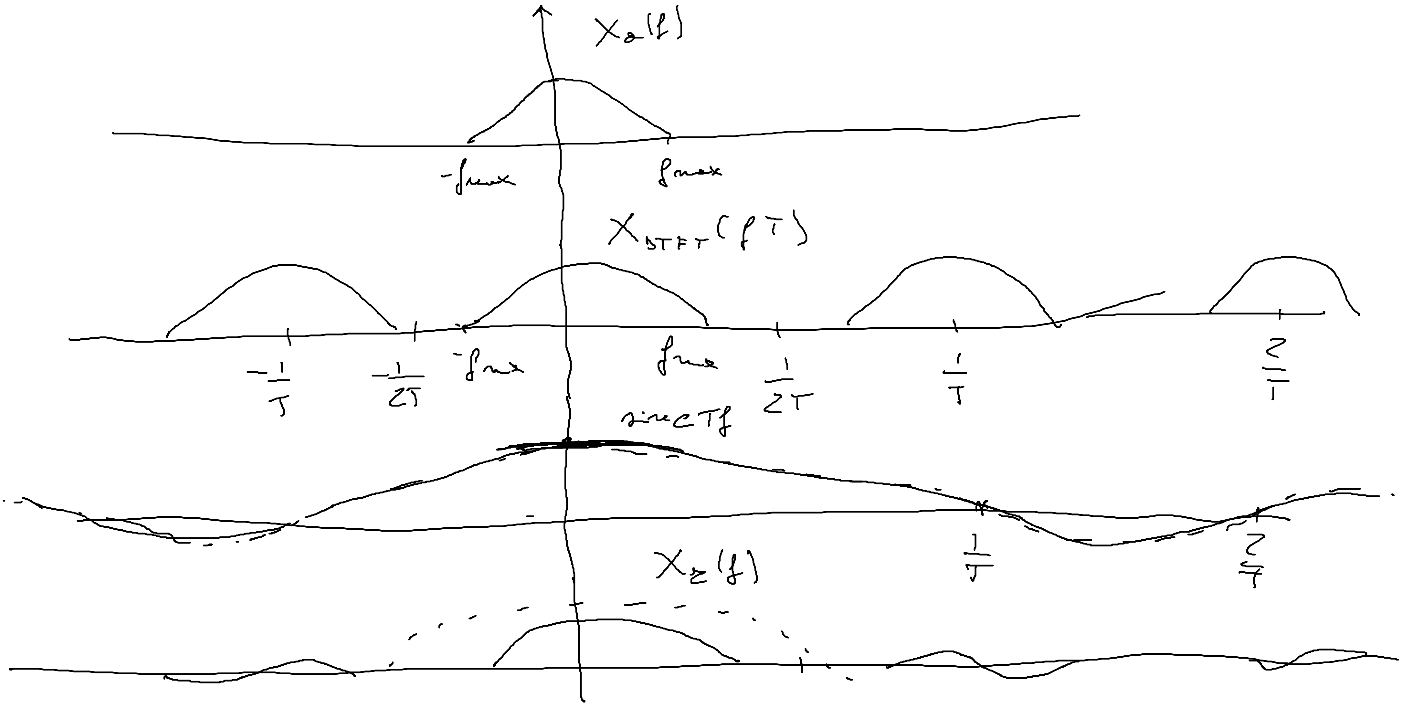


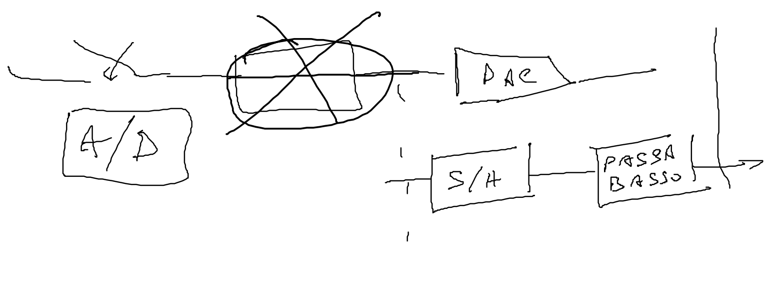
$$x_r(t) = \sum_{n=-\infty}^{+\infty} x[n]\, i(t-nT)$$

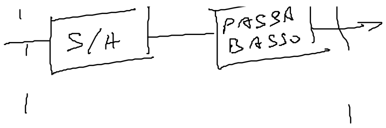


$$\longleftrightarrow \quad I(f) = e^{-j2\pi f T/2}\, T\,\text{sinc}\,Tf$$

$$X_r(f) = I(f)\, X_{DTFT}(fT) = I(f)\,\frac{1}{T}\sum_{n=-\infty}^{+\infty} X_a\left(f-\frac{n}{T}\right)$$



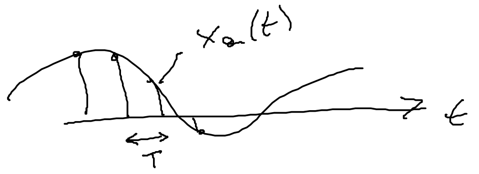




$X[n] \in \mathbb{R}$

QUANTIZZAZIONE

T. del campionamento



$x_a(t)$ limitato in banda a f_{max}

$f_c = \frac{1}{T}$

$$\frac{1}{T} > 2 f_{max}$$

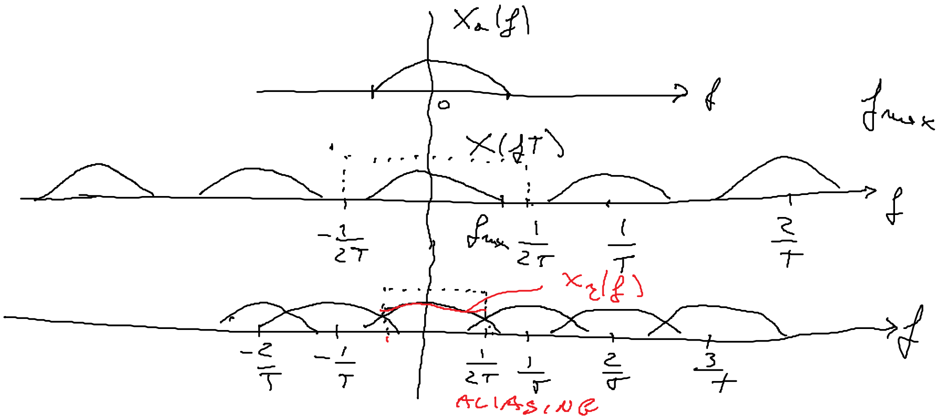
$$x_r(t) = \sum_{n=-\infty}^{+\infty} x_a(nT)\, \mathrm{sinc}\left(t - \frac{n}{T}\right) \;\; \overset{!!}{=} x_a(t)$$

FORMULA DI INTERPOLAZIONE CARDINALE

Relazione tra tempo-continuo e tempo discreto nel D.F

$$x_a(t) \longleftrightarrow X_a(f) \qquad \nu = \frac{f}{f_c} = fT$$

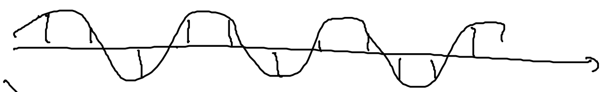
$$x[n] = x_a(nT) \longleftrightarrow X(fT) = \frac{1}{T}\sum_{n=-\infty}^{+\infty} X_a\left(f - \frac{n}{T}\right)$$



