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SOLUZIONI

Prova scritta per il corso
Teoria dei Segnali
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1. Schizzare i seguenti segnali e valutarne l'energia, la potenza e la trasformata di Fourier: $s(t) = \Lambda\left(\frac{t}{2}-1\right)+\Lambda\left(\frac{t}{2}-2\right)$; $s(t) = -1+\cos^2(5\pi t)$; $s(t) = e^t\Pi\left(\frac{t-1}{8}\right)$.

2. Usando il metodo grafico valutare la risposta nel dominio del tempo di un sistema lineare avente risposta impulsiva $h(t) = e^{-t}u(t-3)$ al cui ingresso é posto il segnale $s(t) = \Pi\left(\frac{t-5}{2}\right)$.

3. Si consideri il seguente processo aleatorio

$$Y(t) = [X(t) - X(t-\Delta)]\cos 2\pi f_0 t, \qquad (1)$$

dove Δ é un ritardo deterministico e $X(t)$ é un processo aleatorio SSL avente spettro di potenza $P_X(f) = \Pi\left(\frac{f}{2B}\right)$. Commentare sulla stazionarietá di $Y(t)$ e valutarne e schizzarne autocorrelazione e spettro di potenza.

4. Un segnale aleatorio avente spettro di potenza $P_s(f) = \Lambda\left(\frac{f-B/2}{B/2}\right) + \Lambda\left(\frac{f+B/2}{B/2}\right)$, é trasmesso su un canale avente risposta armonica di energia $|H_c(f)|^2 = \Pi\left(\frac{f}{4B}\right)$ e che introduce rumore additivo avente spettro $P_n(f) = \alpha\left(\Pi\left(\frac{f}{B}\right) + \Pi\left(\frac{f}{4B}\right)\right)$. Derivare e schizzare filtri di enfasi e de-enfasi per il sistema.

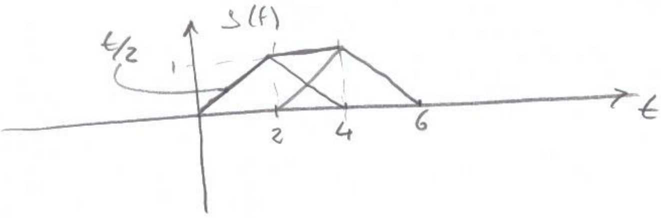
5. Usando la tecnica della serie di Fourier, si progetti un filtro FIR con una caratteristica passa-alto con frequenza di taglio di 2000 Hz. La frequenza di campionamento sia $f_c = 15$ KHz.

6. Si studi la ricostruzione cardinale per il segnale

$$x(t) = \sin^2 5t \qquad (2)$$

campionato alla frequenza $f_c = 2.5$ Hz. (Sugg.: Usare una tabellina per le frequenze)

1. (a) $s(t) = \Lambda\left(\frac{t}{2}-1\right) + \Lambda\left(\frac{t}{2}-2\right) = \Lambda\left(\frac{t-2}{2}\right) + \Lambda\left(\frac{t-4}{2}\right)$



Segnale di energia

$$\mathcal{E}_s = 2\int_0^2 \left(\frac{t}{2}\right)^2 dt + \int_2^4 1\, dt = 2\, \frac{t^3}{4\cdot 3}\Big]_0^2 + 2 = \frac{8}{6} + 2 = \frac{8+12}{6} = \frac{20}{6}$$

$$\mathcal{F}[s(t)] = 2\,\mathrm{sinc}^2 2f\, e^{-j2\pi f 2} + 2\,\mathrm{sinc}^2 2f\, e^{-j2\pi f 4}$$

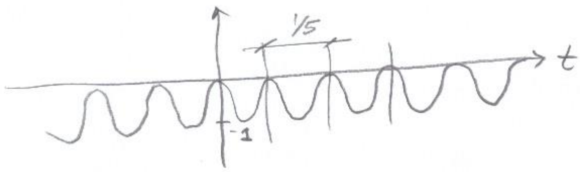
$$= 2\,\mathrm{sinc}^2 2f \left(e^{-j4\pi f} + e^{-j8\pi f}\right)$$

$$= 2\,\mathrm{sinc}^2 2f\, e^{-j6\pi f}\left(\frac{e^{j2\pi f} + e^{-j2\pi f}}{2}\right)\cdot 2$$

$$= 4 e^{-j6\pi f} \cos 2\pi f\, \mathrm{sinc}^2 2f$$

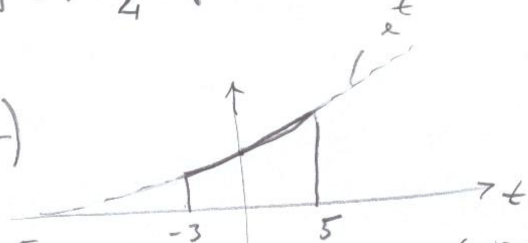
(b) $s(t) = -1 + \cos^2 5\pi t = -1 + \frac{1}{2} + \frac{1}{2}\cos 10\pi t = -\frac{1}{2} + \frac{1}{2}\cos 10\pi t$

