SECONDA UNIVERSITA' DEGLI STUDI DI NAPOLI
SCUOLA POLITECNICA E DELLE SCIENZE DI BASE
Dipartimento di Ingegneria Industriale e dell'Informazione

Teoria dei Segnali/Telecomunicazioni 2
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SOLUZIONI

1. Schizzare i seguenti segnali e valutarne l'energia, la potenza e la trasformata di Fourier: (a) $s(t) = \Lambda(10t - 5)$; (b) $s(t) = \sin^2(4\pi t) - \cos^2(4\pi t + \frac{\pi}{3})$;

2. Usando il metodo grafico valutare la risposta nel dominio del tempo di un sistema lineare avente risposta impulsiva $h(t) = e^{-5t}u(t+3)$ al cui ingresso é posto il segnale $s(t) = u(t-1) - u(t-5)$.

3. Si consideri il seguente processo aleatorio

$$Y(t) = A - \frac{1}{2}X(t)\sin^2(7\pi f_0 t + \frac{\pi}{3}), \qquad (1)$$

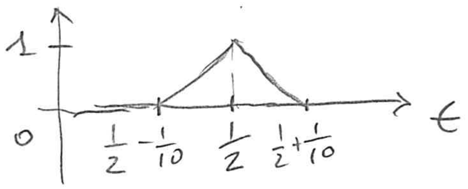
dove $X(t)$ é un processo aleatorio SSL e A una variablile Gaussiana con media $\mu_A = 1$ e varianza $\sigma_A^2 = 4$. Il processo $X(t)$ e la variabile A sono indipendenti. Il processo $X(t)$ ha spettro di potenza $P_{X_1}(f) = \Lambda\left(\frac{f-f_0}{B}\right) + \Lambda\left(\frac{f+f_0}{B}\right)$. Commentare sulla stazionarietá di $Y(t)$ e valutarne e schizzarne autocorrelazione e spettro di potenza.

4. Un segnale aleatorio avente spettro di potenza $P_s = \Pi\left(\frac{f}{2B}\right) - \Pi\left(\frac{f}{2b}\right)$ ($b << B$) é trasmesso su un canale distorcente con funzione di trasferimento dell'energia $|H_c(f)|^2 = \Lambda\left(\frac{f}{B}\right)$. Il canale aggiunge anche rumore avente spettro di potenza $P_n = \eta\left(\Pi\left(\frac{f}{2B}\right) - \frac{1}{2}\Pi\left(\frac{f}{4b}\right)\right)$. Calcolare il rapporto segnale-rumore in uscita e proporre filtri di enfasi e de-enfasi per il sistema.

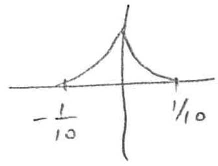
5. Progettare con il metodo del piazzamento poli-zeri un filtro a banda oscura per la banda $[3, 5]KHz$ a frequenza di campionamento di 20 KHz.

6. Un segnale $s(t)$ avente trasformata di Fourier $S(f) = \Pi\left(\frac{f}{2B}\right)$, con $B = 1000$ Hz, é campionato a 1500 campioni/sec. Fornire una espressione per il segnale ricostruito dalla formula di interpolazione cardinale.

1 (a) $s(t) = \Lambda(10t-5) = \Lambda\left(10\left(t-\frac{5}{10}\right)\right) = \Lambda\left(\frac{t-1/2}{1/10}\right)$



