(1)

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Prova Intracorso AA 2021-22 (unica)
Teoria dei Segnali
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giovedi 2 Dicembre 2021

SOLUZIONI

1. Schizzare i seguenti segnali e valutarne l'energia, la potenza e la trasformata di Fourier:

(a)[3 pt] $s(t) = \Pi\left(\frac{t-3}{6}\right) + 2\Lambda(t-3)$;

(b)[3 pt] $s(t) = (e^{-2t} + e^{-4t})u(t)$;

(c)[3 pt] $s(t) = 1 - \cos\frac{\pi}{2}t + \cos^2 2\pi t$.

(d)[4 pt] $s(t) = \sum_{k=-\infty}^{\infty} \Lambda(t-1-3k)$

2.[10 pt] Usando il metodo grafico valutare la risposta nel dominio del tempo di un sistema lineare (anticausale) avente risposta impulsiva $h(t) = e^{\alpha t}u(-t)$ al cui ingresso é posto il segnale $s(t) = u(t-1) - u(t-3)$.

3.[10 pt] Si consideri il segnale aleatorio

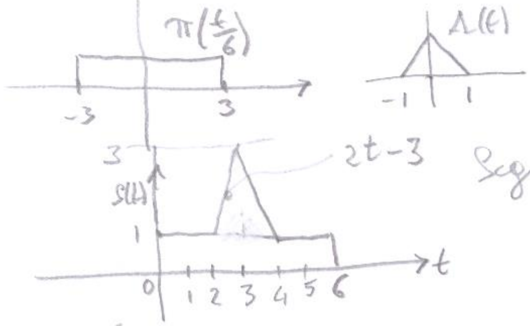
$$s(t) = A + x(t) + x(t)\cos(\pi Bt + \Theta) \quad (1)$$

dove A é una costante deterministica, $x(t)$ é un processo aleatorio SSL a media nulla con spettro di potenza $P_x(f) = \Pi\left(\frac{f}{B}\right)$ e Θ è una variable aleatoria uniformemente distribuita in $[-\pi, \pi]$. Calcolare la autocorrelazione di $s(t)$ e studiarne la stazionarietá. In caso di stazionarietá calcolarne anche lo spettro di potenza.

4.[10 pt] Due segnali passa-basso con frequenza massima B, modulano in fase e in quadratura (QAM) una portante a frequenza f_0. Schizzare lo spettro approssimativo del segnale modulato. Usando l'espressione analitica del segnale modulato, valutare le uscite del demodulatore coerente in presenza di un errore di fase nell'oscillatore locale pari a θ.

1.

a) $s(t) = \Pi\left(\frac{t-3}{6}\right) + 2\Lambda(t-3)$



Segnale di energia, $P_s = 0$

$$\mathcal{E}_s = \int_0^6 s^2(t)\,dt = 2\int_0^2 1\,dt + 2\int_2^3 (2t-3)^2\,dt = 4 + 2\left(\int_2^3 (4t^2 - 12t + 9)\,dt\right.$$

$$= 4 + 2\left[\left.4\frac{t^3}{3}\right]_2^3 - \left.12\frac{t^2}{2}\right]_2^3 + \left.9t\right]_2^3\right]$$

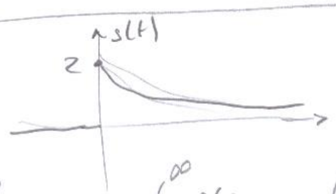
$$= 4 + 2\left[4\,\frac{27-8}{3} - 12\,\frac{9-4}{2} + 9(3-2)\right]$$

$$= 4 + 2\left[\frac{4\cdot 19}{3} - \frac{12\cdot 5}{2} + 9\right] = 4 + \cancel{2}\,\frac{8\cdot 19 - 3\cdot 12\cdot 5 + 54}{\cancel{6}\,3}$$

$$= \frac{12 + 8\cdot 19 - 15\cdot 12 + 54}{3} = \frac{12 + 152 - 180 + 54}{3} = \frac{38}{3}$$

$$S(f) = e^{-j2\pi f3}\, 6\,\mathrm{sinc}\,6f + 2\,e^{-j2\pi f3}\,\mathrm{sinc}^2 f$$