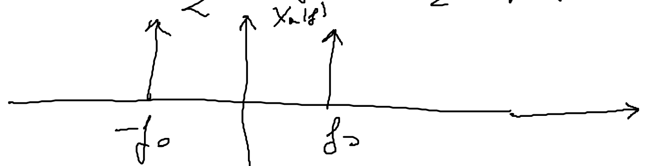
$f_{max} < \frac{1}{2T}$

$\frac{1}{T} > 2 f_{max}$

ESEMPIO $x_a(t) = A\cos 2\pi f_0 t$



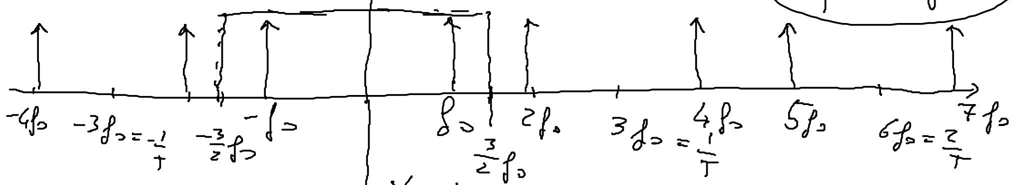
$$X_a(f) = \frac{A}{2}\delta(f - f_0) + \frac{A}{2}\delta(f + f_0)$$



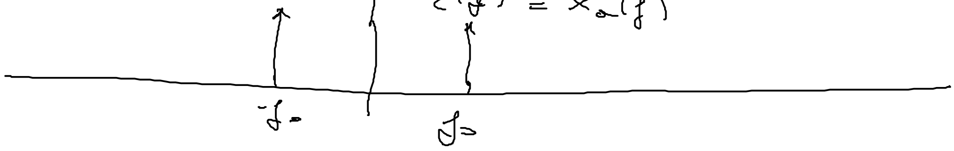
$\frac{1}{T} > 2 f_0$

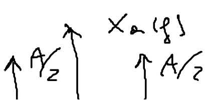
$\frac{1}{T} = 3 f_0$

Ben campionato

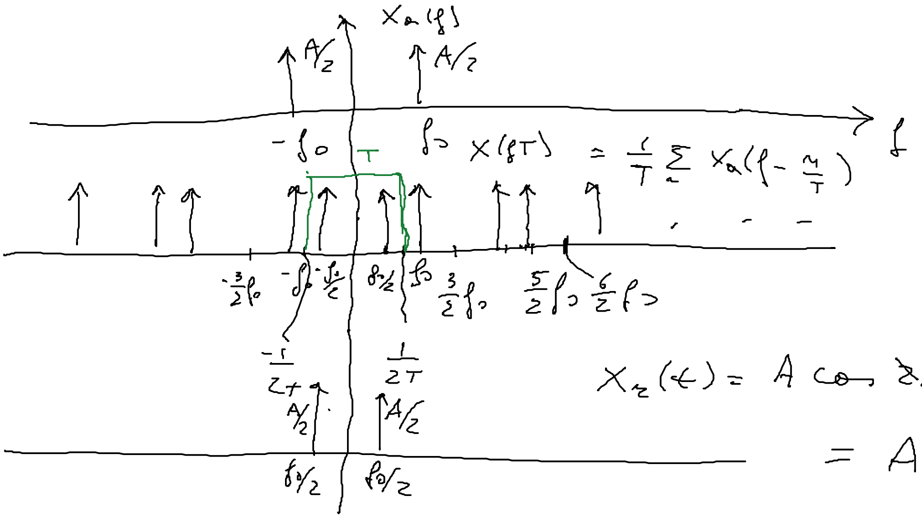


$X_r(f) = X_a(f)$



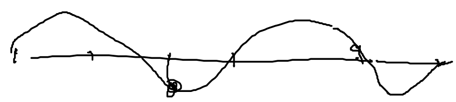


$\cancel{\frac{1}{T} > 2 f_0}$



~~$\frac{1}{T} > 2 f_0$~~

$$\frac{1}{T} = \frac{3}{2} f_0$$

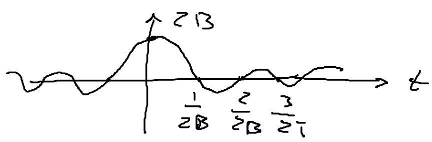


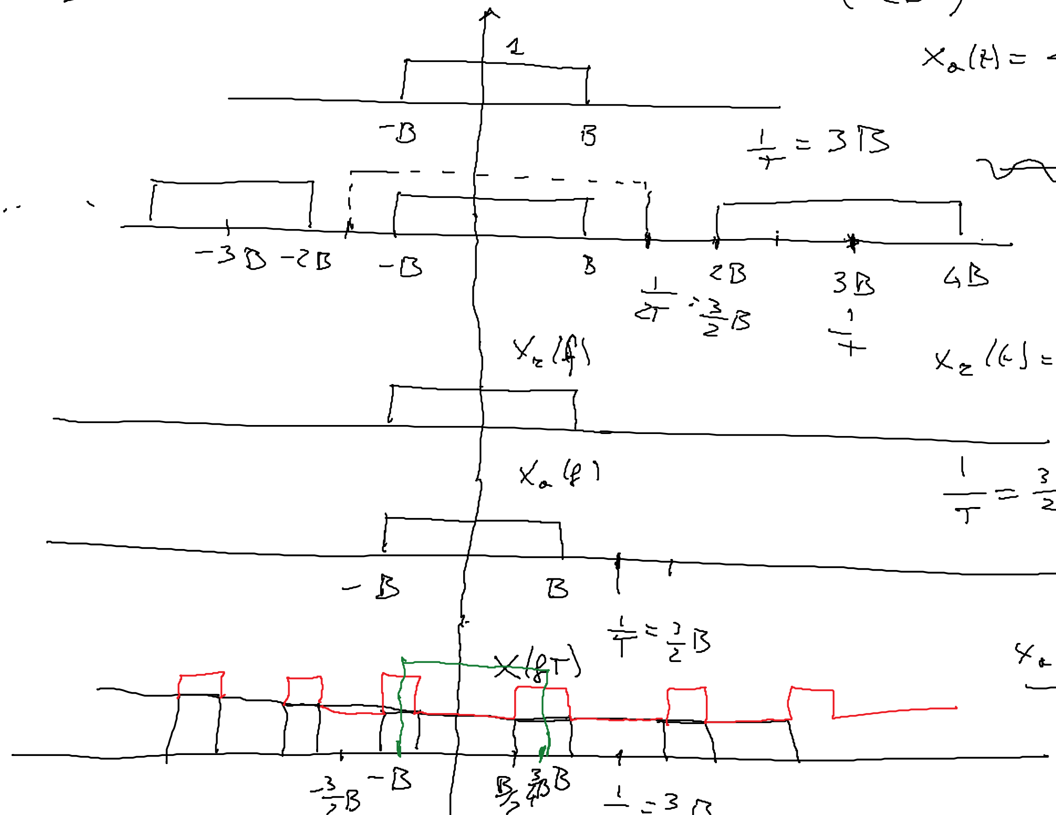
$$x_r(t) = A \cos 2\pi \frac{f_0}{2} t \qquad T = \frac{2}{3} T_0$$