$\mathcal{E}_s = \frac{1}{10}\cdot 1\,\frac{1}{3}\cdot 2 = \frac{1}{15}$



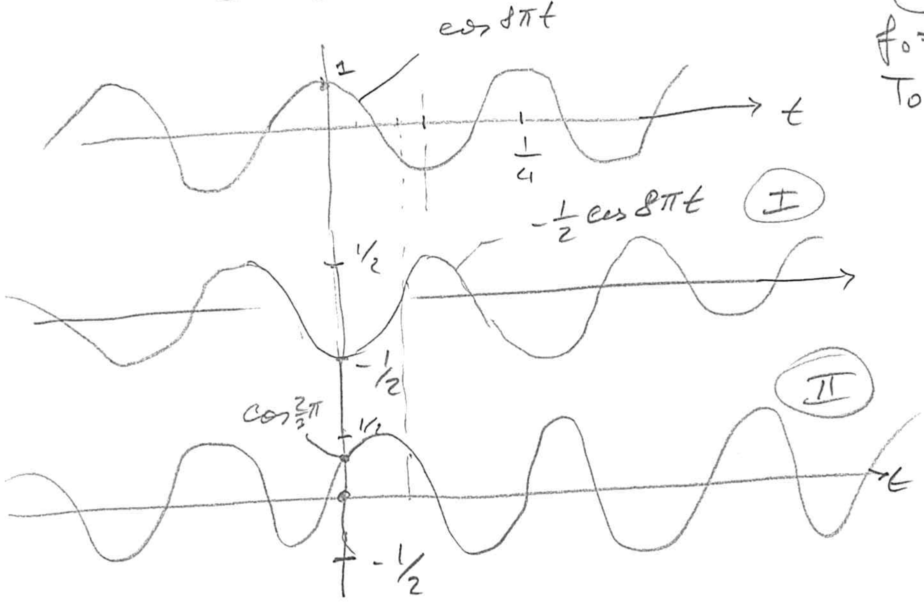
$S(f) = e^{-j2\pi f\frac{1}{2}}\,\frac{1}{10}\,\mathrm{sinc}^2\frac{1}{10}f$

(b) $s(t) = \sin^2 4\pi t - \cos^2\left(4\pi t + \frac{\pi}{3}\right)$

$= 1 - \cos^2 4\pi t - \cos^2\left(4\pi t + \frac{\pi}{3}\right) = 1 - \left(\frac{1}{2} + \frac{1}{2}\cos 8\pi t\right)$

$- \left(\frac{1}{2} + \frac{1}{2}\cos\left(8\pi t + \frac{2}{3}\pi\right)\right) = \underbrace{-\frac{1}{2}\cos 8\pi t}_{\mathrm{I}} \underbrace{-\frac{1}{2}\cos\left(8\pi t + \frac{2}{3}\pi\right)}_{\mathrm{II}}$

$f_0 = 4$
$T_0 = \frac{1}{4}$



$S(f) = -\frac{1}{4}\left(\delta(f-4) + \delta(f+4)\right) - \frac{1}{4}\left(\delta(f-4)\,e^{j\frac{2}{3}\pi} + \delta(f+4)\,e^{-j\frac{2}{3}\pi}\right)$

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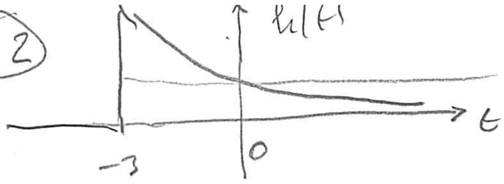
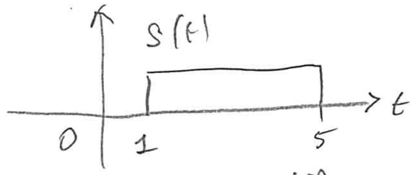
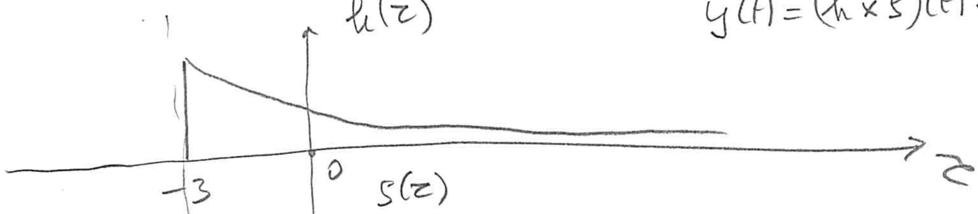


