SECONDA UNIVERSITA' DEGLI STUDI DI NAPOLI
FACOLTA' DI INGEGNERIA

**TEORIA DEI SEGNALI/TELECOMUNICAZIONI 2**
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**Esame scritto**
lunedi 6 febbraio 2012

SOLUZIONI

**1.** Si consideri il seguente segnale

$$x(t) = -1 + \sum_{k=-\infty}^{\infty} \left[ \Pi\left(\frac{2t-6k}{4}\right) + 2\Lambda\left(t-3k\right)\right]. \quad (1)$$

a. Disegnare il segnale
b. Calcolare la trasformata di Fourier
c. Schizzare la sequenza campionata $x[n]$ con frequenza di campionamento di campionamento pari a 2 Hz (sulle discontinuitá si consideri il segnale continuo a destra);
d. Valutare con il metodo grafico il risultato del filtraggio di $x[n]$ con un FIR avente risposta impulsiva $h[n] = \frac{1}{2}(\delta[n] - \delta[n-1])$;
e. Valutare la risposta armonica del filtro FIR.

**2.** Si consideri il processo aleatorio

$$X(t) = a(t)(1 + b(t)), \quad (2)$$

dove $a(t)$ e $b(t)$ sono due processi aleatori indipendenti a media nulla e stazionari in senso lato aventi spettro di potenza rispettivamente

$$P_a(f) = \Pi\left(\frac{f}{2B}\right); \qquad P_b(f) = \Pi\left(\frac{f-f_0}{2B}\right) + \Pi\left(\frac{f+f_0}{2B}\right), \quad (3)$$

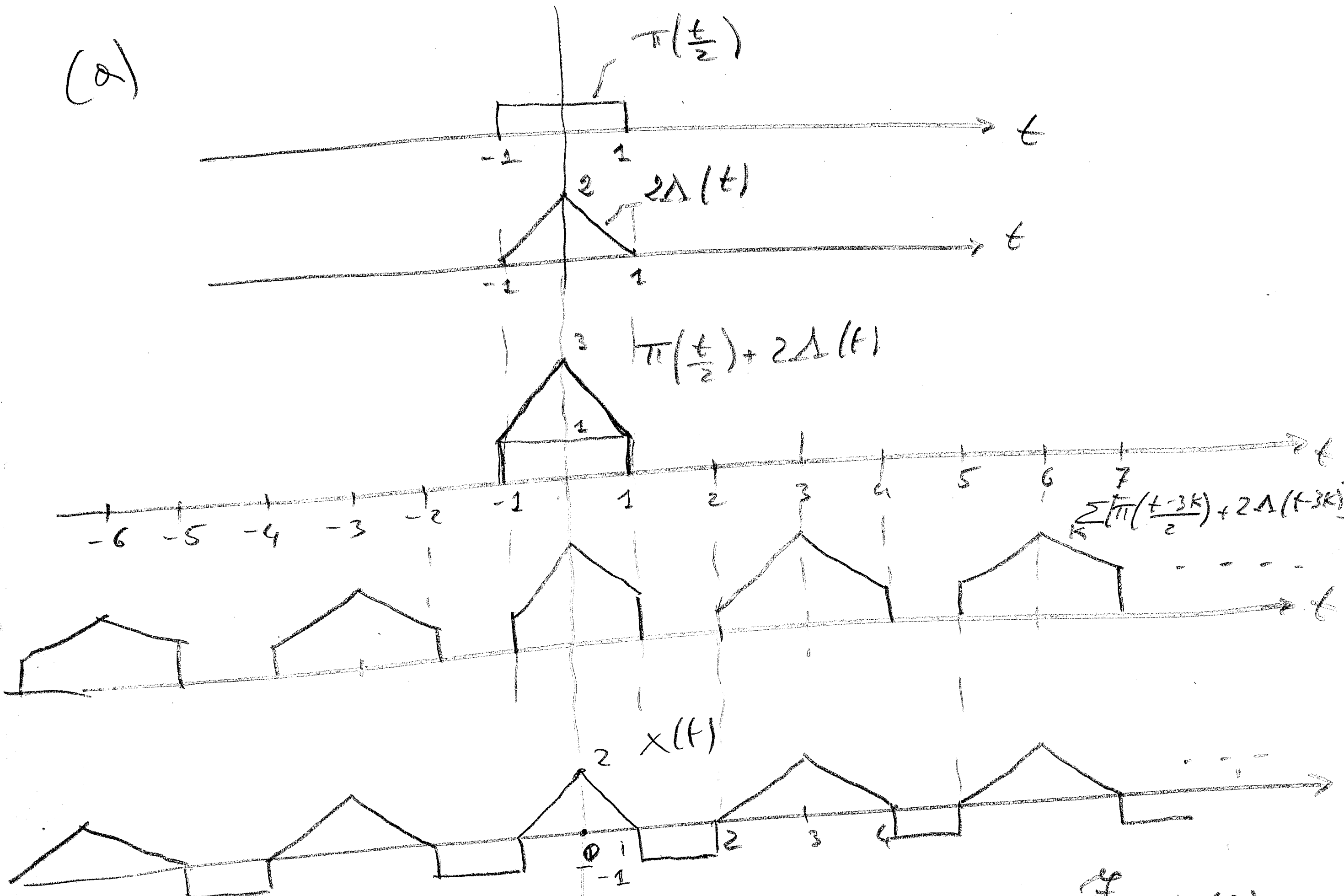
con $f_0 = 12$ MHz e $B = 10$ KHz. Valutare autocorrelazione e spettro di potenza di $X(t)$. Schizzare lo spettro di potenza di $X(t)$.