$f_0 = 5$



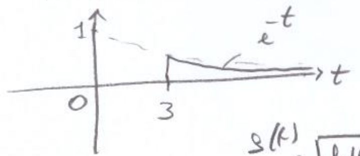
$$S(f) = -\frac{1}{2}\delta(f) + \frac{1}{4}\delta(f-5) + \frac{1}{4}\delta(f+5)$$

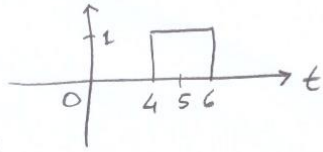
(c) $s(t) = e^t\, \Pi\left(\frac{t-1}{8}\right)$



$$S(f) = \int_{-3}^{5} e^t e^{-j2\pi f t}\, dt = \int_{-3}^{5} e^{(1-j2\pi f)t}\, dt = \frac{e^{(1-j2\pi f)5} - e^{-(1-j2\pi f)3}}{1-j2\pi f}$$

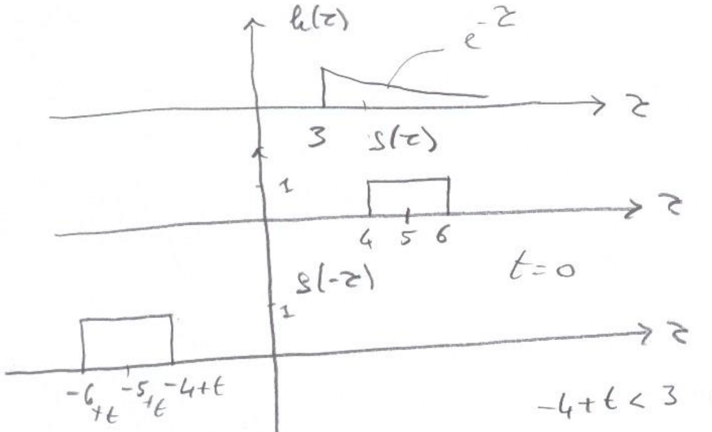
2. $h(t) = e^{-t}u(t-3)$ $\qquad$ $s(t) = \Pi\left(\frac{t-5}{2}\right)$ $\qquad$ 5





$s(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$

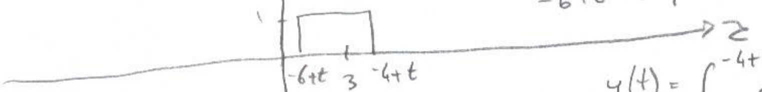
$$y(t) = (h*s)(t) = \int_{-\infty}^{+\infty} h(\tau)\, s(t-\tau)\, d\tau = \int_{-\infty}^{+\infty} h(\tau)\, s(-(\tau-t))\, d\tau$$



$t = 0$

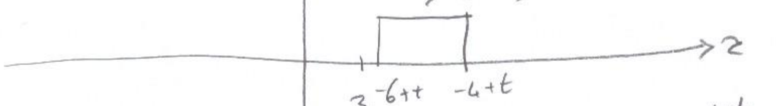
$-4+t < 3 \Rightarrow t < 7 \quad y(t) = 0$

$\left.\begin{matrix}-4+t > 3\\ -6+t < 3\end{matrix}\right\}$ $\begin{matrix}t > 7\\ t < 9\end{matrix}$ $\qquad 7 < t < 9$

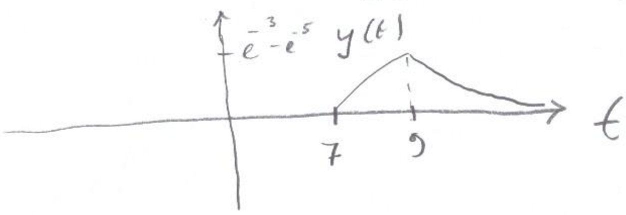


$$y(t) = \int_{3}^{-4+t} e^{-\tau}\, d\tau = \left.\frac{e^{-\tau}}{-1}\right]_{3}^{-4+t} = \frac{e^{4-t} - e^{-3}}{-1} = e^{-3} - e^{4-t}$$