$$= A \cos \pi f_0 t$$

Esempio $x_a(t)$ $\quad X_a(f) = \Pi\left(\frac{f}{2B}\right)$

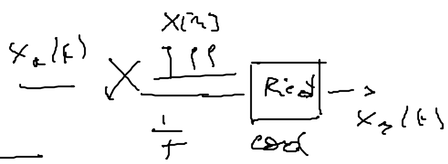
$$x_a(t) = 2B \operatorname{sinc} 2Bt$$



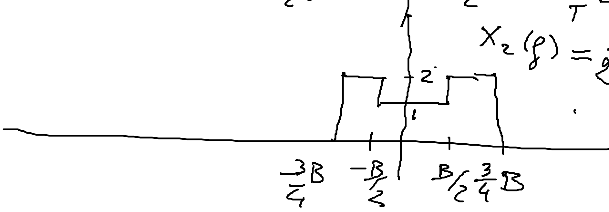


$$x_r(t) = 2B \operatorname{sinc} 2Bt$$

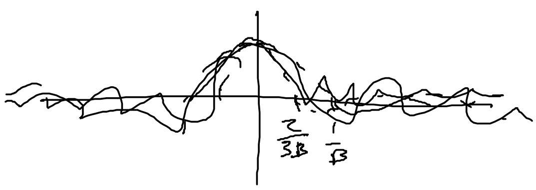
$$\frac{1}{T} = \frac{3}{2} B$$



$$X_r(f) = 2\Pi\left(\frac{f}{3/2\, B}\right) - \Pi\left(\frac{f}{B}\right)$$



$$x_r(t) = 3B \operatorname{sinc} \frac{3}{2}Bt - B \operatorname{sinc} Bt$$



Esempio $\quad x_a(t) = \cos 2\pi f_0 t - \frac{1}{2} \cos 2\pi f_1 t$

$f_0 = 5$ KHZ

$f_1 = 7$ KHZ

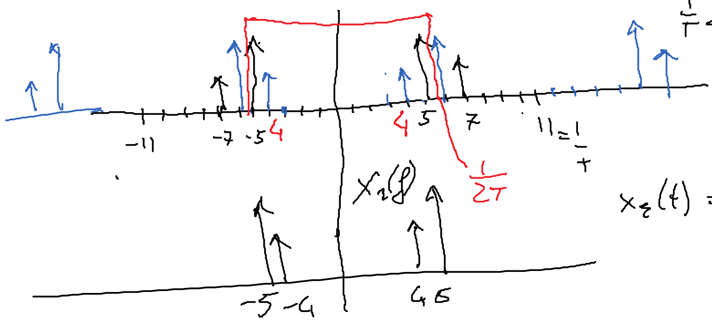
Esempio $x_a(t) = \cos 2\pi f_0 t - \frac{1}{2}\cos 2\pi f_1 t$

$f_0 = 5$ KHZ

$f_1 = 7$ KHZ

$f_c = \frac{1}{T} = 11$ KHZ → Aliasing

$\frac{1}{T}\sum_k X_a\left(f - \frac{k}{T}\right)$



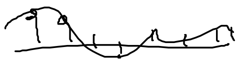
$x_r(t) = \cos 2\pi(5000)t - \frac{1}{2}\cos 2\pi(4000)t$

K					
0	-7	-5	5	7	11
1	+4	6	16	18	
2	15	17	27	29	
-1	-18	-16	-6	-4	-11
-2	-29	-27	-17	-15	

$\frac{1}{2T} = 5.5$

$[-5.5 \,,\ 5.5]$

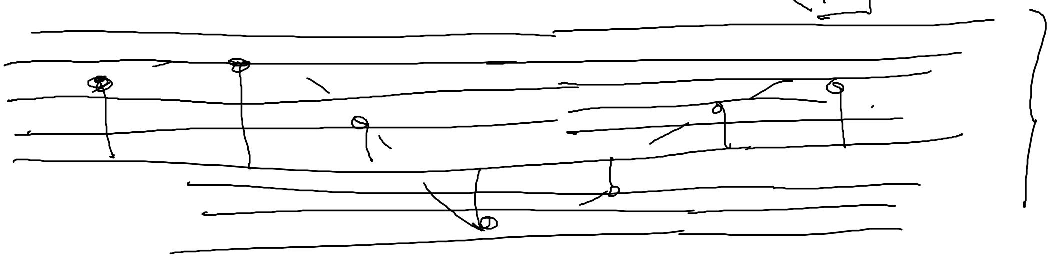
QUANTIZZAZIONE



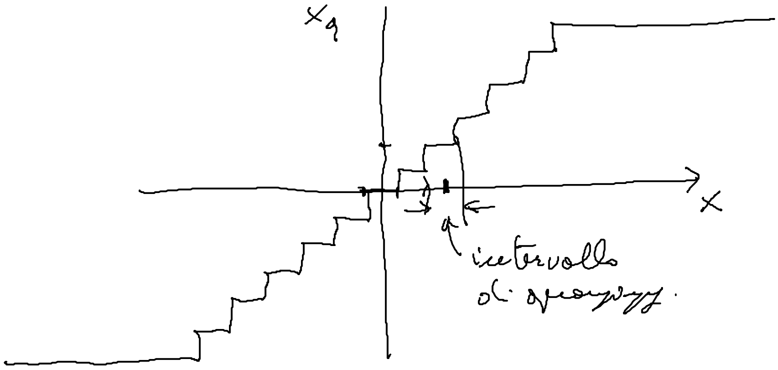
A/D

ANALOGICO

NUMERICO



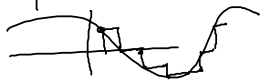
2^n



n bit

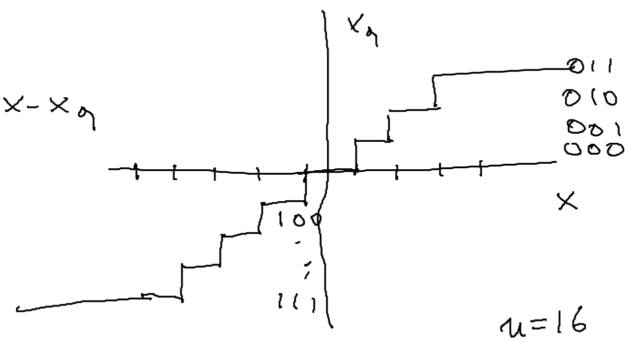
2^n intervalli di quantizzazione

D/A

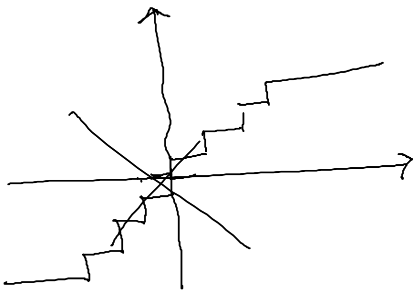


Es $n = 3$ bit 2^3

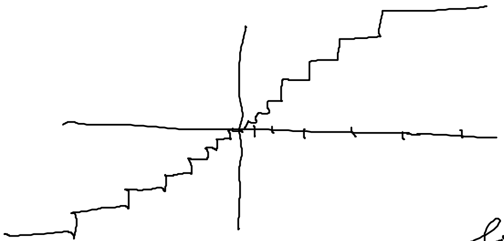
$\varepsilon = x - x_q$

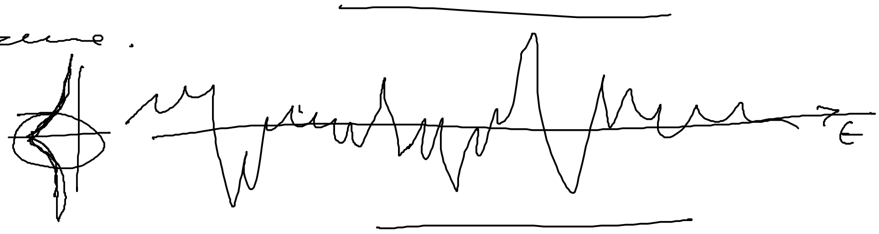


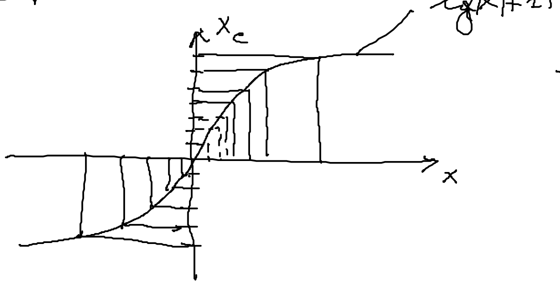
$n = 16$

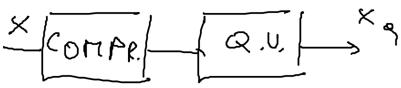


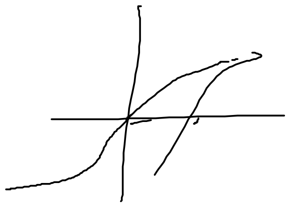
Quant. non uniforme.