(1)

$$x(t) = -1 + \sum_{k=-\infty}^{+\infty}\left[\Pi\left(\frac{2t-6k}{4}\right) + 2\Lambda(t-3k)\right]$$

$$= -1 + \sum_{k=-\infty}^{+\infty}\left[\Pi\left(\frac{t-3k}{2}\right) + 2\Lambda(t-3k)\right]$$

(a)




$$i(t) = \Pi\left(\frac{t}{2}\right) + 2\Lambda(t) \xleftrightarrow{\mathcal{F}} I(f)$$

(b)

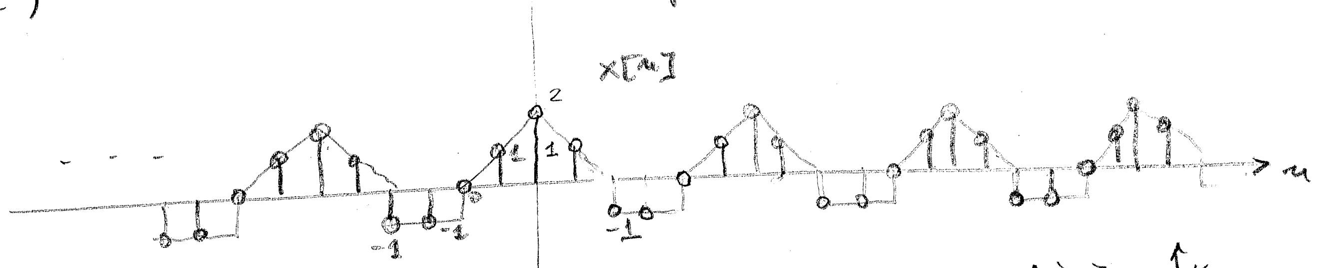
$$X(f) = \mathcal{F}[x(t)] = -\delta(f) + \frac{1}{3}I(f)\sum_{n=-\infty}^{+\infty}\delta\left(f-\frac{n}{3}\right)$$