$-6+t > 3$
$\Rightarrow t > 9$



$$y(t) = \int_{-6+t}^{-4+t} e^{-\tau}\, d\tau = \left.\frac{e^{-\tau}}{-1}\right]_{-6+t}^{-4+t} = \frac{e^{4-t} - e^{6-t}}{-1} = e^{6-t} - e^{4-t} = e^{-t}(e^{6} - e^{4})$$



3. $Y(t) = [X(t) - X(t-\Delta)] \cos 2\pi f_0 t$ (6)

$R_Y(t,\tau) = E[Y(t)Y(t-\tau)] = E\big[(X(t) - X(t-\Delta)) \cos 2\pi f_0 t \cdot$
$(X(t-\tau) - X(t-\tau-\Delta)) \cos 2\pi f_0 (t-\tau)\big]$

$= E\big[(X(t)-X(t-\Delta))(X(t-\tau) - X(t-\tau-\Delta))\big] \cos 2\pi f_0 t \cos 2\pi f_0 (t-\tau)$

$= \big[R_X(\tau) - R_X(\tau-\Delta) - R_X(\tau+\Delta) + R_X(\tau)\big]\left(\frac{1}{2}\cos 2\pi f_0(2t-\tau) + \frac{1}{2}\cos 2\pi f_0 \tau\right)$

Proceso ciclostacionario. Mediando en el periodo $\frac{1}{2f_0}$

$\overline{R_Y(\tau)} = \big[2R_X(\tau) - R_X(\tau-\Delta) - R_X(\tau+\Delta)\big] \frac{1}{2} \cos 2\pi f_0 \tau$

$\overline{P_Y(f)} = \big[2P_X(f) - e^{-j2\pi f\Delta} P_X(f) - e^{j2\pi f\Delta} P_X(f)\big] * \left(\frac{1}{4}\delta(f-f_0) + \frac{1}{4}\delta(f+f_0)\right)$

$= \frac{1}{2} P_X(f-f_0) + \frac{1}{2} P_X(f+f_0) - \frac{1}{4} e^{-j2\pi(f-f_0)\Delta} P_X(f-f_0) - \frac{1}{4} e^{-j2\pi(f+f_0)\Delta} P_X(f+f_0)$

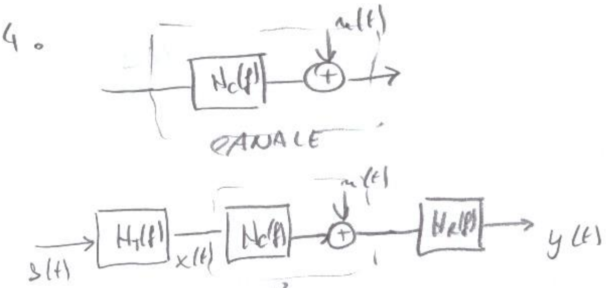
$- \frac{1}{4} e^{j2\pi(f-f_0)\Delta} P_X(f-f_0) - \frac{1}{4} e^{j2\pi(f+f_0)\Delta} P_X(f+f_0)$

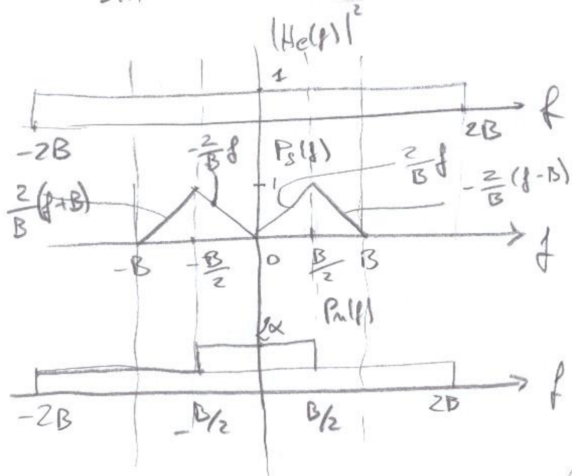
$= P_X(f-f_0)\left[\frac{1}{2} - \frac{1}{4}e^{-j2\pi(f-f_0)\Delta} - \frac{1}{4}e^{j2\pi(f-f_0)\Delta}\right]$
$+ P_X(f+f_0)\left[\frac{1}{2} - \frac{1}{4}e^{-j2\pi(f+f_0)\Delta} - \frac{1}{4}e^{j2\pi(f+f_0)\Delta}\right]$

$= P_X(f-f_0)\left[\frac{1}{2} - \frac{1}{2}\cos 2\pi(f-f_0)\Delta\right]$
$+ P_X(f+f_0)\left[\frac{1}{2} - \frac{1}{2}\cos 2\pi(f+f_0)\Delta\right]$

$= \Pi\left(\frac{f-f_0}{2B}\right)\left[\frac{1}{2} - \frac{1}{2}\cos 2\pi(f-f_0)\Delta\right]$
$+ \Pi\left(\frac{f+f_0}{2B}\right)\left[\frac{1}{2} - \frac{1}{2}\cos 2\pi(f+f_0)\Delta\right]$

4.