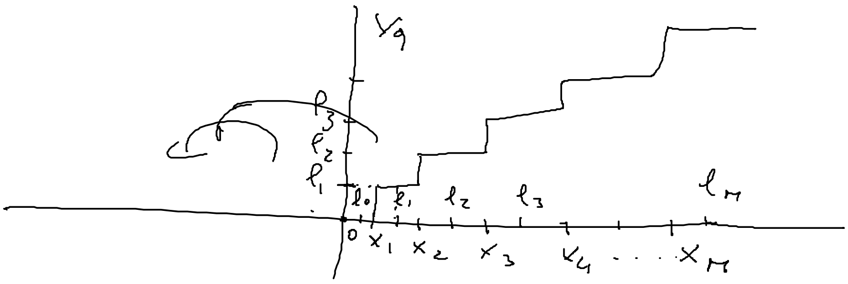


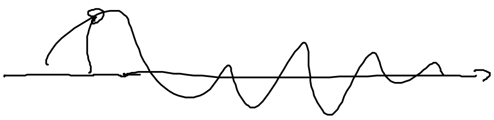


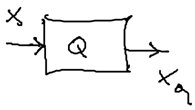












$$\varepsilon_q = x - x_q$$

$$x[n] \in \mathbb{R}$$

$$y[n] = a\, y[n-1] + b\, x[n]$$

Lezione 22

10.24

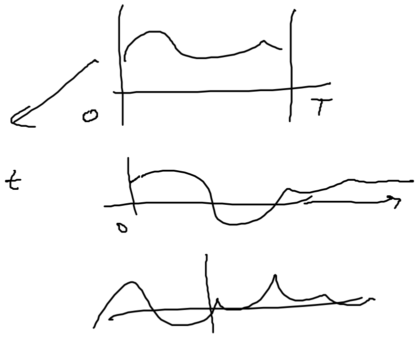
Giovedi 17 Dicembre 9-11

Giovanni Di Gennaro: DFT; Calcolo veloce della DFT (algoritmo FFT)

$\{\varphi_1(t), \dots \varphi_N(t)\}$

$\{e_1 \dots e_N\}$

$$s(t) \doteq \sum_{i=1}^{N} c_i \varphi_i(t)$$



$$\mathcal{E}(\underline{c}) = \| s(t) - \sum_{i=1}^{N} c_i \varphi_i(t) \|^2 = \int_0^T \left(s(t) - \sum_{i=1}^{N} c_i \varphi_i(t)\right)^2 dt$$

$$\underline{c}_0 = \underset{\underline{c} \in \mathbb{R}^N}{\arg\min}\ \mathcal{E}(\underline{c})$$

$$\begin{bmatrix} \int \varphi_1^2 & \int \varphi_1\varphi_2 & \cdots & \int \varphi_1 \varphi_N \\ \int \varphi_2 \varphi_1 & \int \varphi_2^2 & & \int \varphi_2 \varphi_N \\ & & \ddots & \\ \int \varphi_N \varphi_1 & & & \int \varphi_N^2 \end{bmatrix} \begin{bmatrix} c_{01} \\ c_{02} \\ \vdots \\ c_{0N} \end{bmatrix} = \begin{bmatrix} \int s\varphi_1 \\ \int s\varphi_2 \\ \vdots \\ \int s \varphi_N \end{bmatrix}$$

EQ. NORMALE

$\longrightarrow$ φ_i ortogonali

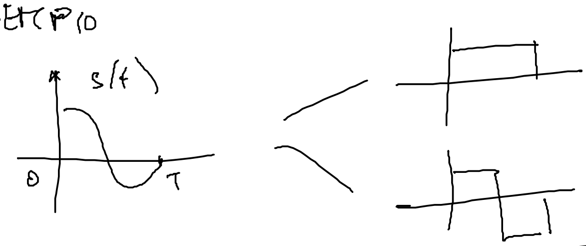
$$\begin{bmatrix} \int \varphi_1^2 & & & 0 \\ & \int \varphi_2^2 & & \\ & & \ddots & \\ 0 & & & \int \varphi_N^2 \end{bmatrix} \begin{bmatrix} c_{01} \\ \vdots \\ \vdots \\ c_{0N} \end{bmatrix} = \begin{bmatrix} \int s\varphi_1 \\ \\ \\ \int s \varphi_N \end{bmatrix}$$

$$c_{0i} = \frac{\int s\varphi_i}{\int \varphi_i^2} \qquad i = 1 \dots N$$

φ_i ortonormali $\left(\int \varphi_i^2 = 1\right)$

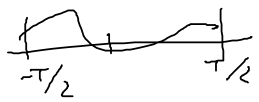
$$c_{0i} = \int s\varphi_i \qquad i = 1 \dots N$$

ESEMPIO



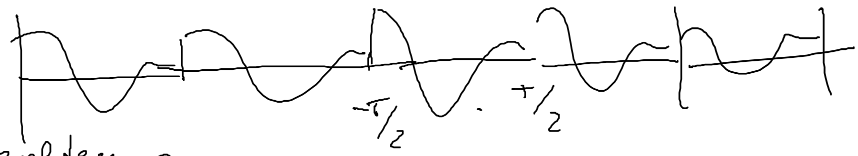
$\Longrightarrow (c_{01}, c_{02})$

cos sin cos + sin



$$\phi_n(t) = e^{+j\frac{2\pi}{T}nt} \qquad s_N(t) = \sum_{n=-N}^{N} c_n e^{j\frac{2\pi}{T}nt} \qquad c_n = \frac{1}{T}\int_{-T/2}^{T/2} s(t)\, e^{-j\frac{2\pi}{T}nt} dt$$

$N \to \infty \qquad S_N(f) \xrightarrow[MQ]{} s(t)$



s(t) - limitato nel tempo
- periodico

$\Rightarrow$ T.F. $\qquad s(t) = \int_{-\infty}^{+\infty} S(f)\, e^{j2\pi f t}\, df$ $\longrightarrow f$

integrale di Fourier

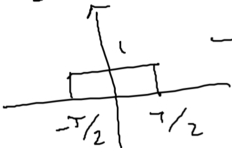
$$S(f) = \int_{-\infty}^{+\infty} s(t)\, e^{-j2\pi f t}\, dt$$

T. di Fourier

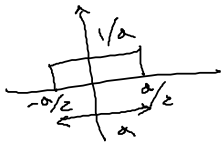
DT $\longleftrightarrow$ DF

Proprietà:
- linearità
- tr. temporale
- convoluzione
- prodotto
- scala
- dualità
- ⋮

$s(t) = \Pi\left(\frac{t}{T}\right)$ $\qquad S(f) = T \operatorname{sinc} T f$



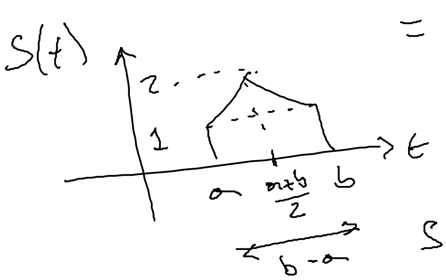
$s(t) = \operatorname{sinc} a t$



$s(t) = \frac{1}{a}(a \operatorname{sinc} a t)$

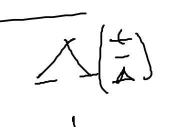
$S(f) = \frac{1}{a}\,\Pi\left(\frac{f}{a}\right)$

$s(t)$



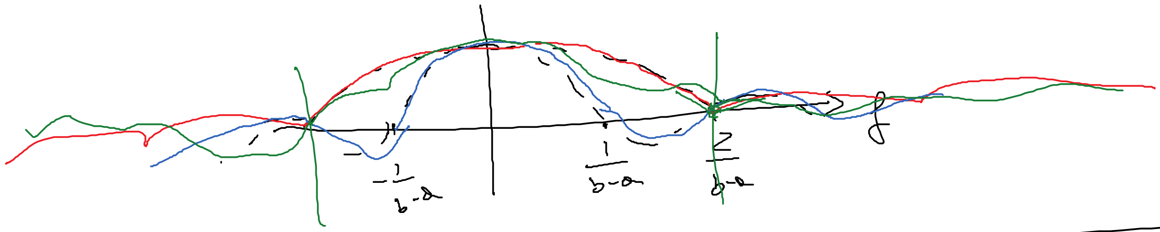
$$= \Pi\left(\frac{t - \frac{a+b}{2}}{b-a}\right) + \Lambda\left(\frac{t - \frac{a+b}{2}}{\frac{b-a}{2}}\right)$$