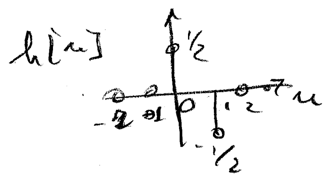
$$= -\delta(f) + \sum_{n=-\infty}^{+\infty}\frac{1}{3}I\left(\frac{n}{3}\right)\delta\left(f-\frac{n}{3}\right)$$

$$I(f) = 2\,\mathrm{sinc}\,2f + 2\,\mathrm{sinc}^2 f$$

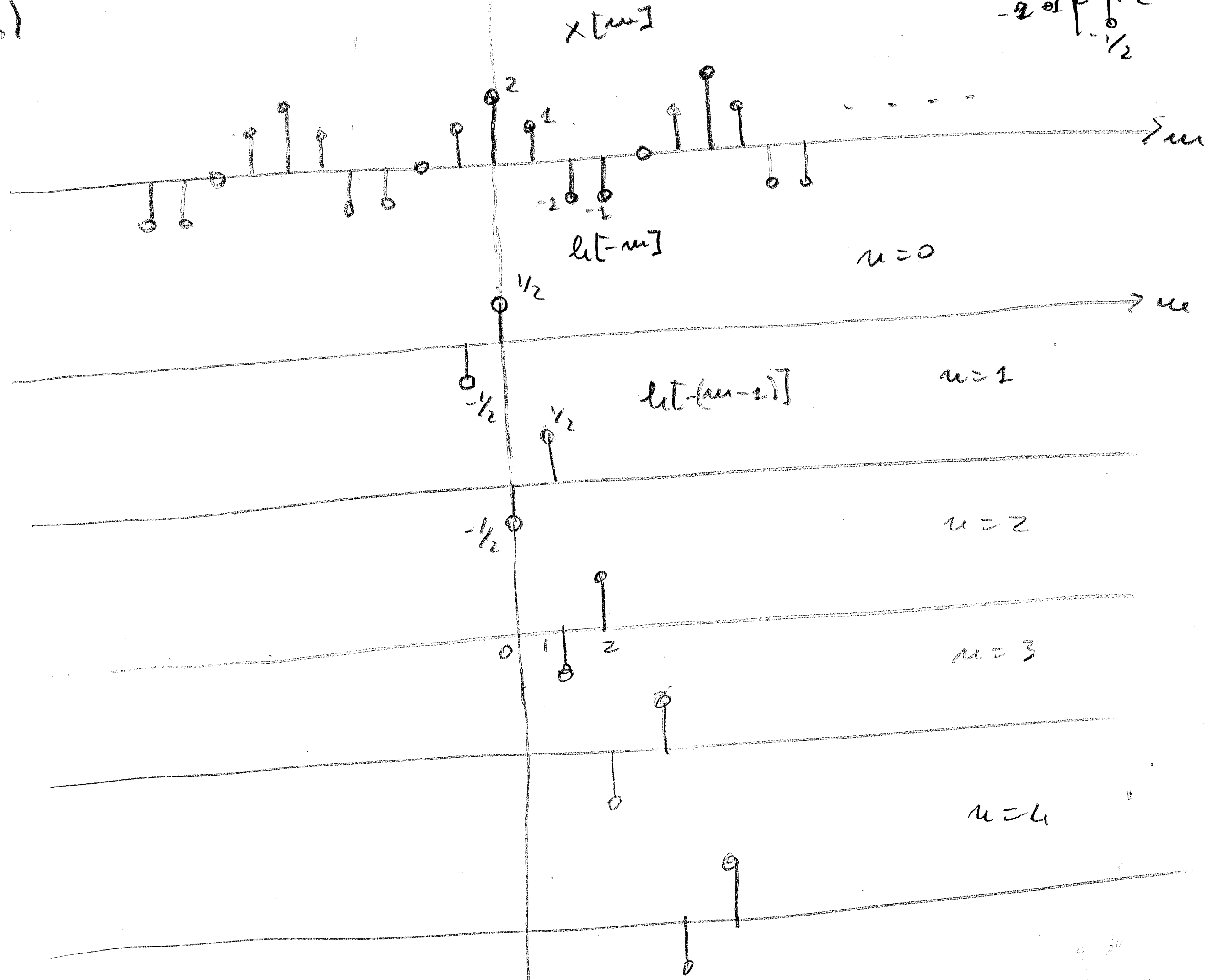
$$X(f) = -\delta(f) + \sum_{n=-\infty}^{+\infty}\left(\frac{2}{3}\,\mathrm{sinc}\,\frac{2n}{3} + \frac{2}{3}\,\mathrm{sinc}^2\frac{n}{3}\right)\delta\left(f-\frac{n}{3}\right)$$