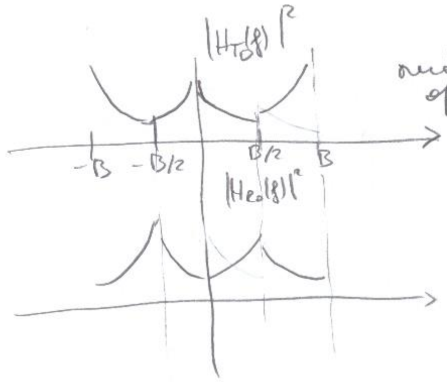
$$
\begin{cases}
|H_{T_0}(f)|^2 = \dfrac{\alpha K}{|H_C(f)|}\,\dfrac{P_n^{1/2}(f)}{P_s^{1/2}(f)} \\[2ex]
|H_{R_0}(f)|^2 = \dfrac{K}{\alpha |H_C(f)|}\,\dfrac{P_s^{1/2}(f)}{P_n^{1/2}(f)}
\end{cases}
\qquad f \in [-B, B]
$$

$$
\begin{cases}
|H_{T_0}(f)|^2 = \alpha K\sqrt{\dfrac{2\alpha}{\frac{2}{B}f}} = \alpha K\sqrt{\dfrac{B\alpha}{f}} \\[2ex]
|H_{R_0}(f)|^2 = \dfrac{K}{\alpha}\sqrt{\dfrac{f}{B\alpha}}
\end{cases}
\qquad f \in [0, B/2]
$$

$$
\begin{cases}
|H_{T_0}(f)|^2 = \alpha K\sqrt{\dfrac{\alpha}{-\frac{2}{B}(f-B)}} = \alpha K\sqrt{\dfrac{B\alpha}{2(B-f)}} \\[2ex]
|H_{R_0}(f)|^2 = \dfrac{K}{\alpha}\sqrt{\dfrac{2(B-f)}{B\alpha}}
\end{cases}
\qquad f \in \left[\frac{B}{2}, B\right]
$$

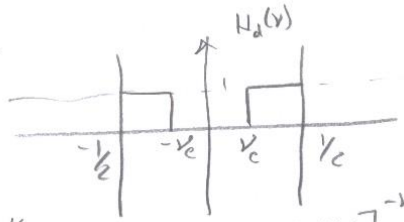
$$
\begin{cases}
|H_{T_0}(f)|^2 = \alpha K\sqrt{\dfrac{2\alpha}{-\frac{2}{B}f}} = \alpha K\sqrt{\dfrac{B\alpha}{-f}} \\[2ex]
|H_{R_0}(f)|^2 = \dfrac{K}{\alpha}\sqrt{\dfrac{-f}{B\alpha}}
\end{cases}
\qquad f \in \left[-\frac{B}{2}, 0\right]
$$

$$
\begin{cases}
|H_{T_0}(f)|^2 = \alpha K\sqrt{\dfrac{\alpha}{\frac{2}{B}(f+B)}} \\[2ex]
|H_{R_0}(f)|^2 = \dfrac{K}{\alpha}\sqrt{\dfrac{\alpha B}{2(f+B)}}
\end{cases}
\qquad f \in \left[-B, -\frac{B}{2}\right]
$$



rendimento approssimativo

5. $\nu_c = \frac{2000}{15000} = \frac{2}{15}$ (6)



$$h_d[n] = \int_{-1/2}^{-\nu_c} e^{j2\pi\nu n}\, d\nu + \int_{\nu_c}^{1/2} e^{j2\pi\nu n}\, d\nu \underset{n \neq 0}{=} \left. \frac{e^{j2\pi\nu n}}{j2\pi n} \right]_{-1/2}^{-\nu_c} + \left. \frac{e^{j2\pi\nu n}}{j2\pi n} \right]_{\nu_c}^{1/2}$$

$$= \frac{e^{-j2\pi\nu_c n} - e^{-j2\pi\frac{1}{2}n}}{j2\pi n} + \frac{e^{j2\pi\frac{1}{2}n} - e^{j2\pi\nu_c n}}{j2\pi n}$$

$$= \frac{e^{-j2\pi\nu_c n} - e^{j2\pi\nu_c n} \cancel{- (-1)^n + (-1)^n}}{2j\pi n} = -\frac{\sin 2\pi\nu_c n}{\pi n}$$