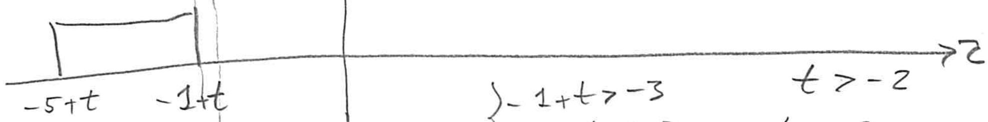


$$y(t) = (h * s)(t) = \int_{-\infty}^{+\infty} h(\tau)\, s(t-\tau)\, d\tau$$



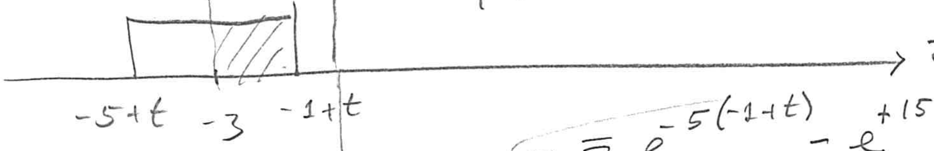
s(−τ) $t=0$

$-1+t<-3 \qquad t<-2 \qquad y(t)=0$



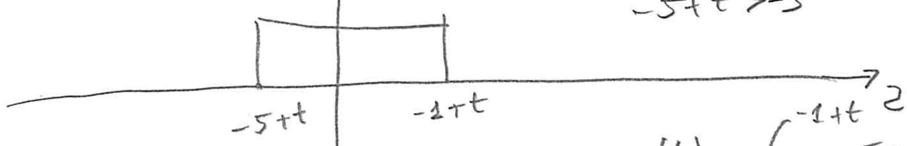
$$\begin{cases} -1+t>-3 \\ -5+t<-3 \end{cases} \qquad t>-2 \qquad t<2$$

$$y(t) = \int_{-3}^{-1+t} e^{-5\tau}\, d\tau = \left. \frac{e^{-5\tau}}{-5} \right]_{-3}^{-1+t} =$$



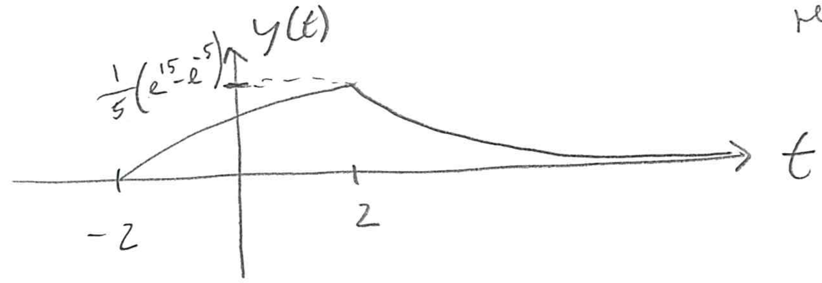
$$= \frac{e^{-5(-1+t)} - e^{+15}}{-5} = \frac{e^{5} e^{-5t} - e^{15}}{-5} = \frac{e^{5}}{5}\left(e^{10} - e^{-5t}\right)$$

$-5+t>-3 \qquad t>2$



$$y(t) = \int_{-5+t}^{-1+t} e^{-5\tau}\, d\tau = \left. \frac{e^{-5\tau}}{-5} \right]_{-5+t}^{-1+t} = \frac{e^{-5(-1+t)} - e^{-5(-5+t)}}{-5}$$

$$= \frac{e^{+5} e^{-5t} - e^{+25} e^{-5t}}{-5} = \frac{e^{-5t}}{5}\left(e^{25} - e^{5}\right)$$



per $t=2$ $\quad \frac{e^{-10}}{5}\left(e^{25} - e^{5}\right)$

$\frac{1}{5}\left(e^{15} - e^{-5}\right)$

(3) $$Y(t) = A - \frac{1}{2}X(t)\sin^2\left(7\pi f_0 t + \frac{\pi}{3}\right)$$

$\sin x^2 = 1-\cos^2 x$
$= 1-\left(\frac12+\frac12\cos 2x\right)$
$= \frac12-\frac12\cos 2x$

$$= A - \frac12 X(t)\left(\frac12 - \frac12\cos\left(14\pi f_0 t + \frac23\pi\right)\right)$$

$$= A - \frac14 X(t) + \frac14 X(t)\cos\left(14\pi f_0 t + \frac23\pi\right)$$

$$R_y(t,\tau) = E[Y(t)Y(t-\tau)] = E\left[\left(A - \frac14 X(t) + \frac14 X(t)\cos\left(14\pi f_0 t + \frac23\pi\right)\right)\left(A - \frac14 X(t-\tau) + \frac14 X(t-\tau)\cos\left(14\pi f_0 (t-\tau) + \frac23\pi\right)\right)\right]$$