(c) $f_c = 2\,Hz$. $T = \frac{1}{2}$

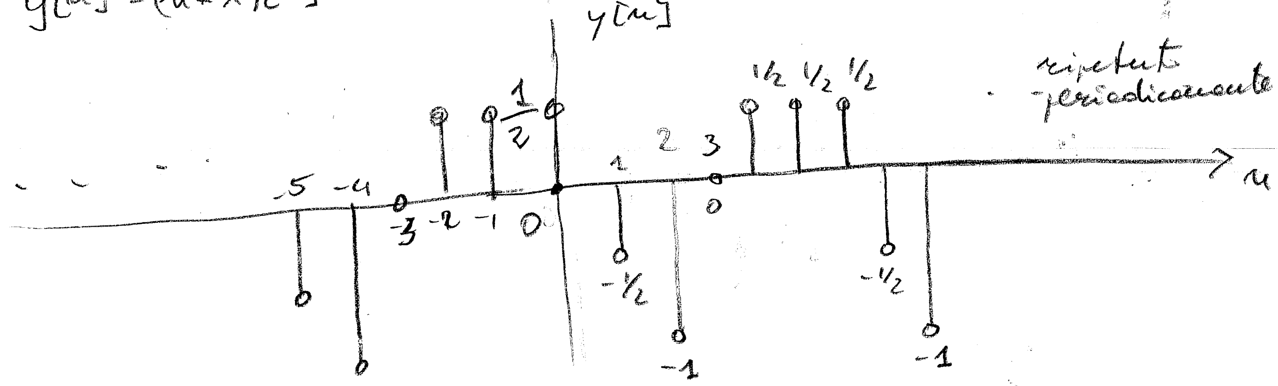



(d)




$y[n] = (h * x)[n]$



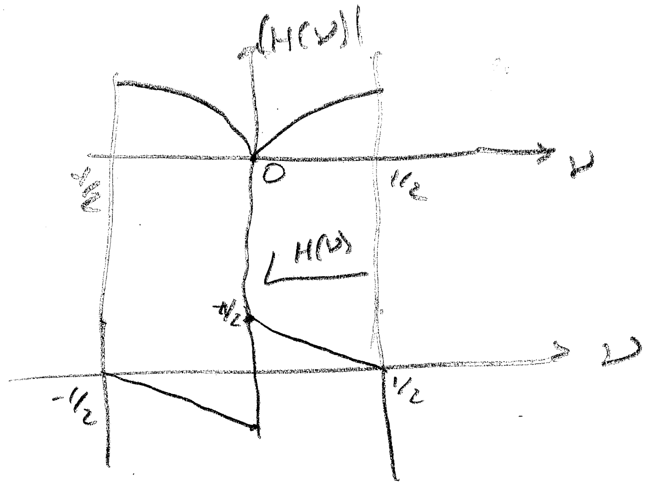

ripetuto periodicamente.

(e) $h[n] = \frac{1}{2}(\delta[n] - \delta[n-1])$

$$H(\nu) = \sum_{n=-\infty}^{+\infty} h[n] e^{-j2\pi\nu n} = \frac{1}{2} - \frac{1}{2} e^{-j2\pi\nu} = e^{-j\pi\nu} j \left( \frac{e^{j\pi\nu} - e^{-j\pi\nu}}{2j} \right)$$

$$= e^{j\frac{\pi}{2}} e^{-j\pi\nu} \sin \pi\nu$$

$$\begin{cases} |H(\nu)| = |\sin \pi\nu| \\ \angle H(\nu) = \frac{\pi}{2} - \pi\nu \pm \pi u[\sin \pi\nu] \end{cases}$$