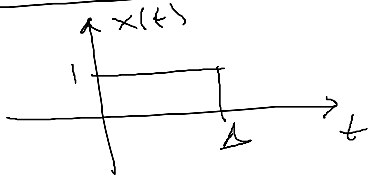
$\Lambda\left(\frac{t}{\Delta}\right)$

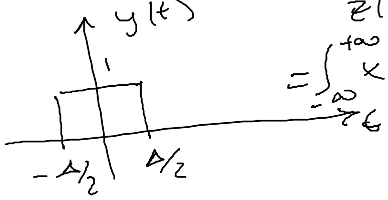


$\Updownarrow \mathcal{F}$

$$S(f) = e^{-j2\pi f \frac{a+b}{2}}\left[(b-a)\operatorname{sinc}(b-a) f + \frac{b-a}{2}\operatorname{sinc}^2 \frac{b-a}{2} f\right]$$



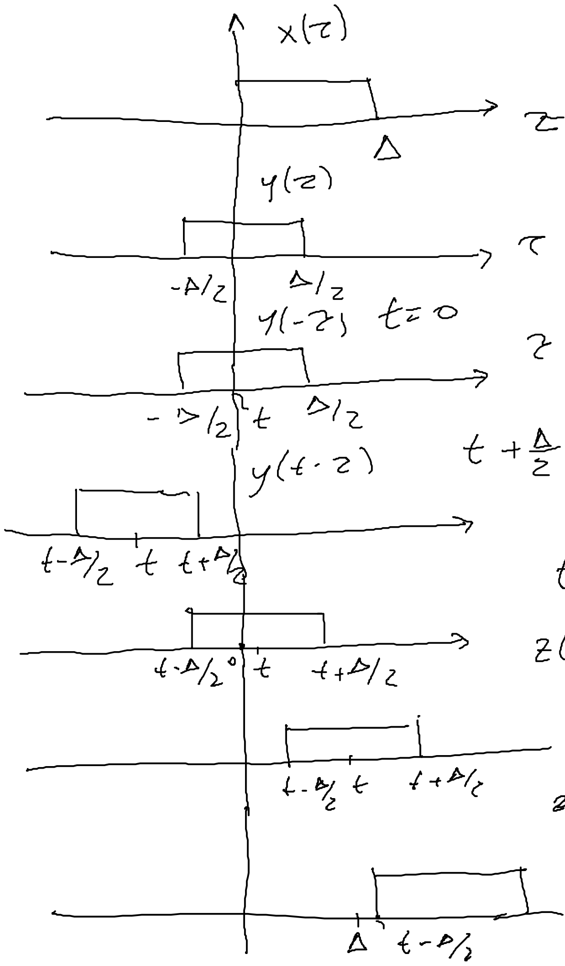




$$z(t) = (x * y)(t)$$

$$= \int_{-\infty}^{+\infty} x(\tau)\, y(t-\tau)\, d\tau$$

$$y(-(\tau - t))$$



$t + \frac{\Delta}{2} < 0$ $\boxed{t < -\frac{\Delta}{2} \qquad z(t) = 0}$

$t - \frac{\Delta}{2} < 0$ $\quad t < \frac{\Delta}{2}$

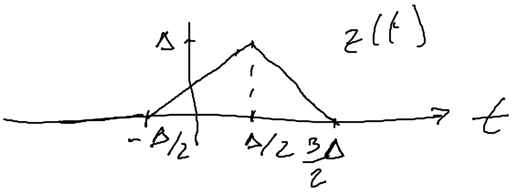
$$z(t) = \int_0^{t+\Delta/2} d\tau = t + \frac{\Delta}{2}$$

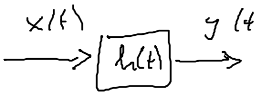
$t - \frac{\Delta}{2} < \Delta \Rightarrow t < \frac{3\Delta}{2}$

$$z(t) = \int_{t-\Delta/2}^{\Delta} d\tau = \Delta - \left(t - \frac{\Delta}{2}\right) = \Delta - t + \Delta/2$$

$$= \frac{3}{2}\Delta - t$$

$t - \frac{\Delta}{2} > \Delta \quad t > \frac{3}{2}\Delta \quad z(t) = 0$





$y(t) = (h * x)(t)$

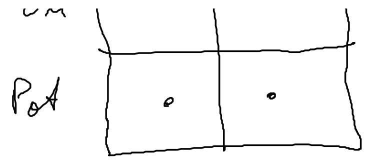
$H(f)$

Autocorrelazione

Densità spettrale

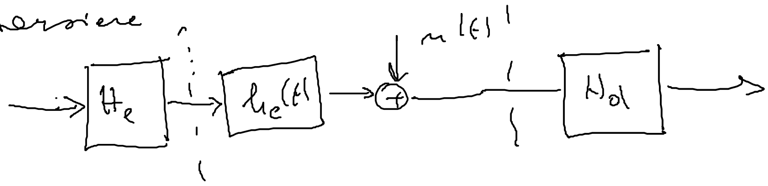
	Det	Aleatori
En	•	•
P.A	•	•

Densità spettrale



Filtri

Rumore
Distorsione

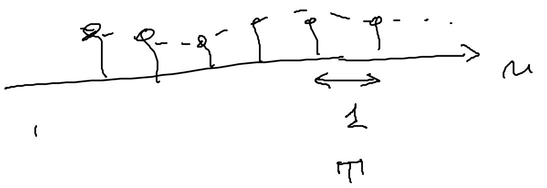


MOD. ANALOGICA

DSB, AM, SSB, VSB, QAM

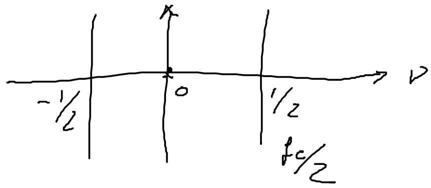
FM, PM

$x[n]$ $\to$ TF $\boxed{\text{DTFT}}$

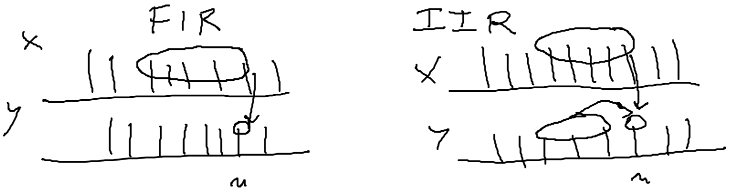


$$X(\nu) = \sum_{n=-\infty}^{+\infty} x[n]\, e^{-j2\pi\nu n}$$

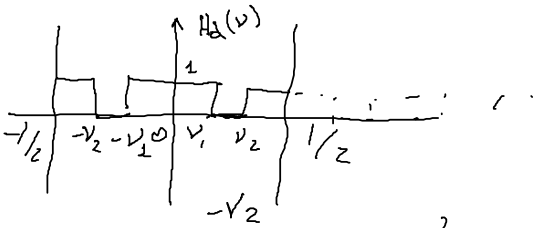
$$\nu = \frac{f}{f_c}$$



FIR IIR



$$H_d(\nu) = \sum_{n=-\infty}^{+\infty} h_d[n]\, e^{-j2\pi\nu n} \qquad h_d[n] = \int_{-1/2}^{1/2} H_d(\nu)\, e^{j2\pi\nu n}\, d\nu$$



T.C.

$f_c = 20\, KHz$.