$$= E[A^2] - \frac14 E[A]E[\cancel{X(t-\tau)}]^{0} + \frac14 E[A]E[\cancel{X(t-\tau)}]^{0}\cos\left(14\pi f_0 (t-\tau) + \frac23\pi\right)$$

$$-\frac14 E[A] + \frac1{16}E[X(t)X(t-\tau)] - \frac1{16}E[X(t)X(t-\tau)]\cos\left(14\pi f_0 (t-\tau) + \frac23\pi\right)$$

$$+\frac14 E[A]E[\cancel{X(t)}]^{0}\cos\left(14\pi f_0 t + \frac23\pi\right) - \frac1{16}E[X(t)X(t-\tau)]\cos\left(14\pi f_0 t + \frac23\pi\right)$$

$$+\frac1{16}E[X(t)X(t-\tau)]\cos\left(14\pi f_0 t + \frac23\pi\right)\cos\left(14\pi f_0 (t-\tau) + \frac23\pi\right)$$

$E[A] = 1 \qquad \sigma_A^2 = E[A^2] - \mu_A^2 = 4 \qquad E[A^2] = \sigma_A^2 + \mu_A^2 = 4 + 1 = 5$

$$= 5 - \frac14 + \frac1{16}R_x(\tau) - \frac1{16}R_x(\tau)\cos\left(14\pi f_0 (t-\tau) + \frac23\pi\right)$$

$$-\frac1{16}R_x(\tau)\left(\cos 14\pi f_0 t + \frac23\pi\right) + \frac1{16}R_x(\tau)\cos\left(14\pi f_0 t + \frac23\pi\right)\cos\left(14\pi f_0 (t-\tau) + \frac23\pi\right)$$

Processo ciclostazionario con $R_x(t,\tau)$ periodica in t con periodo $\frac{1}{2f_0}$. Consideriamo pertanto la autocorrelazione media

$$\overline{R}_y(\tau) = \frac{1}{1/2f_0}\int_{-\frac{1}{4f_0}}^{\frac{1}{4f_0}} R_y(t,\tau)\,dt \overset{def.}{=} \langle R_y(t,\tau)\rangle$$

$$\left\langle \cos\left(14\pi f_0 (t-\tau) + \frac23\pi\right)\right\rangle_{\frac{1}{2f_0}} = 0$$

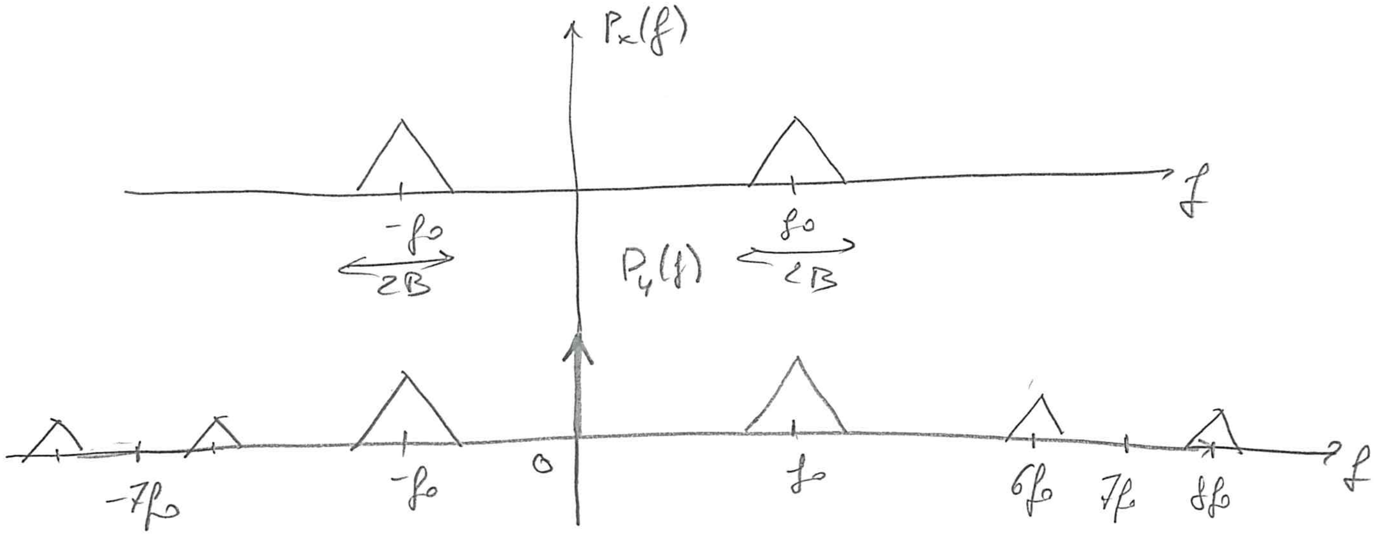
$$\left\langle \cos\left(14\pi f_0 t + \frac23\pi\right)\right\rangle_{\frac{1}{2f_0}} = 0$$