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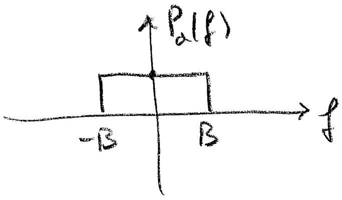
$$X(t) = a(t)(1+b(t))$$

$$R_x(t;\tau) = E[X(t)X(t-\tau)] = E[a(t)(1+b(t))\,a(t-\tau)(1+b(t-\tau))]$$

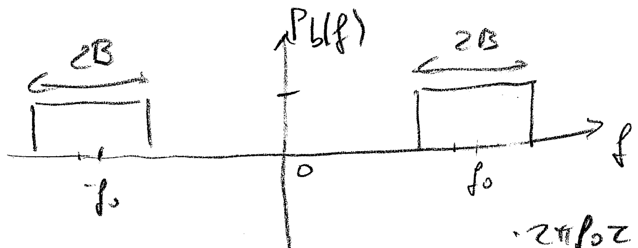
$$= E[a(t)\,a(t-\tau)]\,\left(1 + E[b(t)b(t-\tau)] + \underbrace{E[b(t)]}_{0} + \underbrace{E[b(t-\tau)]}_{0}\right)$$

$$= R_a(\tau)\,(1 + R_b(\tau))$$




$$P_a(f) = \Pi\left(\frac{f}{2B}\right)$$

$$R_a(\tau) = 2B \operatorname{sinc} 2B\tau$$




$$P_b(f) = \Pi\left(\frac{f-f_0}{2B}\right) + \Pi\left(\frac{f+f_0}{2B}\right)$$

$$R_b(\tau) = \mathcal{F}^{-1}[P_b(f)] = e^{j2\pi f_0\tau}\,2B \operatorname{sinc} 2B\tau + e^{-j2\pi f_0\tau}\,2B \operatorname{sinc} 2B\tau$$

$$= 2B \operatorname{sinc} 2B\tau \left(\frac{e^{j2\pi f_0\tau} + e^{-j2\pi f_0\tau}}{2}\right) 2$$