(b) $s(t) = \left(e^{-2t} + e^{-4t}\right) u(t)$



Segnale di energia, $P_s = 0$

$$\mathcal{E}_s = \int_0^\infty s^2(t)\,dt = \int_0^\infty \left(e^{-2t} + e^{-4t}\right)^2 dt = \int_0^\infty e^{-4t}\,dt + \int_0^\infty e^{-8t}\,dt + 2\int_0^\infty e^{-6t}\,dt$$

$$= \left.\frac{e^{-4t}}{-4}\right]_0^\infty + \left.\frac{e^{-8t}}{-8}\right]_0^\infty + \left.\cancel{2}\,\frac{e^{-6t}}{-\cancel{2}3}\right]_0^\infty = \frac{1}{4} + \frac{1}{8} + \frac{1}{3} = \frac{6+3+8}{24} = \frac{17}{24}$$

$$x(t)=e^{-\alpha t}u(t) \overset{\mathcal{F}}{\longleftrightarrow} X(f)=\int_0^{\infty} e^{-\alpha t}e^{-j2\pi f t}\,dt=\int_0^{\infty} e^{-(\alpha+j2\pi f)t}\,dt$$ ③

$$=\left.\frac{e^{-(\alpha+j2\pi f)t}}{-(\alpha+j2\pi f)}\right]_0^{\infty}=\frac{1}{\alpha+j2\pi f}$$

$$S(f)=\frac{1}{2+j2\pi f}+\frac{1}{4+j2\pi f}$$

(c) $s(t)=1-\cos\frac{\pi}{2}t+\cos^2 2\pi t=1-\cos\frac{\pi}{2}t+\frac{1}{2}+\frac{1}{2}\cos 4\pi t$

$s(t)=\frac{3}{2}-\cos\frac{\pi}{2}t+\frac{1}{2}\cos 4\pi t$ Segnale di potenza $\mathcal{E}_s=\infty$

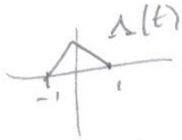
$P_s=\frac{9}{4}+\frac{1}{2}+\frac{1}{8}=\frac{18+4+1}{8}=\frac{23}{8}$

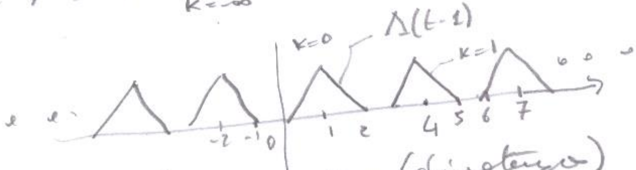
$S(f)=\frac{3}{2}\delta(f)+\frac{1}{2}\delta(f-\frac{1}{4})+\frac{1}{2}\delta(f+\frac{1}{4})+\frac{1}{4}\delta(f-2)+\frac{1}{4}\delta(f+2)$

Nota:
- $\cos\frac{\pi}{2}t$ ha freq. $\frac{1}{4}$

  $2\pi f_0=\frac{\pi}{2}\Rightarrow f_0=\frac{1}{4}$
- $\cos 4\pi t$ ha freq. 2

(d) $s(t)=\sum_{k=-\infty}^{+\infty}\Lambda(t-1-3k)$





$P_s=\frac{\mathcal{E}(\Lambda(t))}{T}=\frac{2}{3}\cdot\frac{1}{3}=\frac{2}{9}$

$\mathcal{E}_s=\infty$

Segnale periodico (di potenza) Periodo T=3

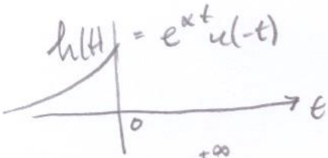
$s(t)=\Lambda(t-1)*\sum_{k=-\infty}^{+\infty}\delta(t-3k)$

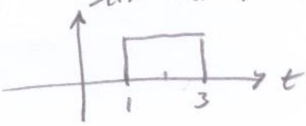
$S(f)=\mathcal{F}[\Lambda(t-1)]\cdot\mathcal{F}\left[\sum_{k=-\infty}^{+\infty}\delta(t-3k)\right]$

$=e^{-j2\pi f}\mathrm{sinc}^2 f\cdot\frac{1}{3}\sum_{n=-\infty}^{+\infty}\delta(f-\frac{n}{3})$

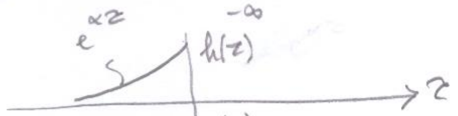
$=\frac{1}{3}\sum_{n=-\infty}^{+\infty}e^{-j2\pi\frac{n}{3}}\mathrm{sinc}^2\frac{n}{3}\,\delta(f-\frac{n}{3})$ Spettro a righe