$$\left\langle \cos\left(14\pi f_0 t + \frac23\pi\right)\cos\left(14\pi f_0 (t-\tau) + \frac23\pi\right)\right\rangle_{\frac{1}{2f_0}} = \frac12\left\langle \cancel{\cos\left(14\pi f_0 (2t-\tau) + \frac43\pi\right)}\right\rangle^{0}_{\frac{1}{2f_0}}$$

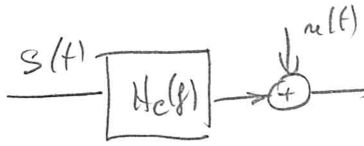
$$+\frac12\cos 14\pi f_0 \tau$$

$$\overline{R_y}(\tau) = \frac{19}{4} + \frac{1}{16} R_x(\tau) + \frac{1}{32} R_x(\tau) \cos 14\pi f_0 \tau$$

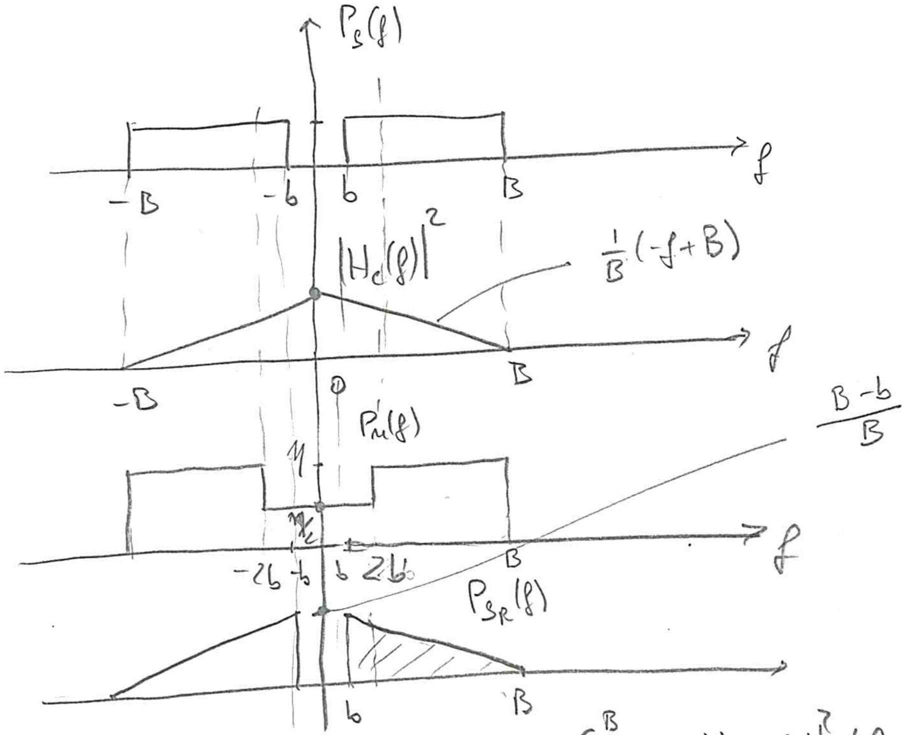
$$\overline{P_y}(f) = \frac{19}{4}\delta(f) + \frac{1}{16} P_x(f) + \frac{1}{64}\left(P_x(f-7f_0) + P_x(f+7f_0)\right)$$



(4)