$[5, 7]\, KHz$

$f_1 \quad f_2$

$$\left[\frac{5000}{20000}\ \frac{7000}{20000}\right]$$

$$h_d[n] = \int_{-1/2}^{-\nu_2} e^{j2\pi\nu n}\, d\nu + \int_{-\nu_1}^{\nu_1} e^{j2\pi\nu n}\, d\nu + \int_{\nu_2}^{1/2} e^{j2\pi\nu n}\, d\nu$$

$$-1/2 \qquad -\nu_1 \qquad \nu_2 \qquad d\nu$$

$$= \left.\frac{e^{j2\pi\nu n}}{j2\pi n}\right]_{-1/2}^{\nu_2} + \left.\frac{e^{j2\pi\nu n}}{j2\pi n}\right]_{-\nu_1}^{\nu_1} + \left.\frac{e^{j2\pi\nu n}}{j2\pi n}\right]_{\nu_2}^{1/2}$$

$$\left[\frac{5000}{20000}\ \frac{7000}{20000}\right] \qquad \nu_1 \quad \nu_2$$

$$\left[\frac{1}{4}, \frac{7}{20}\right]$$

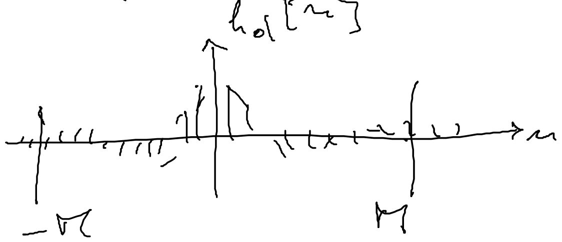
$$= \frac{e^{j2\pi\nu_2 n} - \cancel{e^{-j\pi\frac{1}{2}n}} + e^{j2\pi\nu_1 n} - e^{-j2\pi\nu_1 n} + \cancel{e^{j2\pi\frac{1}{2}n}} - e^{j2\pi\nu_2 n}}{j2\pi n}$$

$$(e^{-j\pi})^n \qquad (e^{j\pi})^n$$

$$= \qquad (-1)^n \qquad (-1)^n$$

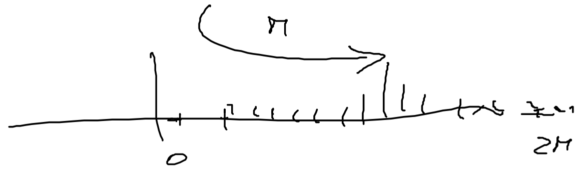
$$= \frac{\sin \nu_2 n + \sin \nu_1 n}{\pi n}$$

$h_0[n]$



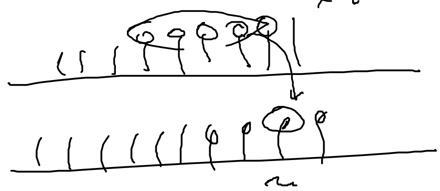
$$h[n] = \frac{\sin \nu_2 (n-M) + \sin \nu_1 (n-M)}{\pi (n-M)}$$

$$n = 0, \dots, 2M$$



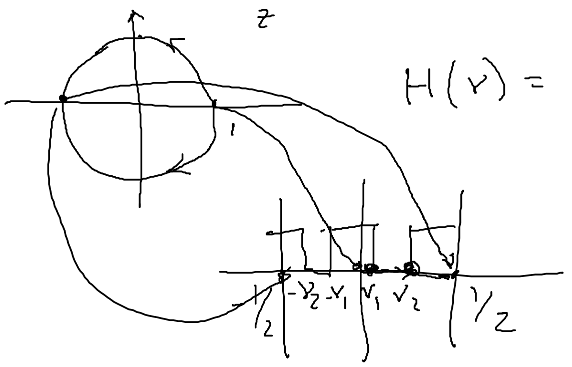
$$y[n] = \sum_{m=0}^{2M} h[m]\, x[n-m]$$

$x[n]$

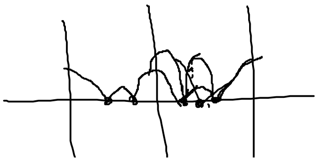


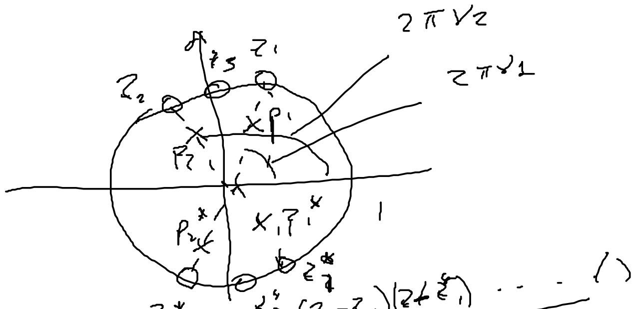
IIR

$$y[n] = \sum_{k=1}^{N} a_k\, y[n-k] + \sum_{k=0}^{M-1} b_k\, x[n-k]$$

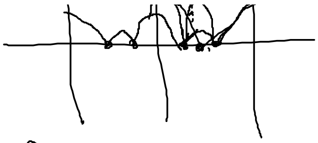


$$H(\nu) = H(z)\Big|_{z=e^{j2\pi\nu}}$$



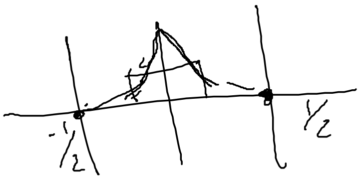


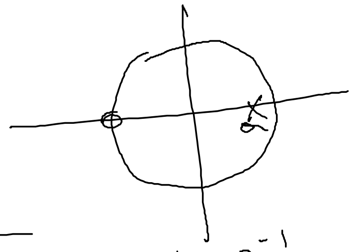
$$H(z) = \frac{(z-z_1)(z-z_1^*) \dots (\ )}{\dots (z-p_1^*)}$$



$$H(z) = \frac{z_2^* \; z_3^* \; (z - z_1)(z + z_1^*) \; \dots \; ()}{(z-p_1)(z-p_1^*)(z-p_2)(z-p_2^*)}$$

PASSA - BASSO





$$H(z) = \frac{z+1}{z-\alpha} = \frac{1+z^{-1}}{1-\alpha z^{-1}}$$

$$\frac{Y(z)}{X(z)} = H(z)$$

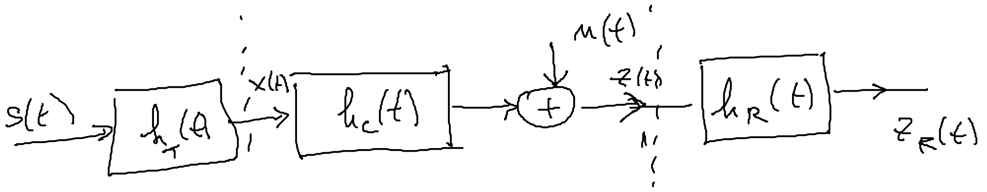
$$\frac{Y(z)}{X(z)} = \frac{1+z^{-1}}{1-\alpha z^{-1}}$$

$$Y(z)\left(1-\alpha z^{-1}\right) = \left(1+z^{-1}\right) X(z)$$

$$Y(z) - \alpha z^{-1} Y(z) = X(z) + z^{-1} X(z)$$

$$y[n] - \alpha y[n-1] = x[n] + x[n-1]$$

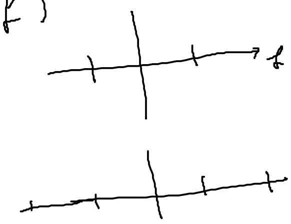
$$\boxed{y[n] = \alpha y[n-1] + x[n] + x[n-1]}$$



s(t) n(t) segnali di potenza

$$z_R(t) = \underbrace{(h_R * h_C * h_T * s)(t)}_{s_R(t)} + \underbrace{(h_R * n)(t)}_{n_R(t)}$$

$$Z_R(f) = \underbrace{H_R(f)\, H_C(f)\, H_T(f)}\, S(f) + H_R(f)\, N_R(f)$$



$$H_R(f)\, H_C(f)\, H_T(f) = K e^{-j2\pi f t_0} \qquad f \in I_s(f)$$

$$P_{Z_R}(f) = \underbrace{|H_R(f)|^2 |H_C(f)|^2 |H_T(f)|^2 P_S(f)}_{P_{S_R}(f)} + \underbrace{|H_R(f)|^2 P_n(f)}_{P_{n_R}(f)}$$

$$\left(\frac{S}{N}\right)_{out} = \frac{P_{S_R}}{P_{n_R}} = \frac{\int_{-\infty}^{+\infty} |H_R(f)|^2 |H_C(f)|^2 |H_T(f)|^2 P_S(f)\, df}{\int_{-\infty}^{+\infty} |H_R(f)|^2 P_n(f)\, df}$$