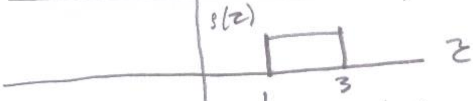
2. $h(t) = e^{\alpha t} u(-t)$ $\quad$ $s(t) = u(t-1) - u(t-3)$ $\quad$ (4)

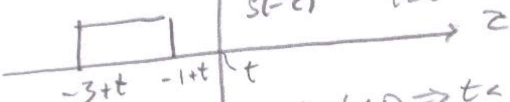




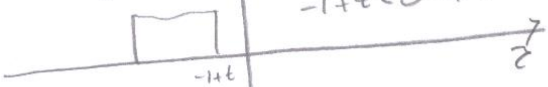
$$y(t) = (h * s)(t) = \int_{-\infty}^{+\infty} h(\tau)\, x(t-\tau)\, d\tau$$







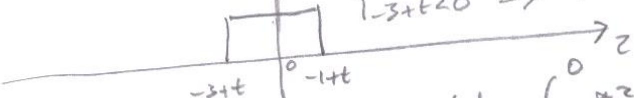
$-1+t<0 \Rightarrow t<1$



$$y(t) = \int_{-3+t}^{-1+t} e^{\alpha\tau} d\tau = \frac{e^{\alpha\tau}}{\alpha}\Bigg]_{-3+t}^{-1+t}$$

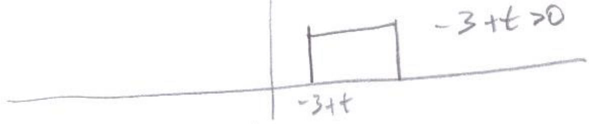
$$= \frac{e^{\alpha(t-1)} - e^{\alpha(t-3)}}{\alpha}$$

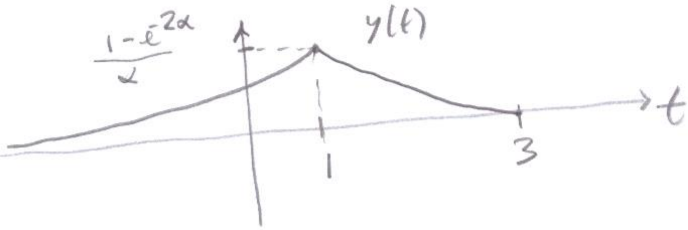
$\begin{cases} -1+t>0 \\ -3+t<0 \end{cases} \Rightarrow 1<t<3$



$$y(t) = \int_{-3+t}^{0} e^{\alpha\tau} d\tau = \frac{e^{\alpha\tau}}{\alpha}\Bigg]_{-3+t}^{0} = \frac{1-e^{\alpha(t-3)}}{\alpha}$$

$-3+t>0 \Rightarrow y(t)=0$





3. $s(t) = A + x(t) + x(t)\cos(\pi B t + \theta)$

cost.

$x(t)$ ssl

$P_x(f) = \Pi\left(\frac{f}{B}\right)$

$\theta \propto U(-\pi,\pi)$

(5)

$R_s(t,\tau) = E[s(t)s(t-\tau)] =$

$E\left[\left(A + x(t) + x(t)\cos(\pi B t+\theta)\right)\left(A + x(t-\tau) + x(t-\tau)\cos(\pi B(t-\tau)+\theta)\right)\right]$

$= E[A^2] + E[A x(t-\tau)] + E[A x(t-\tau)\cos(\pi B(t-\tau)+\theta)]$

$+ E[x(t)A] + E[x(t)x(t-\tau)] + E[x(t)x(t-\tau)\cos(\pi B(t-\tau)+\theta)]$

$+ E[A x(t)\cos(\pi B t+\theta)] + E[x(t)x(t-\tau)\cos(\pi B t+\theta)] + E[x(t)x(t-\tau)\cos(\pi B t+\theta)\cos(\pi B(t-\tau)+\theta)]$

$= A^2 + A\cancel{E[x(t-\tau)]}^{0} + A\cancel{E[x(t-\tau)]}^{0} E[\cos(\pi B(t-\tau)+\theta)]$

$+ A\cancel{E[x(t)]}^{0} + R_x(\tau) + \underbrace{E[x(t)x(t-\tau)]}_{R_x(\tau)} E[\cos(\pi B(t-\tau)+\theta)]$

$+ A\cancel{E[x(t)]}^{0} E[\cos(\pi B t+\theta)] + \underbrace{E[x(t)x(t-\tau)]}_{R_x(\tau)} E[\cos(\pi B t+\theta)] +$