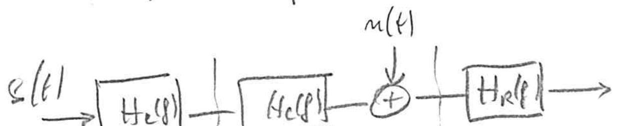


$$y(t) = \underbrace{(h_e * s)(t)}_{s_R(t)} + \underbrace{n(t)}_{n_R(t)}$$



$$\left(\frac{S}{N}\right)_{out} = \frac{P_{S_R}}{P_{N_R}} = \frac{\int_{-B}^{B} P_s(f)|H_e(f)|^2 df}{\int_{-B}^{B} P_n(f)\, df} = \frac{\frac{(B-b)^2}{B}}{4B\frac{\eta}{2} + 2(B-2b)\eta}$$

$$= \frac{\frac{(B-b)^2}{B}}{2B\eta + 2B\eta - 4b\eta} = \frac{(B-b)^2}{B(4B\eta - 4b\eta)} = \frac{(B-b)^2}{B^2\eta 4(1-\frac{b}{B})}$$



$$\begin{cases} |H_e(f)|^2 = \dfrac{\alpha K}{|H_c(f)|} \dfrac{P_n^{1/2}(f)}{P_s^{1/2}(f)} \\ |H_R(f)|^2 = \dfrac{K}{\alpha |H_c(f)|} \dfrac{P_s^{1/2}(f)}{P_s^{1/2}(f)} \end{cases} \quad |f| \in [-b, B]$$

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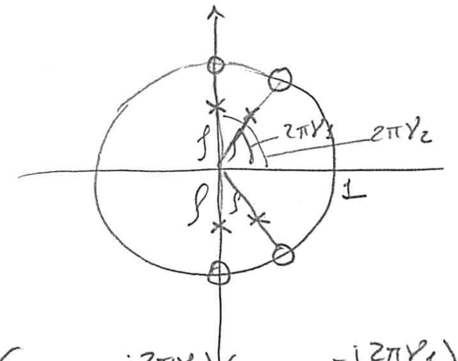
$$[f_1 \;\; f_2] = [3,5] \text{ KHz} \qquad f_e = 20 \text{ KHz}$$

$$\nu_1 = \frac{f_1}{f_e} = \frac{3}{20} \qquad \nu_2 = \frac{f_2}{f_e} = \frac{5}{20} = \frac{1}{4}$$



$$H(z) = \frac{(z - e^{j2\pi\nu_1})(z - e^{-j2\pi\nu_1})(z - e^{+j2\pi\nu_2})(z - e^{-j2\pi\nu_2})}{(z - \rho e^{j2\pi\nu_1})(z - \rho e^{-j2\pi\nu_1})(z - \rho e^{j2\pi\nu_2})(z - \rho e^{-j2\pi\nu_2})}$$

$$= \frac{(z^2 - 2\cos 2\pi\nu_1 \, z + 1)(z^2 - 2\cos 2\pi\nu_2 \, z + 1)}{(z^2 - 2\rho\cos 2\pi\nu_1 \, z + \rho^2)(z^2 - 2\rho\cos 2\pi\nu_2 \, z + \rho^2)}$$

$$= \frac{z^4 + b_1 z^3 + b_2 z^2 + b_3 z^1 + b_4}{z^4 + a_1 z^3 + a_2 z^2 + a_3 z + a_4}$$

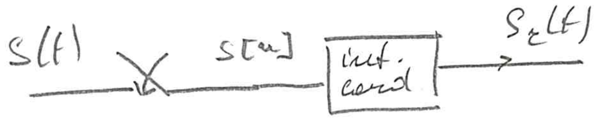
$$= \frac{1 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4}$$

$$y[n] = -a_1 y[n-1] - a_2 y[n-2] - a_3 y[n-3] - a_4 y[n-4]$$
$$+ x[n] + b_1 x[n-1] + b_2 x[n-2] + b_3 x[n-3] + b_4 x[n-4]$$

⑥ $s(t) \longleftrightarrow S(f) = \Pi\left(\frac{f}{2B}\right)$

$B = 1000$ Hz.

$f_c = 1500$ camp./sec.