$$|H_R(f)|^2 |H_C(f)|^2 |H_T(f)|^2 = K^2$$

$$= \frac{K^2 P_S}{\int_{-\infty}^{+\infty} |H_R(f)|^2 P_n(f)\, df}$$

$$\min \rightarrow \frac{P_{n_R}}{P_{S_R}} \cdot P_x = \frac{\int_{-\infty}^{+\infty} |H_R(f)|^2 P_n(f)\, df \cdot \int_{-\infty}^{+\infty} |H_T(f)|^2 P_S(f)\, df}{K^2 P_S}$$

$$\int |V|^2 \quad \int |W|^2 < \infty$$

Dis di Schwartz $\left| \int V W^* \right|^2 \leq \int |V|^2 \int |W|^2$

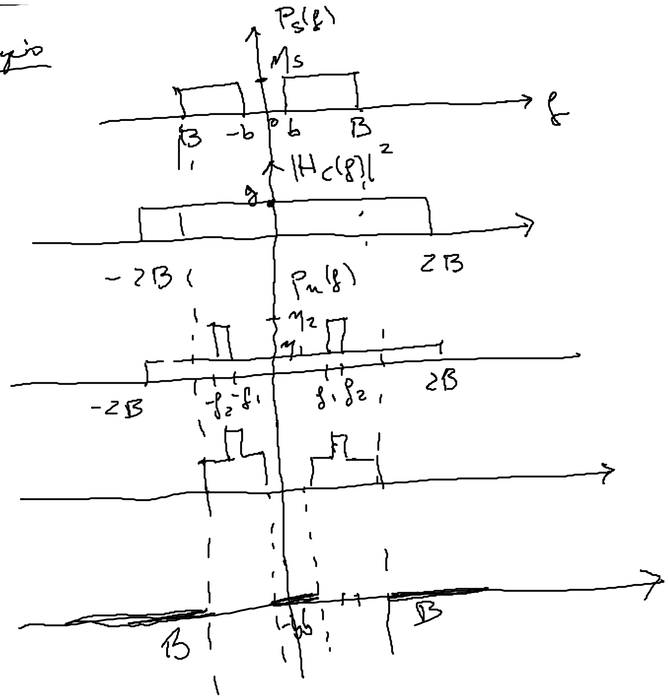
$= \text{se } V = \alpha W$

$$\begin{cases} |H_{R_0}(f)|\, P_n(f)^{1/2} = \alpha\, |H_{T_0}(f)|\, P_S^{1/2}(f) \\ |H_{T_0}(f)|\, |H_C(f)|\, |H_{R_0}(f)| = K \end{cases}$$

Esempio

$f_1 < f < f_2 < B$

Esempio



$$b < f_1 < f_2 < B$$

$$|H_{T0}(f)|^2 = \frac{\alpha K}{|H_c(f)|} \frac{\mathcal{P}_n^{1/2}(f)}{\mathcal{P}_s^{1/2}(f)},$$

$$|H_{R0}(f)|^2 = \frac{K}{\alpha |H_c(f)|} \frac{\mathcal{P}_s^{1/2}(f)}{\mathcal{P}_n^{1/2}(f)},$$

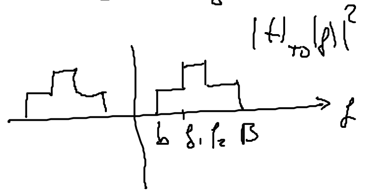
$$|f| \in [b, B]$$

$$|H_{T0}(f)|^2 = \frac{\alpha k}{\sqrt{g}} \frac{P_n^{1/2}(f)}{P_s^{1/2}(f)}$$

$$|H_{R0}(f)|^2 = \frac{K}{\alpha\sqrt{g}} \frac{P_s^{1/2}(f)}{P_n^{1/2}(f)}$$

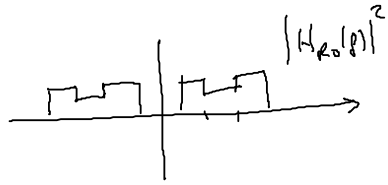
$$f \in [f_1, f_2] \quad |H_{T0}(f)|^2 = \frac{\alpha k}{\sqrt{g}} \frac{\sqrt{\eta_2}}{\sqrt{\eta_s}}$$

$$f \in [b, f_1] \cup [f_2, B] \quad |H_{T0}(f)|^2 = \frac{\alpha k}{\sqrt{g}} \frac{\sqrt{\eta_1}}{\sqrt{\eta_s}}$$



$$f \in [f_1, f_2] \Rightarrow |H_{R0}(f)|^2 = \frac{K}{\alpha\sqrt{g}} \frac{\sqrt{\eta_s}}{\sqrt{\eta_2}}$$

$$f \in [b, f_1] \cup [f_2, B] \Rightarrow |H_R(f)|^2 = \frac{K}{\alpha\sqrt{g}} \frac{\sqrt{\eta_s}}{\sqrt{\eta_1}}$$



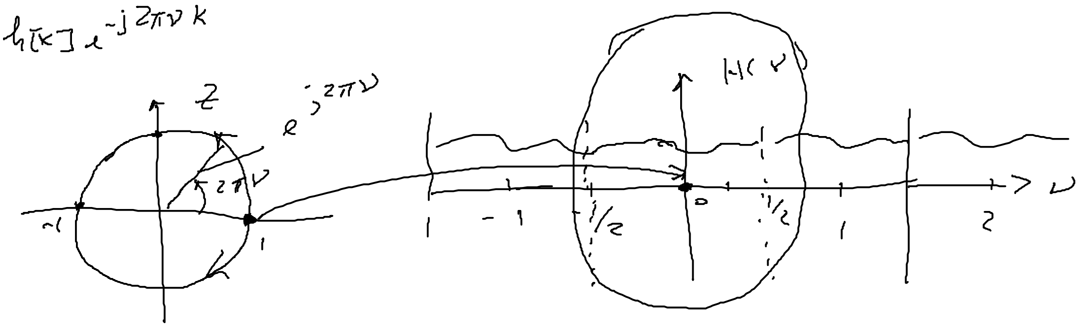
Prog. filtro numerico (Piazzamento Poli - zeri)

$$x[n] \longrightarrow \boxed{H(z)} \longrightarrow y[n] \qquad H(z) = \frac{\text{zeri}}{\text{poli}}$$

$$H(\nu) = H(z)\Big|_{z = e^{j2\pi\nu}}$$

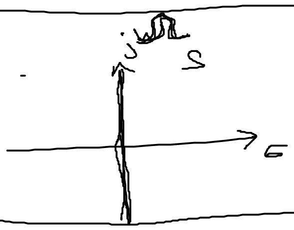
$$H(z) = \sum_{k=-\infty}^{+\infty} h[k]\, z^{-k}$$

$$H(\nu) = \sum_{k=-\infty}^{+\infty} h[k]\, e^{-j2\pi\nu k}$$

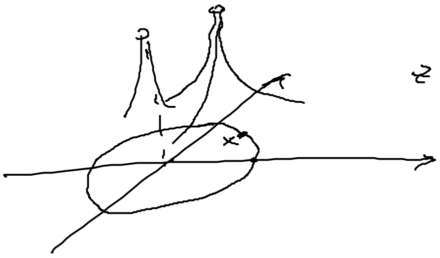


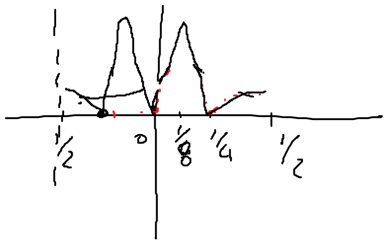
$h(t)$ T.C.