$+ \underbrace{E[x(t)x(t-\tau)]}_{R_x(\tau)} E\left[\frac{1}{2}\cos(\pi B(2t-\tau)+2\theta) + \frac{1}{2}\cos\pi B\tau\right]$

Mediando en θ

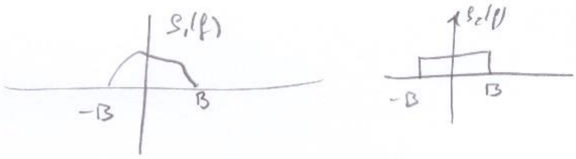
$= A^2 + R_x(\tau) + R_x(\tau)\cancel{E[\cos(\pi B(t-\tau)+\theta)]}^{0} + R_x(\tau)\cancel{E[\cos(\pi B t+\theta)]}^{0}$

$+ \frac{R_x(\tau)}{2}\cancel{E[\cos(\pi B(2t-\tau)+2\theta)]}^{0} + \frac{R_x(\tau)}{2}\cos\pi B\tau$

$= A^2 + R_x(\tau) + \frac{R_x(\tau)}{2}\cos\pi B\tau$ frec. $\frac{B}{2}$ proceso estacionario SL

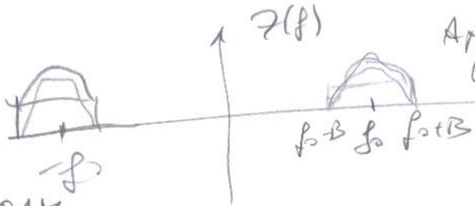
$P_s(f) = A^2\delta(f) + P_x(f) + \frac{P_x(f-\frac{B}{2})}{4} + \frac{P_x(f+\frac{B}{2})}{4}$

4. $s_1(t)$ $s_2(t)$



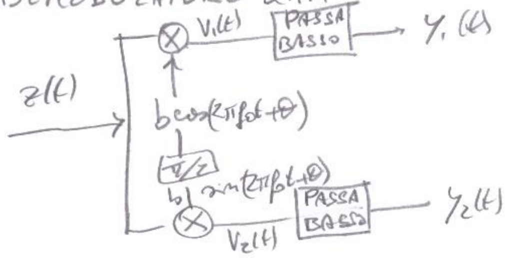
$$z(t) = s_1(t)\cos 2\pi f_0 t + s_2(t)\sin 2\pi f_0 t$$

$$Z(f) = \frac{1}{2} S_1(f-f_0) + \frac{1}{2} S_1(f-f_0) + \frac{1}{2j} S_2(f-f_0) - \frac{1}{2j} S_2(f+f_0)$$



Approssimativo (bisognerebbe disegnare anche la fase)

DEMODULATORE QAM



$$V_1(t) = z(t)\, b\cos(2\pi f_0 t+\theta) = \left(s_1(t)\cos 2\pi f_0 t + s_2(t)\sin 2\pi f_0 t\right) b\cos(2\pi f_0 t+\theta)$$

eliminato dal passa basso — eliminati dal passa basso

$$= \frac{b\, s_1(t)}{2}\cancel{\cos(2\pi f_0 2t+\theta)} + \frac{b\, s_1(t)}{2}\cos\theta + \frac{b\, s_2(t)}{2}\cancel{\sin(2\pi f_0 2t+\theta)}$$

$$+ \frac{b\, s_2(t)}{2}\sin(-\theta)$$

$$\boxed{y_1(t) = \frac{b}{2} s_1(t)\cos\theta - \frac{b}{2} s_2(t)\sin\theta}$$

$$V_2(t) = z(t)\, b\sin(2\pi f_0 t+\theta) = \left(s_1(t)\cos 2\pi f_0 t + s_2(t)\sin 2\pi f_0 t\right) b\sin(2\pi f_0 t+\theta)$$

$$= \frac{b}{2} s_1(t)\sin\theta + \frac{b}{2} s_1(t)\cancel{\sin(2\pi f_0 2t+\theta)} + \frac{b}{2} s_2(t)\cos\theta +$$

$$+ \frac{b}{2} S_2(t)\cancel{\cos(2\pi f_0 2t+\theta)}$$

$$\boxed{y_2(t) = \frac{b}{2} s_1(t)\sin\theta + \frac{b}{2} s_2(t)\cos\theta}$$

I due segnali sono mescolati.
$\theta = 0$ perfettamente separati.
$\theta = \frac{\pi}{2}$ invertiti nelle uscite.