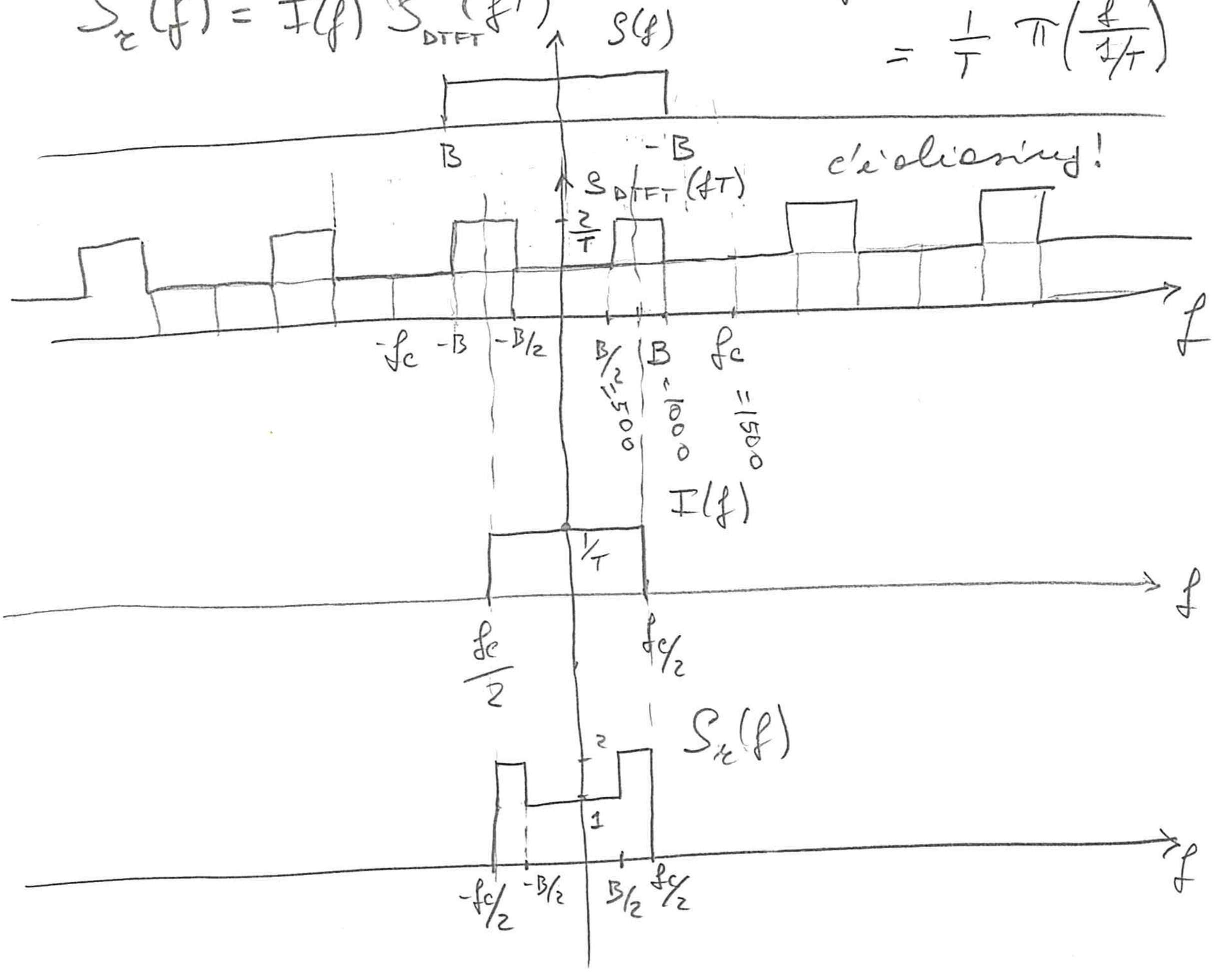


Dalla teoria del campionamento

$$S_r(f) = I(f)\, S_{DTFT}(fT)$$

$$I(f) = \mathcal{F}\left[\text{sinc}\,\frac{t}{T}\right] = \frac{1}{T}\,\Pi\left(\frac{f}{1/T}\right)$$

c'è aliasing!



$$S_r(f) = 2\Pi\left(\frac{f}{f_c}\right) - \Pi\left(\frac{f}{B}\right)$$

$$s_r(t) = 2f_c\,\text{sinc}\,f_c t - B\,\text{sinc}\,Bt$$