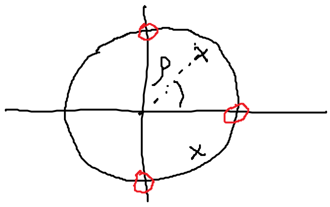
$$H(s)=\int h(t)e^{-st}dt$$



$$H(\Omega)=\int_{-\infty}^{+\infty} h(t)e^{-j\Omega t}\,dt$$

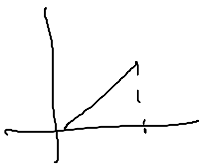






$$H(z)=\frac{(z-1)\left(z-e^{j\pi/2}\right)\left(z-e^{-j\pi/2}\right)}{\left(z-\rho e^{j\pi/4}\right)\left(z-\rho e^{-j\pi/2}\right)}$$

$$=\frac{(z-1)\left(z^2+1-z e^{j\pi/2}-z e^{-j\pi/2}\right)}{z^2+\rho^2-\rho z e^{j\pi/4}-\rho z e^{-j\pi/4}}$$



$$=\frac{(z-1)\left(z^2-2\cancel{\cos\pi/2}\, z+1\right)}{z^2-2\rho\cos\pi/4\, z+\rho^2}=\frac{(z-1)(z^2+1)}{z^2-2\rho\cos\pi/4\, z+\rho^2}$$

$$=\frac{z^3+z-z^2-1}{z^2-2\rho\cos\pi/4\, z+\rho^2}=\frac{z^3-z^2+z-1}{z^2-\rho\sqrt{2}\,z+\rho^2}$$

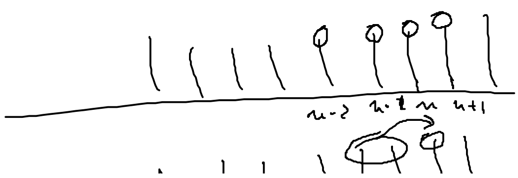
$$=\frac{z^3\left(1-z^{-1}+z^{-2}-z^{-3}\right)}{z^2\left(1-\rho\sqrt{2}\,z^{-1}+\rho^2 z^{-2}\right)}=\frac{z\left(1-z^{-1}+z^{-2}-z^{-3}\right)}{1-\rho\sqrt{2}\,z^{-1}+\rho^2 z^{-2}}$$

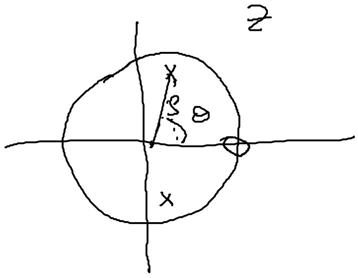
$$\frac{Y(z)}{X(z)}=\frac{z-1+z^{-1}-z^{-2}}{1-\rho\sqrt{2}\,z^{-1}+\rho^2 z^{-2}}$$

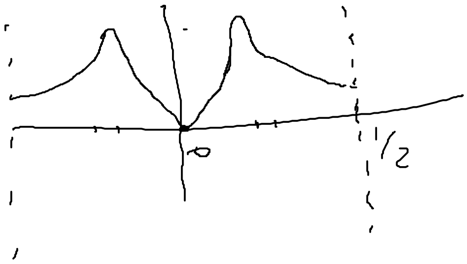
$$Y(z)\left(1-\rho\sqrt{2}\,z^{-1}+\rho^2 z^{-2}\right)=X(z)\left(z-1+z^{-1}-z^{-2}\right)$$

$$y[n]-\rho\sqrt{2}\,y[n-1]+\rho^2 y[n-2]=x[n+1]-x[n]+x[n-1]-x[n-2]$$

$$y[n]=+\rho\sqrt{2}\,y[n-1]-\rho^2 y[n-2]+x[n+1]-x[n]+x[n-1]-x[n-2]$$



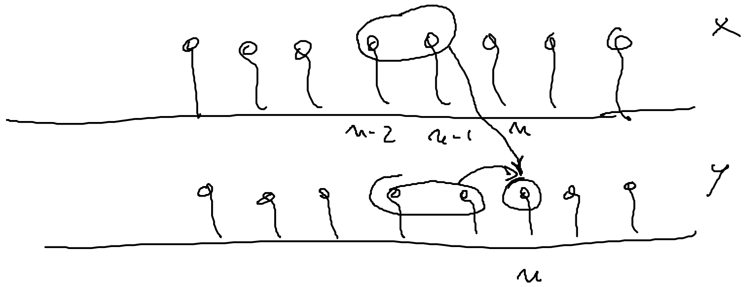


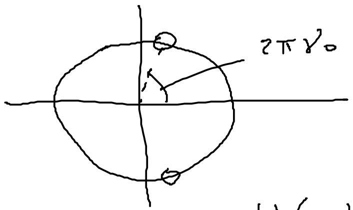


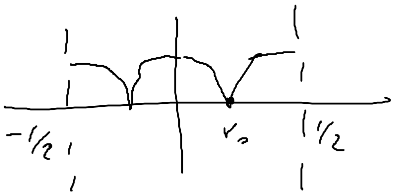
$$H(z) = \frac{z-1}{(z-\rho e^{j\theta})(z-\rho e^{-j\theta})} = \frac{z-1}{z^2 - 2\rho\cos\theta\, z + \rho^2}$$

$$= \frac{z-1}{z^2(1-2\rho\cos\theta\, z^{-1}+\rho^2 z^{-2})} = \frac{z^{-1}-z^{-2}}{1-2\rho\cos\theta\, z^{-1}+\rho^2 z^{-2}} = \frac{Y(z)}{X(z)}$$

$$y[n] = 2\rho\cos\theta\, y[n-1] - \rho^2 y[n-2] + x[n-1] - x[n-2]$$



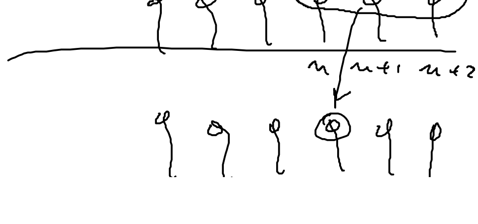


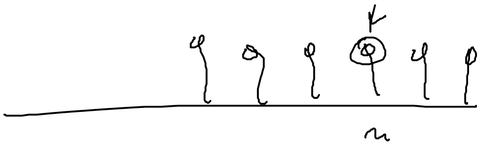


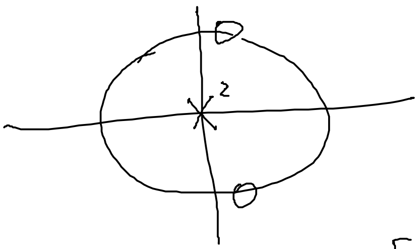
$$H(z) = (z-e^{j2\pi\nu_0})(z-e^{-j2\pi\nu_0})$$

$$= z^2 - 2\cos 2\pi\nu_0\, z + 1 = \frac{Y(z)}{X(z)}$$

$$y[n] = x[n+2] - 2\cos 2\pi\nu_0\, x[n+1] + x[n]$$

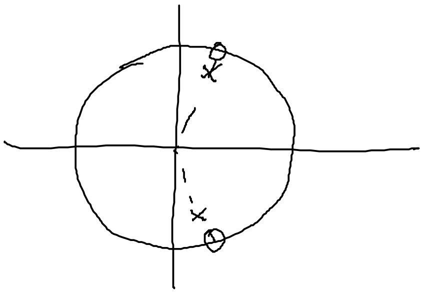


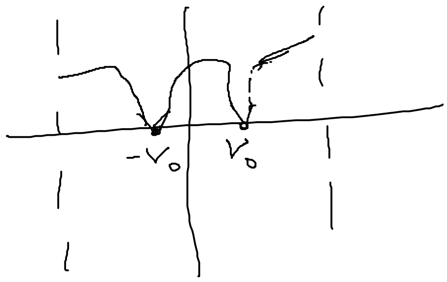




$$H(z) = \frac{z^2 - 2\cos 2\pi\nu_0 z + 1}{z^2} = 1 - 2\cos 2\pi\nu_0 z^{-1} + z^{-2}$$

$$y[n] = x[n] - 2\cos 2\pi\nu_0\, x[n-1] + x[n-2]$$





$$H(z) = \frac{z^2 - 2\cos 2\pi\nu_0 z + 1}{z^2 - 2\rho\cos 2\pi\nu_0 z + \rho^2} = \frac{1 - 2\cos 2\pi\nu_0 z^{-1} + z^{-2}}{1 - 2\rho\cos 2\pi\nu_0 z^{-1} + \rho^2 z^{-2}}$$