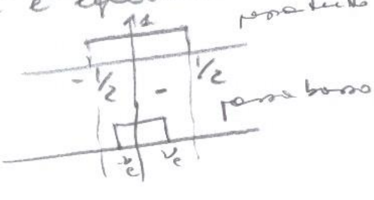
Per $n=0$ $h_d[0] = \int_{-1/2}^{-\nu_c} d\nu + \int_{\nu_c}^{1/2} = -\nu_c + \frac{1}{2} + \frac{1}{2} - \nu_c = 1 - 2\nu_c$

Ma $\left. -\frac{\sin 2\pi\nu_c n}{\pi n} \right|_{n=0} = - \left. \frac{\cos 2\pi\nu_c n \;\; 2\pi\nu_c}{\pi} \right]_{n=0} = -2\nu_c$

Quindi

$$h_d[n] = \delta[n] - \frac{\sin 2\pi\nu_c n}{\pi n}$$

(che è equivalente a



)

$$\boxed{h[n] = \delta[n-M] - \frac{\sin 2\pi\nu_c(n-M)}{\pi(n-M)} \qquad n = 0, \dots, 2M}$$

(Nota: Nell'integrazione con la funzione $e^{j2\pi\nu n}$ controllare il valore del risultato per $n=0$. Ovvero se eseguito con $\int 1\, d\nu$ o con $\int e^{j2\pi\nu n}\, d\nu$)

6. $x(t) = \sin^2 5t = 1 - \cos^2 5t = 1 - \frac{1}{2} - \frac{1}{2}\cos 10t$

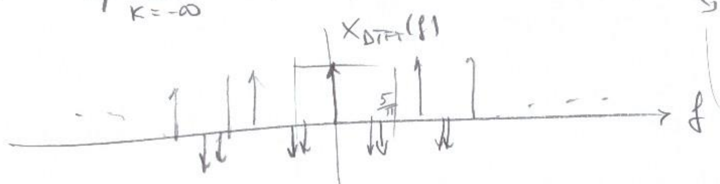
$= \frac{1}{2} - \frac{1}{2}\cos \underbrace{10}_{f_0 =} t \qquad 2\pi f_0 = 10 \Rightarrow f_0 = \frac{10}{2\pi} = \frac{5}{\pi} = 1.59$ Hz

Il campionamento viola la condizione di Nyquist, $f_c > 3.18$ Hz

quindi nella ricostruzione cardinale ci sarà aliasing

$$X(f) = \frac{1}{2}\delta(f) - \frac{1}{2}\delta\left(f - \frac{5}{\pi}\right) - \frac{1}{2}\delta\left(f + \frac{5}{\pi}\right)$$

$$X_{DTFT}(fT) = \frac{1}{T}\sum_{k=-\infty}^{+\infty} X(f - k f_c) \qquad X_r(f) = \underbrace{I(f)}_{\text{filtro di ricostruzione } T\,\Pi\left(\frac{f}{f_c}\right)} X_{DTFT}(fT)$$



Per vedere meglio le frequenze usiamo una tabellina

$K=0$	$-\frac{5}{\pi} = 1.59$	0	$\frac{5}{\pi} = 1.59$
$K=1$	$-\frac{5}{\pi} + 2.5 = 0.91$	$0 + 2.5 = 2.5$	$\frac{5}{\pi} + 2.5 = 4.09$
$K=-1$	$-\frac{5}{\pi} - 2.5 = -4.09$	$0 - 2.5 = -2.5$	$\frac{5}{\pi} - 2.5 = -0.91$
$K=2$	$-\frac{5}{\pi} + 5 = 3.41$	$0 + 5 = 5$	$\frac{5}{\pi} + 5 = 6.59$
$K=-2$	$-\frac{5}{\pi} - 5 = -6.59$	$0 - 5 = -5$	$\frac{5}{\pi} - 5 = -3.41$

Il filtro di ricostruzione taglia a $f_c/2 = \frac{2.5}{2} = 1.25$ Hz

Restano solo le frequenze $-0.91 \quad 0 \quad 0.91$

$$X_r(f) = \frac{1}{2}\delta(f) - \frac{1}{2}\delta(f - 0.91) - \frac{1}{2}\delta(f + 0.91)$$

$$\boxed{x_r(t) = \frac{1}{2} - \sin 2\pi(0.91)t}$$