$$= 4B \cos 2\pi f_0\tau \operatorname{sinc} 2B\tau$$

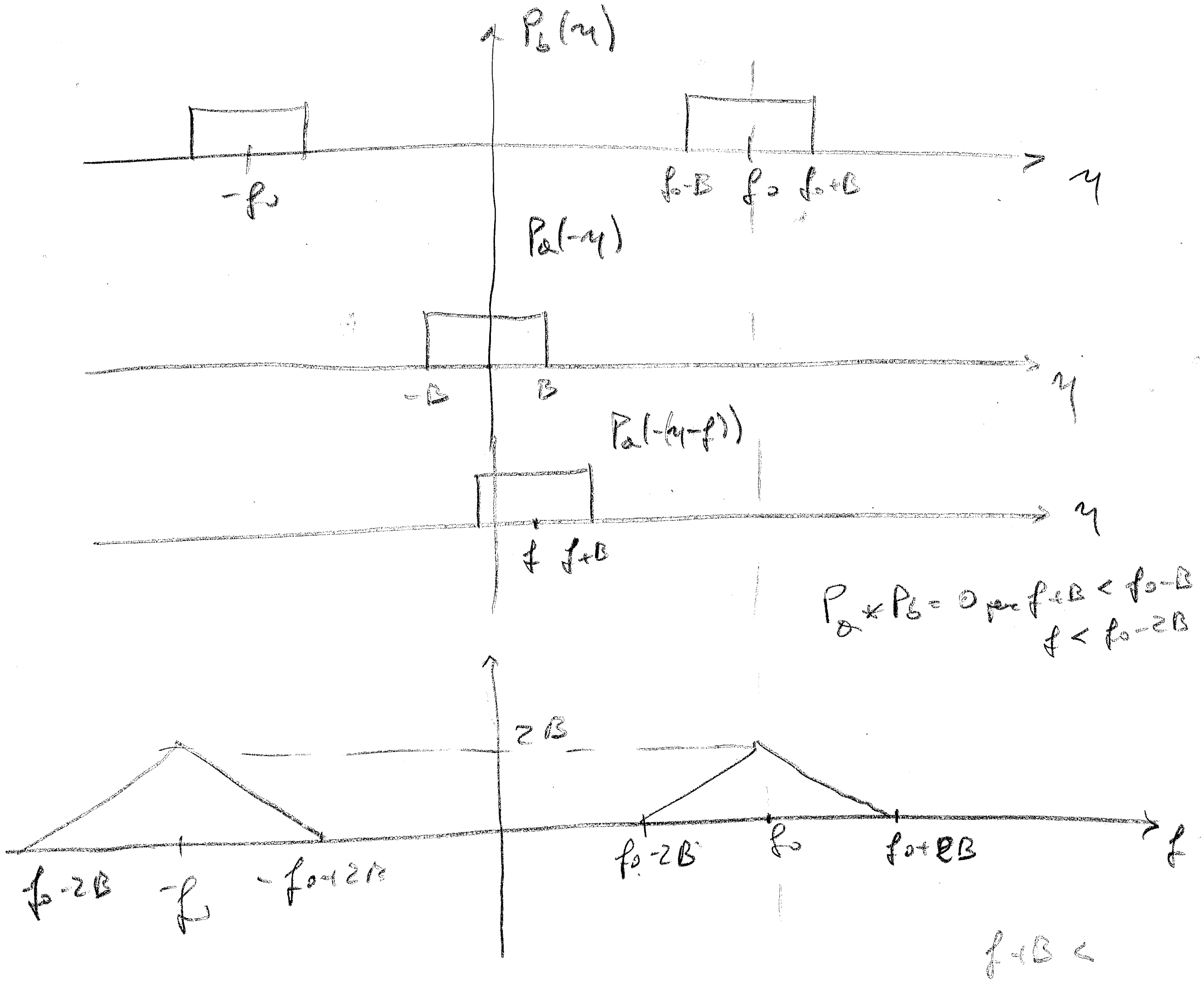
$$R_x(\tau) = R_a(\tau) + R_a(\tau)R_b(\tau)$$

$$P_x(f) = P_a(f) + P_a(f) * P_b(f)$$

$$P_a(f) * P_b(f) = \int_{-\infty}^{+\infty} P_b(\eta)\,P_a(f-\eta)\,d\eta$$

Usiamo il metodo grafico per valutare la convoluzione

↓




$$(P_a * P_b)(f) = 2B\left[\Lambda\left(\frac{f-f_0}{2B}\right) + \Lambda\left(\frac{f+f_0}{2B}\right)\right]$$

Oppure direttamente dalle autocorrelazioni:

$$R_a(\tau)R_b(\tau) = 2B\,\mathrm{sinc}\,2B\tau \;\; 4B\cos 2\pi f_0\tau\,\mathrm{sinc}\,2B\tau$$

$$= 8B^2\cos 2\pi f_0\tau\;\mathrm{sinc}^2 2B\tau \;\; = 2B\,\mathrm{sinc}^2 2B\tau \;\; 4B\cos 2\pi f_0\tau$$

$$\mathcal{F}[R_a(\tau)R_b(\tau)] = \Lambda\left(\frac{f}{2B}\right) * 4B\left[\frac{\delta(f-f_0)+\delta(f+f_0)}{2}\right]$$

$$= 2B\left[\Lambda\left(\frac{f-f_0}{2B}\right) + \Lambda\left(\frac{f+f_0}{2B}\right)\right]$$

In conclusione

$$R_x(\tau) = 2B \operatorname{sinc} 2B\tau + 8B^2 \cos 2\pi f_0 \tau \operatorname{sinc}^2 2B\tau$$

$$P_x(f) = \Pi\left(\frac{f}{2B}\right) + 2B\left[\Lambda\left(\frac{f-f_0}{2B}\right) + \Lambda\left(\frac{f+f_0}{2B}\right)\right]$$